ON THE ORIENTATION PARAMETERS OF NON-METRIC CAMERA

Prof.Dr. Abdel-latif, M.S.,Dr. A. A-R. Mohamed, and A. El-Sonbaty
Civil Engineering Department, Assiut University
Assit, EGYPT

ABSTRACT

The accuracy of the parameters of a camera plays an important role in photogrammetric measurements.

In this paper, a direct mathematical method to determine the exterior and the interior orientation parameters of a non-metric camera is developed. The technique depends on measuring or calculating the lengths of lines connecting one image of a control point to another and also to the principal point. When compared with the collinearity method, the new method has the advantages of higher accuracy with lower time of computation.

INTRODUCTION

One of the most important items in photogrammetry is the determination of the orientation parameters of central perspective photographs taken with non-metric cameras. Collinearity between image point, centre of perspective and object point is often the basic idea behind the linearized form of the mathematical model (AX=L+V). Observations are usually corrected for systematic deviations (film shrinkage, lens distortion etc.) from the collinearity model before adjustment (Torlegard, 1981). Wong (1975) states that in close-range the photogrammetric solution may be divided into two major phases : (1) relative orientation of individual stereoscopic pair of photos, and (2) model linking and absolute orientation to the object-space coordinate system.

This paper introduces a new mathematical approach for determining the external orientation parameters for non-metric cameras. The system depends mainly on using the lengths of lines between the images of the control points. The new method has been programmed on the computer and checked mathematically and practically. When compared with collinearity method, the new method has the advantage of higher accuracy with lower time of computation.

MATHEMATICAL FORMULATION TECHNIQUE

Preliminary Note

Let n control points with known space coordinates be given as :$A_1(X_1,Y_1,Z_1)$, $A_2(X_2,Y_2,Z_2)$, ...$A_n(X_n,Y_n,Z_n)$.

In Fig.(1), images $a_1(x_1,y_1)$, $a_2(x_2,y_2)$, ...$a_n(x_n,y_n)$ of the ground control points appear in an image positive plane Π. At the same time, relative to the space coordinate system O, X, Y, Z, on image point such as a_1 should have space coordinates (X_1,Y_1,Z_1), which can be found as the point of intersection of SA_1 with Π.

The method discussed here depends on deriving the formulas of the distances joining pairs of images of control points in terms of their space coordinates. Equating these formulas to their corresponding distances between image points, which can be measured directly or calculated, will give equations in six unknowns : The space coordinates of the camera station $S(X_s,Y_s,Z_s)$ and the three coefficients of the image plane $\Pi(A,B,C)$. Therefore, at least six independent equations are required for the solution of the problem.

The number of line segments joining n points is given by :-

$$m = n(n-1)/2 \quad (1)$$

From which, only

$$m' = (2n - 3) \quad (2)$$

are independent. This means that for six unknowns, at least five control points are necessary for the solution of the problem, which will yield seven independent equations plus three dependent ones.

Space Coordinate of an Image Point

The parametric equations of the straight line joining $S(X_s,Y_s,Z_s)$ with an object point $A_i(X_i,Y_i,Z_i)$ may be represented by:-

$$\begin{aligned} X &= X_i + (X_s - X_i)t \\ Y &= Y_i + (Y_s - Y_i)t \\ Z &= Z_i + (Z_s - Z_i)t \end{aligned} \quad (3)$$

where t is a parameter.

Let the equation of the image plane Π be:

$$AX + BY + CZ + D = 0 \quad (4)$$

Dividing by one of the coefficients,

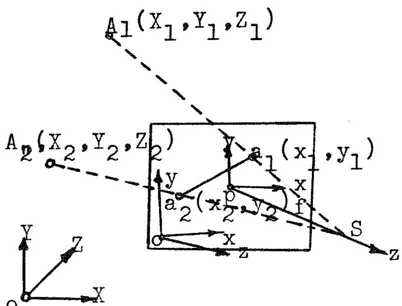

Fig. (1)

assume D, and without loss of generality, the equation of Π may take the form :

$$AX + BY + CZ + 1 = 0 \qquad (5)$$

Substituting (3) into (5), the value of $t_i = t$ associated with the image point a_i is found to be :

$$t_i = \frac{AX_i + BY_i + CZ_i + 1}{A(X_s - X_i) + B(Y_s - Y_i) + C(Z_s - Z_i)} \qquad (6)$$

which can now be substituted into (3) to get the space coordinates of the image $a_i(X_i, Y_i, Z_i)$:

$$\begin{aligned} X_i &= X_i + (X_s - X_i)t_i \\ Y_i &= Y_i + (Y_s - Y_i)t_i \\ Z_i &= Z_i + (Z_s - Z_i)t_i \end{aligned} \qquad (7)$$

where $i = 1,2,3, \ldots ,n$

Now, the distance l_k connecting two image points a_i and a_j may have the form :

$$l_k^2 = (X_j - X_i)^2 + (Y_j - Y_i)^2 + (Z_j - Z_i)^2 \qquad (8)$$

while l_k can, as mentioned before, be calculated or measured.
Equation (8) can take the form :

$$F_k\ (X_s, Y_s, Z_s, A, B, C) = (X_j - X_i)^2 + (Y_j - Y_i)^2 (Z_j - Z_i)^2 - l_k^2 = 0 \qquad (9)$$

with $k = 1,2,3, \ldots .m$.

