VARIANCE REDUCTION BY MULTI-CONTROL VARIATE METHOD

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ABSTRACT

Accuracy improvement is often a desired requirement in many applications of photogrammetry. An extension of this study into the topic of variance reduction in data analyses by use of multi-control variate method among other Monte-Carlo techniques is presented. Variance reduction and the consequent increase of accuracy of the estimate is the motivation for the study. Straight single control variate,regression-adjusted control variate, concomitant control variate are some other variations in common use.

The multiple-control variate method is particularly applicable in photogrammetry as it involves noncon- trol object space coordinate parameters estimation using simultaneously a set of control object space coordinates.

The 3-dimensional coordinates of a point can be considered as functions of photo-coordinates in at least two photographs, interior orientation parameters of the camera and exterior orientation parameters of photographs used.Because of this functional relationship,the coordinates of adjacent points are dependent. Using this information in a judicious way, it is possible to improve accuracy of computed 3-D coordinates of noncontrol object space points.

Control point coordinates could thefore be used in variance reduction by multicontrol variable method. Comparison of resulting accuracy estimates would reveal efficiency and suitability of the selected number and locations of points for use as multiple control variates. Simulation would be the basis of data formulation and analyses.

KEY WORDS: Variance reduction, Monte Carlo, Close range, Simulation, Accuracy improvement.

1. INTRODUCTION

Monte Carlo techniques are often used in many scien- tific disciplines and a considerable part of this effort is in the area of variance reduction in systems analyses. The applicability of some of these techniques to the normal case of terrestrial and close range photogrammetry has been demonstrated in (Nagaraja, 1990,91). Though at first sight it might seem that sampling procedures only apply in case of simulation studies,further reflection should indic- ate that conceptually and practically, it should be possible to incorporate these ideas in reduction of practical data. However, this paper deals with a case study in simulation.

The subject of variance reduction has received, outside photogrammetry,considerable attention and a number of methods have been developed. Therefore, there are a few techniques that help to increase accuracy and hence efficiency of simulations, sometimes substantially, by producing less variable observations. Accordingly, there is a need to study such a possibility. Hillier and Lieberman(1986) have given a long list of successful applications of simulation studies.

Applications of simulation are not new in photogrammetry but new applications are still possible.Variance reduction techniques seek either an increase in precision(decreased variance) for a fixed sample size or a decrease in the sample size required to obtain a given degree of precision. Several authors have cautioned us in using these techniques.If properly used, these techniques can provide tremendous increase in the efficiency of the model; however,if the intuition is faulty and analyst does not use a reasonable design, the technique can also be very unpredictable and actually increase variance for some techniques.Because of this reason,a systematic and thorough study of the selected method is essential in adapting it for any specific application.

The control variables technique applies very well when there is an equivalent to the process we are simulating that can be treated theoretically. For example,in the normal case of close range photogrammetry we have an equivalence between computation of coordinates of a noncontrol object space point and that of a control object space point. We can then simulate the Y-coordinates (say) of a selected object space point and that of the known control point simultaneously, using same random numbers in both computations. Then the difference in the known and computed coordinate of the control object space point is an estimate of the correction to be applied to the computed results of the selected object space point. This procedure,therefore cuts out the variance due to common parameters in the two processes leaving only the variance of the error of approximation. Obviously, this should be of a lower order of magnitude. In photogrammetry, as control point information is generally available, it can be used as a control variate. This same idea can be further extended to a multi-control variable(MCV) technique and an attempt is made to present this extended application study in this paper.

2. A SIMULATION EXPERIMENT BASED ON NORMAL CASE

In order to understand the capabilities of the MCV technique,it is necessary to set up a framework for the simulation study. This aspect is reported in this section. The assumed/true object space coordinates (to nearest mm), and the corresponding photo coordinates in the left and right photographs is listed in table 1. The calculated photo coordinates conform to the 'normal case' in terrestrial and close range photogrammetry.

