AUTOMATIC COMPILATION OF CONTOUR LINES BY LOCAL DTM METHOD

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Abstract

With the progress of computers applied to plotters, it has become available to obtain digital contour line data instead of conventional raw manuscripts. Therefore, it became possible to edit contour lines with the help of numerical calculation instead of manual redrawing.

Usually computer aided editing is realized by interactive method, however, it also takes long time as manual editing. In order to reduce editing time, an automatic method to compile digital contour lines has been studied.

Compilation of contour lines is a kind of smoothing of the surface plane expressed by contour lines. Therefore, simple curved surface (named local DTM, DTM=digital terrain model) is calculated to represent the local terrain within the neighborhood of the concerned point on a contour line. Then the point is to be moved horizontally so that the point is dropped on the computed surface. Local DTM of the neighboring points are so strongly correlated to each other that we can obtain smooth contour lines consist of the moved points.

As an application of this method, generation of DEM(=digital elevation model) raster data is mentioned in this paper. The elevation on each intersection of the concerned grid is calculated out by generating its local DTM using neighbouring points.

KEY WORDS: Local DTM, Editing, Contour lines, DEM

1. THEORY OF EDITING

Plotting work can be considered as observation on terrain surface by aerial photographs and a plotter. The observed data are presented on a mapping sheet by an analogue plotter, while they are recorded on a digital media as a list of coordinates by an analytical plotter. Observations always have errors. In the case of plotting contour lines, both of altimetric and planimetric values are observed at the same time, and both observations have errors. These observing errors cause undesirable flutter or illegal crossings of contour lines.

The editing process can be considered as correcting these observation errors for the real terrain. When editing work is done by hand, firstly the editor chooses a small area and he/she "guesses" the real terrain of the area from the drawn contour lines. After that he/she redraws contour lines and corrects them against the terrain image generated in his/her mind.

The Local DTM editing method is developed to replace this job by a computer. Instead of guessing the real terrain, a local DTM is generated around every point which was recorded by encoders. Each point which has planimetric and altimetric errors are adjusted on the local DTM, and whole contour lines have been redrawn when all points are moved(Table 1).

2. THE BASIC THEORY OF THE LOCAL DTM METHOD

2.1 Concept of the Local DTM

The coordinate stream (x_i,y_i) of a contour line with the elevation z, obtained by a plotter, can be considered as a group of planimetric and altimetric observations at each point. The contour lines consist of numbers of observations with various elevations, all over the area. A local DTM (z=f(x,y)) is generated around each point collecting planimetric and elevation data

Table 1 Contrast between manual and Local DTM editing

	Manual	Local DTM
Observation data	Plotted manuscript	Obtained coordinates
Terrain model	An image guessed from the manuscript	Local DTM (simple curved surface)
Data correction	Redrawing lines by hand	Calculating corrections of observations

as observations within the neighborhood.

Since the local DTMs of the neighboring points are strongly correlated to each other, the whole terrain model consists of those local DTMs will form a smooth surface. Contour lines redrawn as dropped on this surface are expected to be fine and beautiful(Fig 1). This redrawing process is carried out moving only the planimetry of every point which forms contour lines. In a strict way, change of elevation should be concerned, however, the accuracy of elevation observation in normal situation is good enough to ignore the effect of its correction.