Taking into consideration all point pairs, we get $m = n(n-1)/2$ equations similar to (9), which form a system of nonlinear equations in the six unknowns (X_s, Y_s, Z_s, A, B, C).

Determining the Unknowns

For the case when n=5 (min. value), the system (9) contains 10 nonlinear equations in the six unknowns and can be solved using the least squares technique to compensate for any probable measuring errors.

To simplify the procedure, each equation in (9) can be linearized using Taylor's theorem as follows :-

$$F_k = (F_k)_o + \left(\frac{\partial F_k}{\partial X_s}\right)_o dX_s + \left(\frac{\partial F_k}{\partial Y_s}\right)_o dY_s + \left(\frac{\partial F_k}{\partial Z_s}\right)_o dZ_s + \left(\frac{\partial F_k}{\partial A}\right)_o dA + \left(\frac{\partial F_k}{\partial B}\right)_o dB + \left(\frac{\partial F_k}{\partial C}\right)_o dC + \text{terms of higher order} \qquad (10)$$

where $(F_k)_o$, $\left(\frac{\partial F_k}{\partial X_s}\right)_o, \ldots \ldots$ are value of F_k and its partial derivatives at suitable chosen initial values of the unknowns : $X_{so}, Y_{so}, Z_{so}, A_o, B_o, C_o$. The derivatives can be found by differentiating (9) partially with respect to the unknowns. For example:

$$\frac{\partial F_k}{\partial X_s} = 2(X'_j - X'_i)\left(\frac{\partial X'_j}{\partial X_s} - \frac{\partial X'_i}{\partial X_s}\right) + 2(Y'_j - Y'_i)\left(\frac{\partial Y'_j}{\partial X_s} - \frac{\partial Y'_i}{\partial X_s}\right) + 2(Z'_j - Z'_i)\left(\frac{\partial Z'_j}{\partial X_s} - \frac{\partial Z'_i}{\partial X_s}\right) \qquad (11)$$

The new derivatives in (11) can be found from (7), such as :

$$\frac{\partial X'_i}{\partial X_s} = (X_s - X_i)\frac{\partial t_i}{\partial X_s} + t_i \qquad (12)$$

t_i is previously found in (6), then :

$$\frac{\partial t_i}{\partial X_s} = \frac{-A(AX_i + BY_i + CZ_i + 1)}{[A(X_s - X_i) + B(Y_s - Y_i) + C(Z_s - Z_i)]^2} \qquad (13)$$

Similar formulas can be found for other derivatives.

The system of equation (10) can now be approximated by considering only the linear term. The new system will have the following matrix form :

$$[A][X] = [B] \tag{14}$$

where [A] is the coefficient matrix which contains the initial values of the derivatives :

$$[A] = \begin{bmatrix} \left(\frac{\partial F_1}{\partial X_s}\right)_o & \left(\frac{\partial F_1}{\partial Y_s}\right)_o & \cdots\cdots & \left(\frac{\partial F_1}{\partial C}\right)_o \\ \left(\frac{\partial F_2}{\partial X_s}\right)_o & & & \\ \vdots & & & \\ \left(\frac{\partial F_m}{\partial X_s}\right)_o & & & \left(\frac{\partial F_m}{\partial C}\right)_o \end{bmatrix}$$

[X] is the unknown vector where
$[X]^T = [dX_s, dY_s, dZ_s, dA, dB, dC]$, and
[B] is the constant term vector
$[B]^T = [-(F_1)_o, -(F_2)_o, \ldots\ldots -(F_m)_o]$, whose components are found by substituting the initial values in (9).

Equation (14) are solved to obtain corrections $dX_s, dY_s, \ldots\ldots dC$. These corrections are added to $X_{so}, Y_{so}, \ldots\ldots C_o$ to obtain improved estimates and the procedure is continued until the corrections are negligible. It is to be noted that the iteration problem is convergent if the initial values are near the solution (similar to Newton's method for solving a system of nonlinear equations).

