SYSTEM CALIBRATION, USING A TEST FIELD WITH ONLY KNOWN HEIGHTS

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Abstract
Establishment of a three-dimensional test field, its survey and maintenance is very expensive.

Using a standard bundle adjustment procedure with added parameters, it is possible to calibrate a camera system using a test area with only known height. In the calibration are determined tangential, radial, affine and shear deformations caused by decentering of lenses, non-corrected distortion and refraction, mechanical deformations etc. Only the principal distance and shift of principal point is not found.

The only condition is that the area is covered by 3-4 photos, flown in different directions. Any open town area can be used, where signals can easily be painted. Sufficiently precise levelling (maybe .05 °/0, H) is much easier to achieve than x-y-coordinates, thus making it a very cheap method.

Introduction
For more than ten years it has been possible to make a calibration of a camera system - including atmospheric correction - based on test field measurement and bundle adjustment. This method has never been popular due to the heavy expenses of establishing and maintaining a test field with more than 100 points coordinated in x, y and z. System calibration without using any test field at all would cut down these expenses drastically.

As described in /1/ and /2/ it is possible to make a system calibration using only parallax measurements. But unfortunately not all systematic deformations can be determined by this calibration method. A calibrating method using a "cheaper" test field, but still with the possibility of determining all significant deformations, is therefore of great interest. Using a test field with only given heights makes it possible to cut down the expenses of establishing and maintaining a test field, and with a special flight arrangement all deformations can still be determined.

Bundle Adjustment sANA
For more than six years the bundle adjustment programme sANA has been used for research and practical work in Denmark. This work as well as international tests - ISP Jämijärvi, OEEPE Gross Error - have shown that usual types of systematic errors can be described by three types of added parameters: tangen-
tial, radial and affine components. All three types can be explained by physical conditions including camera errors, refraction and laboratory conditions (development, drying, etc.)

One of the crucial points for a system calibration is that all three types of added parameters should be determined with a high accuracy. The added parameters are given by four tangential parameters: \( t_1, t_2, t_3 \) and \( t_4 \), three radial parameters: \( r_1, r_2 \) and \( r_3 \), and two affine parameters: \( af \) and \( sh \).

The deformation formulas can be described as follows:

1. **Tangential deformation:**
   
   \[
   \begin{align*}
   dt &= r^{1.85} (t_1 \cos \alpha + t_2 \sin \alpha + t_3 \cos 2\alpha + t_4 \sin 2\alpha) \\
   x'' &= x' + dt/100000 \cdot \sin \alpha \\
   y'' &= y' - dt/100000 \cdot \cos \alpha
   \end{align*}
   \]

2. **Radial deformation:**
   
   \[
   \begin{align*}
   dr &= r_1 (r_1^3 - r_0^2) + r_2 r_0 (\sin (r_1 / r_0))^{2} + r_3 \sin (2r_1 / r_0) \\
   x'' &= x' + dr/100000 \cdot \cos \alpha \\
   y'' &= y' + dr/100000 \cdot \sin \alpha
   \end{align*}
   \]

3. **Affinity:**
   
   \[
   \begin{align*}
   x'' &= x' \\
   y'' &= y' - (af \cdot y' + sh \cdot x')/100000
   \end{align*}
   \]

\( x' \) and \( y' \) are image coordinates in dm

\( r' = \sqrt{x'^2 + y'^2} \)

\( \alpha \) = grid bearing

\( r_0 \) = second (or first if only one) intersection of the radial lens distortion curve with the 0-axis of the coordinate system.

The parameters can be treated as elements as well as observations with individual weights.
**Test Field with only Given Heights**

A test field with only given heights can be established for only a fraction of the costs of a traditional test field. In many cases even targeting can be avoided! But let us take a closer look at the special conditions - or lack of such - for the test field and the flight arrangement. We want to compute all nine deformation parameters.

**Height Differences for the Test Field**

No height differences are necessary. The area can even be flat.

**Size of the Test Field**

The size of the test field is limited to the area covered by one photo. If the area is to be used for different photo scales and different camera constants, it must be designed correspondingly.