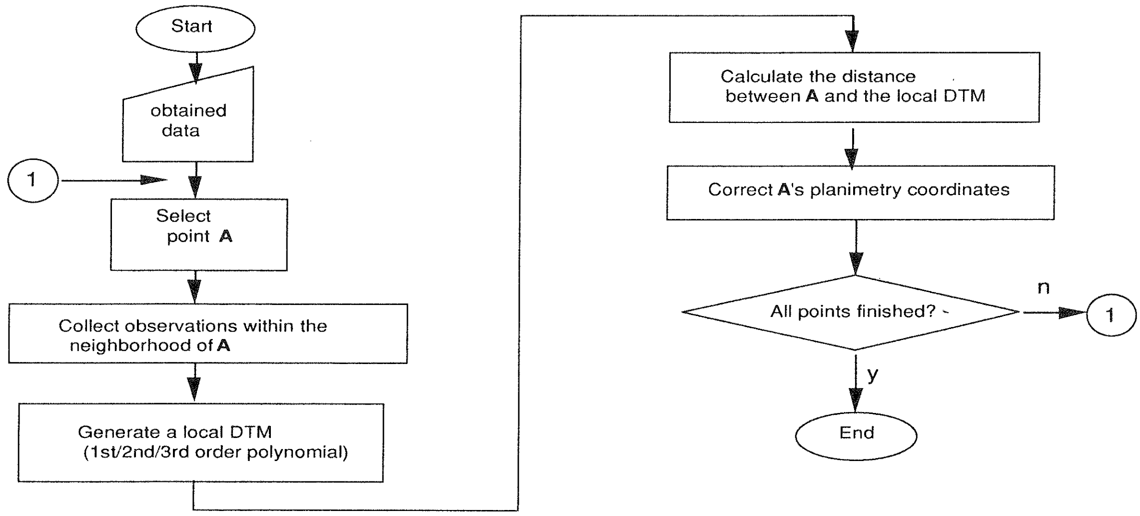

Fig 1 The flow chart of the Local DTM editing

2.2 Mathematical model of Local DTM

In the following, n is used as the number of observed points which are found within the area of radius r around a concerned point A(x_0,y_0,z_0). By using the least square method, the surface function of DTM is determined which approximates those observations best. Before calculation, all these observed data are transformed into local coordinates so as to let A be the origin(Fig 2).

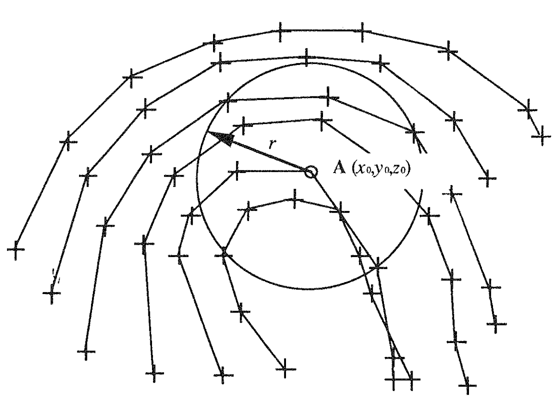

Fig 2 Local area around point A

2.2.1 Type of the Local DTM The local DTM of the point A is expressed as an equation of elevation z functioned by its planimetry as follows:

$$z = f(x,y) \qquad (2.1)$$

In each local DTM type of 1st, 2nd and 3rd order polynomial will be expressed as:

$$z = a+bx+cy \qquad (2.2)$$

$$z = a+bx+cy+dx^2+exy+fy^2 \qquad (2.3)$$

$$z = a+bx+cy+dx^2+exy+fy^2+gx^3+hx^2y+ixy^2+jy^3 \qquad (2.4)$$

In the following explanation, the type of 2nd order DTM(2.3) is used.

2.2.2 Observation equations In order to determine the best fit DTM polynomial, the least square method using the coordinates of points in the local area is adopted. Let $[x_i,y_i,z_i]_{i=1,n}$ be observations of elevation at each point, v_i be corrections for them, and function (2.3) be the type of local DTM. The observation equations are given as:

$$\mathbf{L} \cdot \mathbf{X} = \mathbf{C} + \mathbf{V} \qquad (2.5)$$

where:

$$\mathbf{L} = \begin{bmatrix} 1 & x_1 & y_1 & x_1^2 & x_1y_1 & y_1^2 \\ 1 & x_2 & y_2 & x_2^2 & x_2y_2 & y_2^2 \\ & & & \vdots & & \\ 1 & x_n & y_n & x_n^2 & x_ny_n & y_n^2 \end{bmatrix}$$

$$X = \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix}, \quad C = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix}, \quad V = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

2.2.3 Weighing factors of observation Contribution of each observation point around A should depend on the distance between the point and A. Weight of each observation must be larger when the point is closer to A. Thus, the following function (2.6) giving a proper weight value at every position of distance is introduced, as Fig 3.

$$p_i = \exp\{-(d_i/d_0)^2\} \qquad (2.6)$$

where d_0:standard distance

$$d_i = (x_i^2+y_i^2)^{1/2}$$

Fig 3 Weighing function for observation

2.2.4 Normal equations The normal equation of the least square method is set up from (2.5),(2.6) as:

$$\mathbf{SX} = \mathbf{K} \qquad (2.7)$$

where: $\mathbf{S} = \mathbf{L}^T\mathbf{PL}$

$\mathbf{K} = \mathbf{L}^T\mathbf{PC}$

$$\mathbf{P} = \begin{bmatrix} p_1 & & 0 \\ & p_2 & \\ & & \vdots \\ 0 & & p_n \end{bmatrix}$$