Determination of the Focal Length f and the Principal Point p

After determining the six external orientation elements (X_s, Y_s, Z_s, A, B, C), the focal length can be calculated as the length of the perpendicular dropped from S onto Π and is given by :

$$f = \frac{|\ AX_s + BY_s + CZ_s + 1\ |}{(\ A^2 + B^2 + C^2\ )^{0.5}} \tag{15}$$

The space coordinates of the principal point $p(X_o', Y_o', Z_o')$ can be found as the point of intersection of the just mentioned perpendicular with the image plane Π as follows :

$$X_o' = X_s - \frac{A(AX_s + BY_s + CZ_s + 1\ )}{A^2 + B^2 + C^2}$$

$$Y_o' = Y_s - \frac{B(AX_s + BY_s + CZ_s + 1\ )}{A^2 + B^2 + C^2} \tag{16}$$

$$Z_o' = Z_s - \frac{C(AX_s + BY_s + CZ_s + 1\ )}{A^2 + B^2 + C^2}$$

To find the coordinates of $p(x_o, y_o)$ in Π, it is necessary to rely on at least two image points, e.g. a_1 and a_2, whose space and image plane coordinates are known.

The space coordinates $a_1(X_1', Y_1', Z_1')$ and $a_2(X_2', Y_2', Z_2')$ can be found from (6) and (7), while the image plane coordinates $a_1(x_1, y_1)$ and $a_2(x_2, y_2)$ are originally given. As shown in Fig. (2) the lengths of the line segments l_1, l_2 and l_3 are given by :

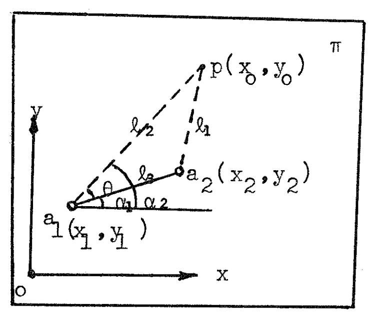

Fig. (2)

$$l_1 = \overline{a_2 p} = \sqrt{(X_o' - X_2')^2 + (Y_o' - Y_2')^2 + (Z_o' - Z_2')^2}$$

$$l_2 = \overline{a_1 p} = \sqrt{(X_o' - X_1')^2 + (Y_o' - Y_1')^2 + (Z_o' - Z_1')^2} \tag{17}$$

$$l_3 = \overline{a_1 a_2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The slope of $a_1 a_2$ in Π

$$\tan \alpha_1 = \frac{y_2 - y_1}{x_2 - x_1} \tag{18}$$

The angle $\theta = \measuredangle\ Pa_1a_2$ can be found from the cosine law :

$$\cos \theta = \frac{l_2^2 + l_3^2 - l_1^2}{2\ l_2 l_3} \tag{19}$$

There are two solutions for θ :

a) $\theta = \theta_1$, $0^\circ < \theta_1 < 180^\circ$

and b) $\theta = -\theta_1$

The coordinates of $P(x_o, y_o)$ can now be found :

$$x_o = x_1 + l_2 \cos(\alpha_1 + \theta)$$
$$y_o = y_1 + l_2 \sin(\alpha_1 + \theta) \qquad (20)$$

To reveal the ambiguity about the values of θ, the above procedure is repeated using a third point a_3 with a_2. One of the new solution of (x_o, y_o) should coincide with one of the first ones. This is chosen to be the correct answer. It is to be noted that if all image points are taken into consideration, the least squares technique can be applied to find the most probable position of the principal point p.

Determination of The Angular Orientation Parameters ω, ϕ and $\varkappa$

There are three rotations necessary to transform the space coordinate system O, X, Y, Z into a position parallel to the image plane system o, x, y, z, fig.(3).

At first the space system is rotated about the Y-axis through an angle ϕ to the position X_1, $Y_1 = Y$, Z_1. The X_1 axis is parallel to d, the line of intersection of Π with the XZ-plane. Next, the new system is rotated about the X_1-axis through an angle ω to the position $X_2 = X_1$, Y_2, Z_2, such that Z_2 is parallel to Z. Finally, the second system is rotated through an angle $\varkappa$ about the Z_2 axis to the position $X_3 // x$, $Y_3 // y$, $Z_3 : Z_2 // z$. Hence

$$\phi = \measuredangle (d, X)$$
$$\varkappa = \measuredangle (d, x) \qquad (21)$$
$$\omega = \measuredangle (Z_1, z)$$

The values of these angles depend on the direction of the vectors along the different lines.