Table 1:True object space and photo coords.(mm)

Pt. No.	Lt. photocoord. x	y	Rt. photocoord. x'	y'	Ob.spa.coord. X	Y	Z
11	10.000	10.000	-10.000	10.000	600	3840	600
12	20.000	10.000	0.000	10.000	1200	3840	600
13	20.000	0.000	0.000	0.000	1200	3840	000
14	20.000	-10.000	0.000	-10.000	1200	3840	-600
15	10.000	-10.000	-10.000	-10.000	600	3840	-600
16	0.000	-10.000	-20.000	-10.000	000	3840	-600
17	0.000	0.000	-20.000	0.000	000	3840	000
18	0.000	10.000	-20.000	10.000	000	3840	600
19	10.000	0.000	-10.000	0.000	600	3840	000
21	7.934	7.934	-07.934	7.934	600	4840	600
22	15.868	7.934	0.000	7.934	1200	4840	600
23	15.868	0.000	0.000	0.000	1200	4840	000
24	15.868	-7.934	0.000	-7.934	1200	4840	-600
25	7.934	-7.934	-7.934	-7.934	600	4840	-600
26	0.000	-7.934	-15.868	-7.934	000	4840	-600
27	0.000	0.000	-15.868	0.000	000	4840	000
28	0.000	7.934	-15.868	7.934	000	4840	600
29	7.934	0.000	-7.934	0.000	600	4840	000
31	6.575	6.575	-6.575	6.575	600	5840	600
32	13.151	6.575	0.000	6.575	1200	5840	600
33	13.151	0.000	0.000	0.000	1200	5840	000
34	13.151	-6.575	0.000	-6.575	1200	5840	-600
35	6.575	-6.575	-6.575	-6.575	600	5840	-600
36	0.000	-6.575	-13.151	-6.575	000	5840	-600
37	0.000	0.000	-13.151	0.000	000	5840	000
38	0.000	6.575	-13.151	6.575	000	5840	600
39	6.575	0.000	-06.575	0.000	600	5840	000
41	10.200	10.200	-10.280	10.200	598	3750	598
42	19.200	10.200	0.000	10.200	1200	4000	638
43	18.071	0.000	0.000	0.000	1200	4250	000
44	20.400	-10.200	3.330	-10.200	1434	4499	-717
45	10.200	-10.200	-6.000	-10.200	756	4741	-756
46	0.000	-10.200	-15.360	-10.200	000	5000	-797
47	0.000	0.000	-14.629	0.000	000	5250	-837
48	0.000	10.200	-13.984	10.200	000	5492	875
49	10.200	-1.000	-3.800	-1.000	874	5486	-086

B=1200 mm,f=64.0mm,Format=90x65 mm, Xo=Yo=Zo=0.0
S.E.:SB=0.1 mm,Sf=0.01 mm,Sx=Sy=Sx'=Sy'=0.005 mm
Points 41 to 48 are used as MCV variable points.
Pt.49 was used as MCV only in case L of table.2.

3. THE MULTI-CONTROL VARIATE (MCV) METHOD

The idea of a single-control variable Monte Carlo technique for reduction of variation of sample observations, which thereby increases the precision of the estimate, can be readily extended to a multiple control variates technique. The computational procedure to be followed in this multiple case is explained in (Kobayashi, 1981). Following this idea, we proceed to define a new random variable Z, by

$$Z(R)=Y(R)-\sum bi(Xi(R)-E[Xi]), \; i=1,2,..,k. \quad ...(3.1)$$

If Q denotes the covariance matrix of X = [x1,x2...Xk] and if C denotes the cross covariance vector between Y and X;

$$Qij=Cov[Xi,Xj], \; i,j = 1,2,...k. \quad ...(3.2)$$

$$\text{and } Ci= Cov[Y,Xi], \; i=1,2,...k. \quad ...(3.3)$$

then the optimal value for B=[b1,b2,...,bk] is

$$Bo = C\,Q^{-1} \quad ...(3.4)$$

which leads to

$$Var[Z]=Var[Y]-C\,Q^{-1}C^{T}=Var[Y](1-Ryx^{2}) \quad ...(3.5)$$

where Ryx is the multiple correlation coefficient between Y and X.The square of the correlation coefficient is often called the coefficient of determination as it represents the fraction of the total variation of Y explained by variation of X. Here,as E[Z]=E[Y],computed value of Z is used as Y. The idea behind the multiple-control variate variance reduction is similar to regression analysis(special case of analysis of covariance). However, in the regression analysis we usually wish to investigate the power of a set of predictive variables X in explaining the variation of a response variable Y, whereas in variance reduction by the multi-control variate method, we evaluate the additional reduction in the variance against the additional computation involved. We should bear in mind that it is possible to achieve any desired reduction of variance by using the mean of a sufficiently long simulation run, i.e., we could use the arithmetic mean in place of each observation. The MCV method has been successfully applied in studying the queueing system. Referring to (Graver, 1969),(Kobayashi, 1981) reports that multi-control variate method (three control variates) cuts the variance to about 8% (that is by a factor of 12.5) of the initial value. It is interesting to note that the mean value of Z and Y would still be the same when the negative value before the summation in eq.3.1 is changed to a positive sign. This fact has been used to modify eq.3.1 as follows.