The coefficients of the local DTM are obtained as the solution of (2.7) as follow:

$$\mathbf{X} = \mathbf{S}^{-1}\mathbf{K} \qquad (2.8)$$

2.3 New coordinates of the point A

After the DTM is determined, the point A is moved horizontally, until it is dropped on the DTM surface, and accordingly, the contour line which A belongs to are renewed. To calculate correction for coordinates of A, a condition that "the new position of A should be on the DTM surface" is used. In this step, the coordinates of A are considered as observation data on the DTM surface. Each of x,y and z coordinates is an observation, and correction for each will be calculated setting approximate weight to each of three coordinates. Then the planimetry of point A will be changed by adding those calculated corrections to x and y.

The accuracy of calculated DTM is also important. In case the DTM is not accurate enough, it is not a proper way that to move the point A perfectly onto the DTM surface. Therefore the accuracy of local DTM generation should be concerned when the corrections are calculated(Fig 4).

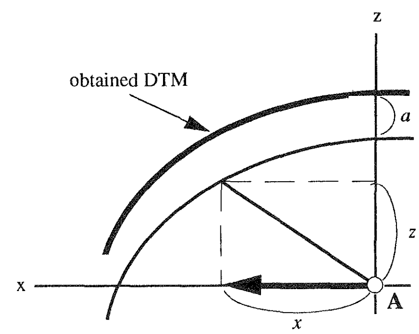

Fig 4 Movement of point A

2.3.1 Observing accuracy of point A and calculating accuracy of the local DTM Surmising from study, observation of elevation using 1/40,000 scale aerial photographs has about σ_h=2m (s.d.) accuracy, and that of planimetry has about σ_v=5m. However, they are likely influenced by the quality of the photographs and operator's skill. Also it is difficult to estimate reliable accuracy of generated DTM. In this study, the standard deviation of residuals among observed elevations and generated local DTM surface at all of the collected points are taken into account in determining a weighing factor.

2.3.2 Condition equations The local DTM generated for point A is described as:

$$z = f(x,y) = a+bx+cy+dx^2+exy+fy^2 \qquad (2.9)$$

If the point A locates right upon the DTM surface, z_0-f(x_0,y_0) = 0 should have been satisfied. It is not satisfied in general, therefore, coordinates of point A and the coefficients of the DTM are to be corrected by solving the condition equations. Actually, the coefficients except 'a' can be ignored because of their little effect to the solution. (In addition, since the local coordinates are used, so $[x_0, y_0, z_0] = [0,0,0]$).

In the following, Ma, Mx_0, My_0, Mz_0 are used as the real values of a, x_0, y_0, z_0 (unknowns), and $\Delta a, \Delta x_0, \Delta y_0, \Delta z_0$ are used for their correction. The condition equation is set up as:

$$Mz_0 - Ma - b \bullet Mx_0 - c \bullet My_0 - d \bullet Mx_0^2 - e \bullet Mx_0 y_0 - f \bullet My_0^2 = 0 \tag{2.10}$$

Using $Ma = a + \Delta a$, $Mx_0 = x_0 + \Delta x_0$, $My_0 = y_0 + \Delta y_0$, $Mz_0 = z_0 + \Delta z_0$,

$$z_0 + \Delta z_0 - (a + \Delta a) - b(x_0 + \Delta x_0) - c(y_0 + \Delta y_0) - d(x_0 + \Delta x_0)^2 - e(x_0 + \Delta x_0)(y_0 + \Delta y_0) - f(y_0 + \Delta y_0)^2 = 0 \tag{2.11}$$