The direction ratios of $\vec{Z}$ is the same as the normal to Π, hence

$$\vec{Z} = (A : B : C) \qquad (22)$$

Those of d are perpendicular to $\vec{z}$ and $\vec{Y}$:

$$\vec{d} = \vec{Y} \times \vec{z} = (C : 0 : A) \qquad (23)$$

$\vec{Z_1}$ is also perpendicular to both Y and d, then

$$\vec{Z}_1 = d \times Y = (A : 0 : C) \qquad (24)$$

Let the direction ratios of the x-axis relative to the space system be

$$\vec{x} = (a : b : c) \qquad (25)$$

which are unknown till now. This vector is perpendicular to $\vec{z}$ and subtends an angle = α_1 with the vector $\vec{v}$ along $a_1 a_2$ given by

$$\vec{v} = ((X_2' - X_1') : (Y_2' - Y_1') : (Z_2' - Z_1')) \qquad (26)$$

The two equations

$$\vec{x} \cdot \vec{v} = |\vec{x}| \cdot |\vec{v}| \cos \alpha_1 \qquad (27)$$

$$\text{and } \vec{z} \cdot \vec{x} = 0 \qquad (28)$$

can be solved simultaneously for a : b : c. They yield a quadratic equation, which can be easily solved.

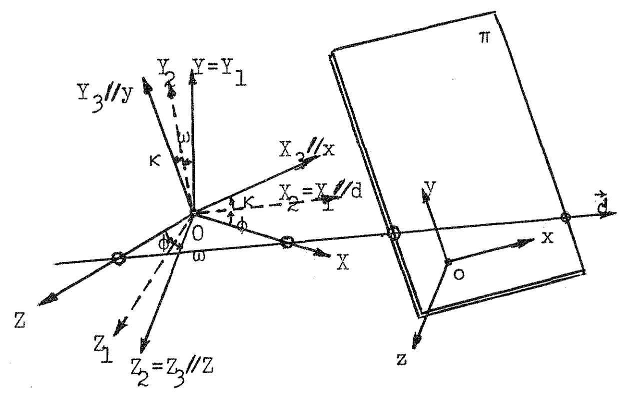

Fig. (3)

Finally, the rotation angles can be calculated as follows :

$$\cos \phi = \frac{\vec{d} \cdot \vec{X}}{|d| \cdot |X|} = \frac{C}{\sqrt{A^2 + B^2}} \qquad (29)$$

$$\cos \omega = \frac{\vec{z} \cdot \vec{Z_1}}{|\vec{z}| \cdot |\vec{Z_1}|} = \frac{A^2 + C^2}{\sqrt{A^2 + C^2} \sqrt{A^2 + B^2 + C^2}} \qquad (30)$$

and

$$\cos \varkappa = \frac{\vec{d} \cdot \vec{x}}{|\vec{d}| \cdot |\vec{x}|} = \frac{aC - cA}{\sqrt{a^2 + b^2 + c^2} \sqrt{A^2 + C^2}} \qquad (31)$$

EVALUATION OF THE ACCURACY

The developed method has been verified using both a mathematical model and a practical approach. A mathematical simulation of a photograph of an object (model) consists of 30 points with six control points has been done. The interior and exterior orientation parameters for both left and right photos were assumed. the geometrical dimensions were : the base length (B) = 5.00 m, and the object distance (H) = 10.00 m. Random errors were added to each photo coordinate (0 to ± 0.05 mm) to simulate the measuring errors and random image deformations (e.g. lens distortion, film deformation,...etc.). Table (1) shows the standard deviation for object (check points) space coordinates (in cm).

Stand. dev.	Metric case	Non-metric case
σ_x	0.1635	0.1270
σ_y	0.1115	0.1103
σ_z	0.3983	0.4976
σ_{xyz}	0.4440	0.5252

Table (1)

A practical test was carried out by Ebrahim (1992) using a phototheodolite 19/1318 metric camera. The control field consisted of 50 check points, among which 8 control points were chosen. The object distance was taken as H= 10.00 m. Image coordinates were taken for 15 stereopairs of photographs using Zeiss Jena stereocomparator 1818 in the photogrammetric Lab. in Assiut University. The standard deviation for the object space coordinates were determined using the proposed method. The results were compared with those calculated by the collinearity method as a standard method. Table (2) shows an example of the results of the standard deviation for check and control points (in cm) using both the developed method and the collinearity method.

Stand. dev.	Check points		Control points	
	New	Collin.	New	Collin.
σ_x	0.057	0.059	0.00017	0.047
σ_y	0.061	0.071	0.00018	0.050
σ_z	0.166	0.193	0.00060	0.232
σ_{xyz}	0.185	0.214	0.00065	0.242

Table (2)

CONCLUSION

The developed method described here has proved an increase in the accuracy of the orientation parameters for both metric and non-metric cameras, which affected directly the accuracy of the calculated space coordinates for both check and control points. This increase in accuracy may be referred to the reduction in the number of the unknowns in the equations of the developed method when compared to the collinearity method, (6 unknowns instead of 9 in the case of non-metric cameras, and 5 unknowns instead of 6 in the case of metric cameras). This means that the number of iterations and consequently the computing time is reduced.

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