**Number of Given Z-Coordinates in the Test Field**

75-200 points are to be targeted (targeting can in many cases be avoided, see later) randomly all over the area covered by one photo. The RMSE of the computed deformation parameters is depending on the number of points covered by one photo. The graph shows this dependency. The RMSE of the other elements will vary in the same manner. No absolute values are put on the graph, as the RMSE will vary according to the quality of the z-coordinates, the number of the photos in the adjustment and the measuring quality.

![Graph](image)

**Fig. 1.**

**Quality of Z-Coordinates**

It is mainly the quality of the given z-coordinates that influences the quality of the deformation parameter $r_1$ (see (2)). This is demonstrated in the following graph. The RMSE of the z-coordinates is given in $\%$ of the flying height. The graph will vary slightly according to the camera constant. In this example 15 cm is used. From the graph you can see that by a normal levelling procedure it is easy to obtain the required RMSE. Even with a photo scale of 1:1500 a RMSE of 1 cm for the z-coordinates is well over the required value.
Number of Photos

Only three photos exposed over the test field are necessary. The overlap and flight arrangement is illustrated in the figure below. The two strips are flown orthogonal to each other. Two photos in the first strip and only one in the second. To improve the result two photos in the second strip can be exposed. In this case an overlap of 60% – as in the first strip – will be optimal. The second strip shall cover the first one. Two photos in strip two will improve the computed RMSE of the deformation parameter \( r \) (see (2)) with about 25%. All other parameters will only have an improvement of less than the half.

Special Requirements

The same point on the ground should be measured in all 3 or 4 photos!

Disadvantages

With this method it is not possible to estimate the camera constant and the displacement of the principal point. These values could be of interest in mountainous terrain.
Example

To demonstrate the fine results obtained by this method an example is given with the following conditions:

- Test field with 150 targeted points
- Z-coordinates measured for all points. RMSE 2 cm
- Camera: 23 x 23/15
- Photo scale: 1:10000
- Three photos exposed as shown in fig. 3 (60% overlap and 90% overlap)
- All targeted points measured in a monocomparator. RMSE 2 µm
- After adjustment a $\sigma_0$ of 3.5 µm came out.

The RMSE of the nine deformation parameters computed in the bundle adjustment programme sANA are shown in the table below:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$t_1$</th>
<th>$t_2$</th>
<th>$t_3$</th>
<th>$t_4$</th>
<th>$r_1$</th>
<th>$r_2$</th>
<th>$r_3$</th>
<th>$af$</th>
<th>$sh$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE (µm)</td>
<td>0.7</td>
<td>0.7</td>
<td>0.5</td>
<td>0.4</td>
<td>1.8</td>
<td>1.7</td>
<td>0.4</td>
<td>0.8</td>
<td>0.6</td>
</tr>
</tbody>
</table>

This table shows that it is possible to compute the systematic errors with a RMSE less than two microns using only three photos and a test field with only given heights. A better determination of the systematic errors will only be of academic interest as the camera and flight conditions will vary from flight mission to flight mission. Maybe even the same day. In the bundle adjustment programme sANA it is possible to introduce the systematic errors as observations with different weights. The result from a system calibration can in this way be used in later bundle adjustments equal to image observations.

Terrestrial Cameras

The method is not limited to airborne cameras. At the University of Aalborg we have used the method with great success for calibrating terrestrial cameras using a photo scale of 1:10.

Test Field without Targeted Points

As mentioned above it is even possible to use the method without targeted points. Targeted points are used to obtain that

1) the same terrestrial point is measured in all 3-4 photos
2) the photogrammetrically measured point is the same as the terrestrially measured point.

With a modern analytical plotter and a relatively flat terrain the above-mentioned two crucial points can be fulfilled without any targeting.

re 1) With stereo measurements in an analytical plotter it is possible to transfer points from one photo to another. The same terrestrial point can in this way be measured in all 3-4 photos.

re 2) If a flat urban area is used as a test field it is possible to define areas for z-coordinates, e.g. the middle
of a footpath, where variations in elevations are less than required for a precise determination of the systematic errors.

**Conclusion**

A test field without targeted points and with only given z-coordinates is a very low-cost test field with the same possibilities for determination of systematic errors as a standard "three-dimensional and targeted" test field can give. If targeting cannot be avoided, the method is still much cheaper, as no x- and y-coordinates are to be measured.

**References**