$\Delta a, \Delta x_0, \Delta y_0, \Delta z_0$ should satisfy the following condition;

$$[p \Delta \Delta] = p_a(\Delta a)^2 + p_{x0}(\Delta x_0)^2 + p_{y0}(\Delta y_0)^2 + p_{z0}(\Delta z_0)^2 = \min. \tag{2.12}$$

where: $p_a = (1/\sigma_d)^2$, $p_{x0} = p_{y0} = (1/\sigma_v)^2$, $p_{z0} = (1/\sigma_h)^2$

$$\sigma_d = [\Sigma \{z_i - f(x_i, y_i)\}^2 / n]^{1/2}$$

By solving (2.12), we obtain:

$$\begin{aligned} \Delta x_0 &= -(a \bullet b \bullet \sigma_v^2) / \{(b^2+c^2) \bullet \sigma_v^2 + \sigma_h^2 + \sigma_d^2\} \\ \Delta y_0 &= -(a \bullet c \bullet \sigma_v^2) / \{(b^2+c^2) \bullet \sigma_v^2 + \sigma_h^2 + \sigma_d^2\} \\ \Delta z_0 &= (a \bullet \sigma_v^2) / \{(b^2+c^2) \bullet \sigma_v^2 + \sigma_h^2 + \sigma_d^2\} \end{aligned} \tag{2.13}$$

Then the new coordinates of A are given as:

$$[x,y,z]_{new} = [x,y,z]_{old} + [\Delta x_0, \Delta y_0, 0] \tag{2.14}$$

2.4 Collection of observations

Considering the contour line data as observations of terrain surface, they are not equally located in general. The density of observed points in an area depends on that of contour lines. At the process of local DTM calculation around point A, if observations are collected simply from the closer one to A until the number of observations becomes enough for calculation, the distribution of them may be biased. To prevent this, the searching area is divided into eight parts by direction from A, and observations are collected equally from every part. In this process, in case there is no observations found in more than three neighboring parts, the accuracy of DTM generation may not be enough because of inequality of observation distribution. Therefore, correction for this point will not be done. The accuracy of DTM will be also low in case the total of observations found in searching area is too small, therefore, the point is corrected only when found observations are more than as twice as the number of normal equations (it is 12 when 2nd order DTM is applied).

2.5 Iteration

One calculation cycle is finished when every point is corrected by its local DTM. Generally, generated DTMs at the first calculation are not very accurate. Besides, σ_d in (2.12) may be still large so correction of points may not be enough. Therefore, the whole step of DTM generation and correction at every point must be done several times. The new coordinates calculated by previous round are used as observation data at the next round.

3. EXAMINATION

The results are influenced by many kinds of case factors, and mostly it is difficult to estimate the best value for them by theoretical way. Therefore, examinations under variable conditions are carried out to find the best parameters to represent fine and proper expression of the terrain.

3.1 Sample data

Fig 5 is the sample data obtained from a contour lines sheet of 1:25,000 topographic map plotted by an analytical plotter using 1:40,000 scale aerial photographs. It is 2km by 2km wide, and the elevation interval of contour lines is 10m. Easy mistakes at plotting such as illegal breaks and dust data are already removed in advance. The contour lines consist of points that observed in 10m(0.4mm on map) pitch. Only in the case 9, one in 5m(0.2mm) is used. The sample data includes about 80 contour lines, which consist of about 20,500 points (case 9:about 38,000 points).

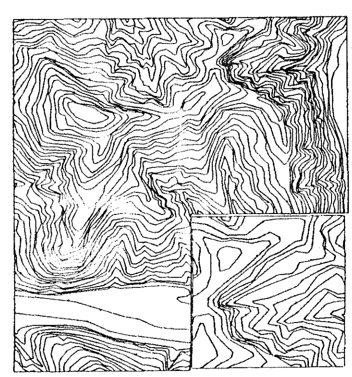

Fig 5 Sample data

3.2 Case study with various conditions

Major factors of Local DTM editing are listed in the following. There are numbers of combinations of them, and many of them are correlated to each other. In order to compare the effect of each factor, a standard combination of these factors is chosen at first, then change factors one by one (Table 2).

Table 2 Case factors

factors	standard value	case	changed value
(a)	2nd order	1	3rd order
		2	1st order
(b)	50m	3	25m
		4	100m
(c)	50m	5	100m
(d)	σ_v=5m, σ_h=2m	6	σ_v=1m, σ_h=1m
		7	σ_v=5m, σ_h=1m
(e)	concerned	8	not concerned
(f)	10m	9	5m
(g)	3	10	5
		11	1

(a) Observation equations in local DTM generation

The order of the polynomial used for observation equations is changed in the case 1 and 2. The higher order polynomial is used, the more flexible DTM is generated, however, the data are not corrected very much.