$$Z(R)=Y(R)-\sum bi(Xi(R)-E[X]) \text{ for } bi<0 \quad ...(3.6a)$$

$$Z(R)=Y(R)+\sum bi(Xi(R)-E[X]) \text{ for } bi>0 \quad ...(3.6b)$$

SIGNIFICANT RESULTS

Some combinations / variations of experimentation involving MCV parameters were carried out through computer programs written in MS Fortran and implemented on Casio FP4100 micro- computer (IBM XT compatible) and the results obtained are presented in some detail in this section in table 2.

ANALYSES OF RESULTS

The results listed in table 2 are quite useful in finding answers to basic questions related to experimental design. The analyses of results, in general, follow this scheme.

Q1. Does MCV technique improve accuracy of estimate of Y and how does it compare with simple Monte Carlo technique?. To begin with, the results of case A and case H may be compared. The reduction in variance is of the order of 90%. Additional computer time required for this is only about 25%. The answer is thus clearly affirmative.

Q2. Which statistical scheme/eq. is best suited to the normal case in close range photogrammetry? We can find the answer to this through step by step comparisons. Result of cases B and C indicate that it is better to use the same random number stream for processing the variances at the various points and further that it is preferable to adjust the random numbers to yield a mean of zero. Again, comparison of case A or case B with case D indicates that it is better to use a computed value using eq.3.4 for bi vector rather than use a value of 1 for each of the elements in the vector. If we compare the results of cases D or F or G against result of case H, we understand that use of eq. 3.6 a & b combined in one would be preferable to use of either 3.6a alone or 3.6b alone or eq.3.1. This is attributable to the maximum reduction in the magnitude of the residuals realizable in case H.

Table 2: Variances obtained by use of different methods

CASE ID.	VARIANCES pl.1	pl.2	pl.3	DESCRIPTION/REMARKS
A	2.5	5.7	11.6	Simple Monte Carlo tech. SS=54. No MCV Para. used. Run time = 2M 10S. Rewind used. File:MCV01A.OUT
B	112.4	90.3	57.8	bi=1.MCV technique used.MCV=8.Different ran.numbers used at each step. Rand. nos. not adj. to mean 0. All MCV can be located in one plane. No rewind used. MCV01B.OUT. Ref.eq.3.1
C	103.6	79.0	62.4	MCV tech. MCV = 8.Rand. nos. adj. to mean 0. bi=1.Pts.18, 19, 23,29, 31,34,37 reveal large diff.bet.calc. & true values. All MCV Y are equal. No Rewind.MCV01C.OUT. Ref. eq. 3.1.
D	2.5	5.7	11.3	MCV tech. SS=54. MCV=8. Five pts show neg cov. bet Y & X. Correction computed as in Kobayashi.Ref.eq.3.1.Var. is not reduced. MCV01D.OUT.
E	2.5	5.9	12.1	MCV tech.8 points all in one plane. Diff. rand. nos. used for each control variable. MCV01D.2UT. Ref.eq. 3.1.
F	0.5	2.3	6.2	MCV tech.SS=54.MCV = 8. In computing corrections contributions arising from pos. bi values only are taken into account. Var. red. is quite good. MCV01E.OUT. Ref. eq. 3.6b.
G	0.4	2.1	5.9	MCV = 8. They are unequal Y coord. of control points. Var.red. obtained by use of neg. bi values only. MCV01F.OUT.Ref.eq. 3.6a.
H	0.04	0.4	2.6	MCV = 8. bi values calc. from C*QINV. Var red obtd. by consideration and seperate treatment of positive and neg. bi values.Run time: 2M 45S. MCV01G.OUT. Ref. eq. 3.6a & 3.6b.
I	36.0	28.4	21.0	MCV = 3*8 =24.No improvement in var. of comp.Y-coords.SS=54.Run time:4M10S.MCV02A.OUT Ref. eq. 3.6a & 3.6b.
J	1.2	3.6	8.2	MCV= 4.Only Y-coord of cont. pts used.Var.red. = 20%. The 4 pts are situated similar to mid-side fiducial marks in MCV03A.OUT.The Four pts are situated at corner of cont. field in MCV03A.2UT. Run time=2M 37S.MCV03A.OUT & .2UT.Ref. eq. 3.6a & 3.6b.
K	0.7	2.8	7.0	MCV = 2. Significant red. of var. Run time=2M 30S.MCV03A .OUA,.OUB,.OUC,.OUD. Ref. eq. 3.6a & 3.6b.
L	1.5	4.1	9.1	MCV = 1. Significant red. of var. is observed. Run time = 2M 23S. MCV03A.OU1 TO .OU9. Ref. eq. 3.6a & b.
M	5.4	2.3	0.3	Same as case H except that the ref. pt. for calc. of bi is changed to 32. MCV01G.2UT
N	1.3	0.1	0.5	Same as case H except that The ref. pt. for calc. is changed to 22. MCV01G.3UT

