

EVALUATION OF THE BETA-SPLINE INTERPOLATOR FOR DIGITAL
ELEVATION MODELS WITH DIFFERENT BETA PARAMETERS ON EACH PATCH

by

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ABSTRACT

The use of the Beta-spline interpolator in Digital Elevation Models permits to shape the interpolated surface to approximate the terrain shape without changing its control points. This interpolator has a user defined set of parameters that enables one to control the bias and the tension of the interpolated surface. The results are shown in graphical form.

KEY WORDS: Digital Elevation Models, DEM, Numerical Interpolation, Beta-splines.

1. INTRODUCTION

Sometimes certain terrain features are known a priori, but when an interpolator is used, on a Digital Elevation Model, DEM, these features are masked by the interpolator properties, like the smoothing of break lines.

The solution would be to use an interpolator whose shape could be controlled by the user. The beta-spline interpolator has such capability. It is possible, by means of the two parameters β_1 and β_2 , to control, respectively, the bias (to "push" or to "pull" the interpolated surface toward or away to the vertices of the control polygon, as will be seen shortly).

This work presents a case study about a terrain in which some information about it is known. The original data is used for comparison (considered as the "real" terrain). The original data is sampled and then interpolated by Beta-splines. It is used for different patches, different sets of beta parameters, thus changing the shape according to the knowledge available.

Session 2 presents a brief description of the Beta-splines, the following session the results obtained, and finally a conclusion. The computer environment used consisted of Workstations for the generation of the plots, and IBM PC-like computers for the calculations. A simple program, written in turbo C provided the files to be visualized in the workstations and plotted in a laserjet printed. It was determined that the Beta-splines were

fast enough to run efficiently in this unsophisticated environment.

2. BETA-SPLINES

The Beta-spline interpolator (Barsky, 1987), is an extension of the Bézier family of interpolators (Foley and VanDam, 1984). The main property is that the interpolated surface lies inside the convex hull determined by the polygon control vertices, thus the interpolator does not pass by the control vertices, as do other interpolators.

The mathematical form is as a series, in which the control vertices are weighted by base functions. Parametrically it can be written as:

$$z(u,v) = \sum_{i=0}^{n-1} \sum_{j=0}^{m-1} B_{ij}(u,v,\beta_1,\beta_2) V_{ij} \quad (1)$$

where S stands for summation, u and v are the variable parameters, i, j , the summation indexes, m and n the number of lines and columns, V_{ij} the position of the polyhedron control vertices, $B_{ij}(\)$ the base functions. These base functions are dependent on i, j, u, v , and the user defined bias, β_1 and tension, β_2 , but are independent on the position of the control vertices.

The action of β_1 is the following: for β_1 equal to one the bias is neutral, for β_1 from 0 to plus infinity the bias is toward the initial values of v and u , from 0 to minus infinity toward the opposite side. The tension, as the

name implies, is the "force" that plus the surface toward the control vertices.

3. RESULTS

The interpolator was tested in two consecutive patches. The cases bellow were studied:

A) Linear case, with $\beta_1=1$ (no bias), and $\beta_2=90$ (strong tension), both patches.

B) Moderate bias, no tension. Patch 1 bias $1/8$, patch2 bias $+8$. This case models a river or valley.

C) Moderate bias, no tension. Patch 1 bias $+8$, patch2 bias $1/8$. This case models a ridge.

Figure 1 is the original data set, taken from aerial photography, as courtesy of AERODATA S/A. Figure 2 presents the original figure undersampled; it will be used as our control polyhedron.

Next set of figures are the terrain representations after the Beta-spline interpolator has been applied. Figure 3 for case A, Figure 4 for case B, and Figure 5 for case C.

4. CONCLUSIONS

As seen from the figures of the previous session, the use of the Beta-splines can improve the terrain representation, especially if some knowledge about its the shape is known previously. This is a preliminar work, so there is much more study to be done, but the indications are that this interpolator has its place as another tool for the goal of obtaining better Digital Elevation Models.

5. ACKNOWLEDGEMENTS

The authors would like to thank INPE, AERODATA S/A, and the Paraiba Valley University, UNIVAP, for use of their equipment. IBM Brasil deserves special thanks for helpfull discussions with the colleagues from IBM Rio Scientific Center.

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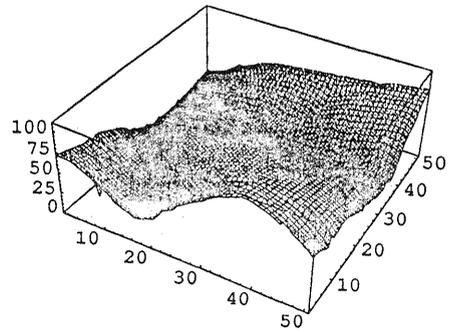


Fig. 1 - Original Data Set.

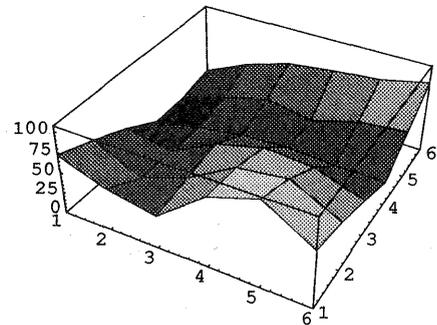


Fig. 2 - Original Undersampled.

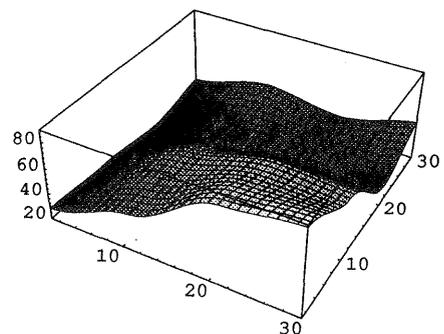


Fig. 3 - $\beta_1=1$, $\beta_2=90$.

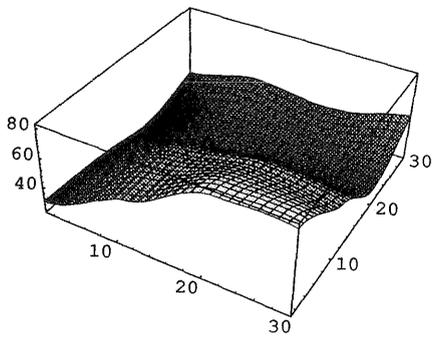


Fig. 4 - $\text{Beta}_1=1/8, \text{Beta}_2=8.$

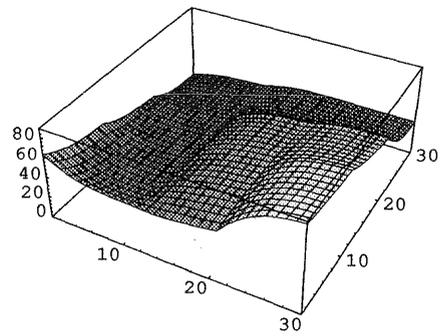


Fig. 5.- $\text{Beta}_1=8, \text{Beta}_2=1/8$