(b) Weights of observations in local DTM generation

Weights of observations are calculated by (2.6). The parameter d_0 is changed in the case 3 and 4. The larger standard distance is taken, the more observations affect the movement on each point, and the gentler drawing will be made as the result.

(c) Area for collection of observations

The radius of the area for correction of observations (r in Fig 2) in local DTM generation is changed in the case 5.

(d) Observation accuracy in planimetry and elevation

Observation accuracy is assumed from observing situation (σ_v, σ_h in (2.12)). The less accurate observation in planimetry in comparison with one in elevation is assumed, the larger movement will be given to each point.

(e) Concern about the accuracy of generated DTM

In the case 8, the accuracy of local DTM(σ_d) is not concerned in the step of correction (2.10).

(f) Point pitch of contour lines

The point pitch affects the number of observations in DTM calculation. Besides, the higher density data present the smoother drawing as the result, however, calculation will take longer.

(g) Iteration

Number of iteration is changed in the case 10 and 11.

3.3 Results

The edited drawing by each case is shown in Fig 6. Calculation time was about 15 minutes for case 9, and 8 minutes for each of other cases. [computer: Sun sparc station 2 (28 mips), CPU time]

4. CONCLUSION OF LOCAL DTM EDITING

Local DTM editing by the standard condition presents properly smoothed drawing as the result, which would be acceptable as the base map of 1:25,000 topographic map. The expression of the terrain by manual editing would not be far different from one of this method. Considering about its effect for reducing of editing work, this Local DTM method is valuable. Actually editing by this method has already put into practice at the Geographical Survey Institute of Japan.

In addition, following facts appeared over these examinations.

(a) 1st order DTM editing does not fit on some parts of terrain such as where a deep valley or a steep peak exists, on the other hand 3rd order DTM editing still have some unsmoothed parts left.

(b) Assumed accuracy of observations is especially affective for the results. In the case 6 in which standard deviations of both planimetry and elevation observations are assumed as very small(1m), edited drawing has only a little change, on the contrary in the case 7 in which the s.d. of elevation observation is assumed to be 5 times large as one of planimetry observation, edited drawing has greatly moved to be very much loose.

(c) Area size for collecting observations at local DTM generation does not cause much difference as far as it is between 25m and 100m in radius.

(d) The drawing becomes looser every after one cycle of iteration, however, there is little change over 3rd iteration.

The results mostly satisfy our demands, however, there are some tendencies and problems to be concerned by followers.

(a) By local DTM editing, ridges in the drawing have been a little widened and valleys have been narrowed.

(b) Position of mountain peaks is not fixed during calculation, therefore edited drawing may present those peaks shifted a bit.

(c) In observation, neighboring points on the same contour line are correlated to each other, however, each of them is concerned independently at the correction process. That may make