Notes: unit for variances is mm**2. SS=Sample size used in estimating variances. MCV : No.of Multi-control var. used. MCV0??.?U? refer to output file names. In case of methods K & L, as the variations were very small for the different cases, only one representative value is listed. pl. stands for plane.

The results contained in case L may be studied to help us get some insight into the geometrical nature of the problem of variance reduction. We proceed to tabulate the results of bi values for the various variations in case L. These are -0.195, -0.174, -0.156, -0.141, -0.128, -0.117, -0.107, -0.098 & -0.098 corresponding to the use of points 41, 42, ...,49 as the MCV parameters. These values were computed specifically for point 12 only, situated in plane 1, in all 9 cases.For this particular location, the covariance between the Y-coordinate of the point and the MCV parameter in each case is negative, and hence also the computed values of B obtained by use of eq.3.4. It could be that, if this simplifying assumption were not made and the covariance and hence the bi value computed seperately for each plane and used on a plane by plane basis ,then we should have obtained greater variance reduction for points in planes 2 & 3 also. Inspection of cases M & N shows that this is indeed true. It might also be interesting to compare how the bi values change when one switches from the individual control variates of case L to MCV control variates of case H.Thus, the bi values of case H are: -0.248, -0.024, -0.168, 0.118, -0.076, 0.013, 0.097, 0.094. Obviously, inspection reveals that there is no apparent relationship between the two sets of values. Another point of interest may lie in comparison of residul systematic errors in cases A vs H. These values for points in planes 1, 2 and 3 are 0.043, 0.019, 0.021 for case A and 0.02, -0.004 and -0.044 for case H. Obviously, with this limited amount of data, no specific conclusions may be drawn. Further data analyses would be necessary.

Q3. Is the use of complete three dimensional coordinates for the MCV parameters preferred over use of Y-coordinate only?. Comparison of results tabulated against cases H and I reveal that it is advisible to use only the Y-coordinate information. Further, from the computational point of view it should be easily possible to invert the Q matrix. This would be the case when all used Y- coordinates are unequal and considerably different in value from each other.

Q4. Which location in the control field is preferred for the selected MCV parameters?. To arrive at an answer to this question, It is necessary to look at the variations between the various results grouped under case L. It was noted that this variation did not exceed 0.1 mm**2 for any of the cases. Accordingly, it should be inferred that the location of the MCV parameter in the control field need not be restricted.

Q5. In data reduction, how many MCV parameters should one use?. To suggest an answer to this question, it is necessary to compare results given in cases L, K, J and H. The variances at mid-plane location for these cases in order are 4.1, 2.8, 3.6 and 0.4 mm**2 respectively. It may be noted that the variance reduction is by a factor of 10 when the MCV parameters are increased from 1 to 8. The results for other cases can be obtained by interpolation.

CONCLUSIONS

The inferences arrived at in the previous section may now be summerized in a nutshell.

1. The MCV technique is a powerful tool which is quite readily applicable in terrestrial and close range photogrammetry.

2. Considering some possible variations given earlier the mathematical structure represented by eq.3.6 a & b is to be preferred.

3. Using just the known Y-coordinates for MCV para- meters is better than using all the three dimensional coordinates for the MCV parameters.

4. As only the Y-coordinates are effective MCV parameters there is no preferred location for such points.

5. The MCV control points should all lie in diff- erent XY planes or stated in another equivalent way, the Y-coordiate of any MCV point should not be equal to Y-coordinate of any other MCV point.

6. As a thumb rule, it may be stated that the variance reduction of approximately 10% per MCV parameter used may be expected.

7. Further data analyses would be desirable in confirming the above conclusions.

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