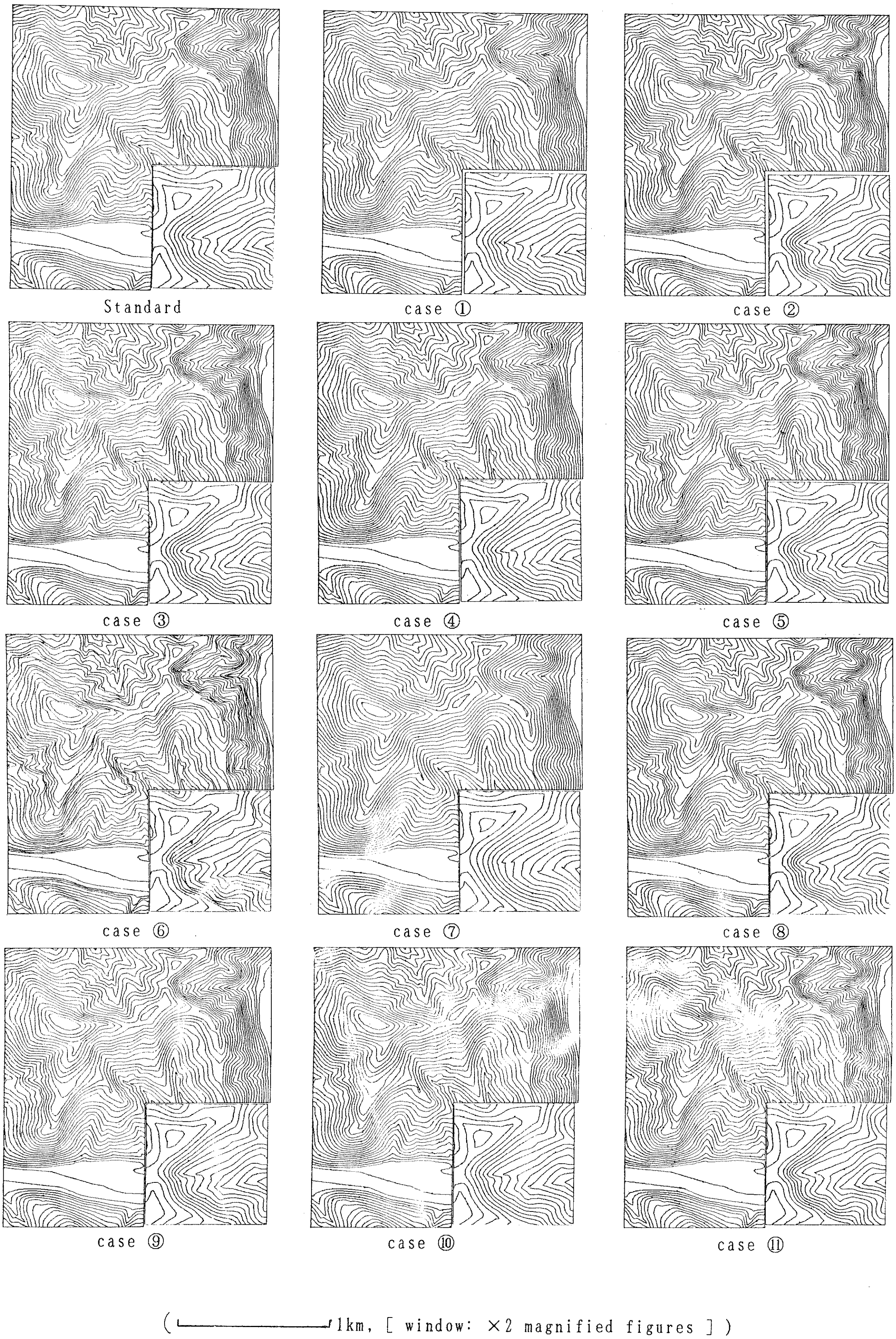

Fig 6 Results of Local DTM editing

rows of points to be disrupted.

The aim of this method is to get as good results as by manual editing easily with the help of a computer. Therefore the quality of the results is mainly concerned, on the other hand each process of calculation has not been explained in detail. It will become important to get theoretical bases about this method for solving problems in above.

4. APPLY TO DEM RASTER DATA GENERATION

A Digital Elevation Model is a series of elevation data usually measured on a grid system. In the process of Local DTM editing, the accurate elevation at the concerned point is calculated. Instead of selecting the point from the contour polyline nodes, local DTM calculation is applied at the each intersection of the aimed grid, then elevation raster data of the grid area will be created(Fig 7). Many methods have been studied to create DEM raster data from contour polyline data, however, generally those methods interpolate elevation value from only a few contour line nodes. As a merit of Local DTM method, the elevation value is calculated concerning general terrain shape around the point, so it's unlikely the whole data includes abnormal or irregular height points. Actually, examinations got pretty fine DEM data by this method.

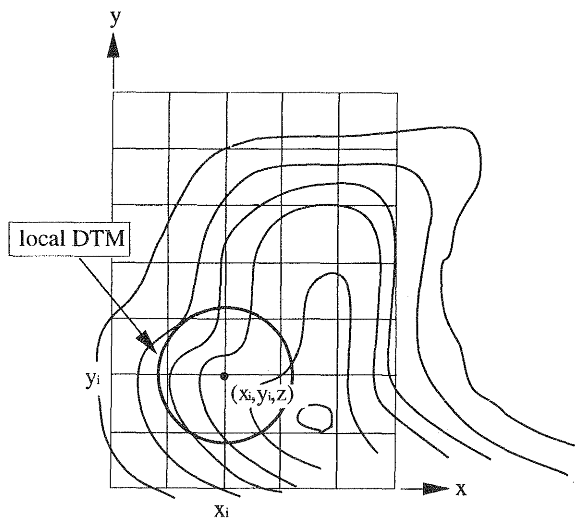

Fig 7 DEM raster data generation by Local DTM method

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