The estimated formulas for the precision of the attitude of the stellar camera determined by photogrammetry are presented in this paper, and compared with the corresponding empirical formulas in "Manual of Photogrammetry" (the fourth edition) by ASP. The tests show that these formulas in this paper are fit for the practical case.

In the satellite photogrammetry, the attitude of the stellar camera is often used to determine that of the terrain camera. The angular precision of the space direction of the stellar camera will therefore have a direct effect on the precision of the later operation. In order to pre-estimate the precision of the attitude of the stellar camera determined by photogrammetry, a set of practical estimated formulas is presented in this paper.

It is well known that for the photos by stellar camera, the equations of the collinearity condition take the following forms:

\[
\begin{align*}
    x &= -f \frac{a_x A + b_y B + c_z C}{a_z A + b_z B + c_z C} \\
    y &= -f \frac{a_y A + b_y B + c_y C}{a_z A + b_z B + c_z C}
\end{align*}
\]

where

- \((x, y)\) = photographic coordinates of the star image;
- \((a_x, b_y, c_z, \ldots, a_y, b_y, c_y)\) = direction cosines of the image-space coordinate system, as a function of the attitudes;
- \(A = \cos \alpha \cdot \cos \delta, B = \sin \alpha \cdot \cos \delta, C = \sin \delta\);
- \(\alpha = \) right ascension of the star;
- \(\delta = \) declination of star;

After linearizing Eqs. (1) by Taylor series, the general matrix forms of observation equations are:

\[
\begin{align*}
    \mathbf{B} \mathbf{\Delta} - \mathbf{L} &= \mathbf{V} \\
    \mathbf{\Delta} &= (\mathbf{B}^T \mathbf{B})^{-1} \mathbf{B}^T \mathbf{L}
\end{align*}
\]

and then the normal equations can be written as

\[
\begin{align*}
    \mathbf{B}^T \mathbf{B} \mathbf{\Delta} &= \mathbf{B}^T \mathbf{L}
\end{align*}
\]

Then, the least squares solution is

\[
\begin{align*}
    \mathbf{\Delta} &= (\mathbf{B}^T \mathbf{B})^{-1} \mathbf{B}^T \mathbf{L}
\end{align*}
\]

and the standard error of the ith unknown parameter is

\[
\begin{align*}
    \sigma_i &= \sqrt{Q_{ii} \cdot \mathbf{\Delta}}
\end{align*}
\]

where

- \(Q_{ii}\) = the ith diagonal element in matrix \(\mathbf{Q}\);
- \(\mathbf{Q} = (\mathbf{B}^T \mathbf{B})^{-1}\), which is variance-covariance matrix;
- \(\sigma_i\) = the standard error of unit weight.
It is a well known method for computing the theoretical precision, but this classical method can only be used with the computation of attitude simultaneously.

Now we intend to derive the formulas suitable for pre-estimating the attitude precision with the above method.

At first, we analyse the theoretical precision of attitude in the case of 4 and 9 stars. Assume that the stars are of symmetric distribution as shown in Fig. 1.

4 stars used in computation

9 stars used in computation

Fig. 1.

By a lot of derivation, according to the above classical method, we can get:

in the case of 4 stars

\[
\begin{align*}
Q_{ll} &= \frac{m_{x0}^2}{m_o^2} = \frac{1}{\cos^2 \delta_0} \cdot Q_{22}, \\
Q_{22} &= \frac{m_{x0}^2}{m_o^2} = \frac{4(2d^2 + f^2 + 2f^2d^2)}{f^2}, \\
Q_{33} &= \frac{m_{x0}^2}{m_o^2} = \frac{1}{f^2 + t g^2 \delta_0} \cdot Q_{22}
\end{align*}
\]

(5)

in the case of 9 stars

\[
\begin{align*}
Q_{ll} &= \frac{m_{x0}^2}{m_o^2} = \frac{1}{\cos^2 \delta_0} \cdot Q_{22}, \\
Q_{22} &= \frac{m_{x0}^2}{m_o^2} = \frac{10d^4 + 9f^4 + 12f^2d^2}{f^2}, \\
Q_{33} &= \frac{m_{x0}^2}{m_o^2} = \frac{1}{12f^2 + t g^2 \delta_0} \cdot Q_{22}
\end{align*}
\]

(6)

The varying rule of precision for the calculated attitude in Eqs. (5) and (6) is different from that shown in the corresponding empirical formulas in "Manual of Photogrammetry" [3].

These empirical formulas are:

\[
\begin{align*}
\delta_\phi &= \frac{\sqrt{2}}{2n^3} \cdot \frac{m_x}{f}, \\
\delta_\lambda &= \frac{\sqrt{2}}{2n^3} \cdot \frac{m_x}{L}
\end{align*}
\]

(7)
It will be seen from this that eqs. (7) seem to be not so ideal to provide estimated precision and show its varying rule.

But the given formulas (5) and (6) are only suitable for the above case. In practice, there need be a set of general formulas that can be suitable for all cases. In fact, it's impossible to derive the rigorous formulas, we can only get approximate formulas. Assume that $n$ stars are distributed as shown in Fig. 2, and then derive the estimated formulas for $n$ stars. These formulas can be used instead of the general ones approximately.

In Fig. 2, the locations of $n$ stars takes the style of Fig. 1, with different values $d$, we get

$$Q_{11} = \frac{m_{x_0}^2}{m_0^2} = \frac{1}{\cos^2 \delta_0} \cdot Q_{22}$$

$$Q_{22} = \frac{m_{y_0}^2}{m_0^2} = \frac{15n^2f^2}{2d^2(n+2)(n+4)[(3n^2+12n-16)d^2+5n^4f^2]+15n^2f^4}$$

$$Q_{33} = \frac{m_{x_0}^2}{m_0^2} = \frac{3n}{2(n+2)(n+4)d^2+9n^2f^2}$$

(8)

If we don't consider the principal distance is much longer than $d$, we may get

$$Q_{22} = \frac{3n}{2(n+2)(n+4)d^2+9n^2f^2}$$

(9)

In order to check the exactness of these formulas, some tests are made in Electronic Computer DJS-220. The results are listed in the following table.
### Comparison of the Estimated Precision

<table>
<thead>
<tr>
<th>Number of Used Stars</th>
<th>Mode</th>
<th>$M''_{\alpha_o}$</th>
<th>$M''_{\delta_o}$</th>
<th>$M''_{\kappa_o}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>theoretic value</td>
<td>4.263</td>
<td>3.820</td>
<td>17.496</td>
</tr>
<tr>
<td>(7)</td>
<td>1.766</td>
<td>1.766</td>
<td>5.297</td>
<td></td>
</tr>
<tr>
<td>(8)</td>
<td>4.213</td>
<td>3.818</td>
<td>18.093</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>theoretic value</td>
<td>2.964</td>
<td>2.686</td>
<td>14.274</td>
</tr>
<tr>
<td>(7)</td>
<td>0.774</td>
<td>0.774</td>
<td>2.322</td>
<td></td>
</tr>
<tr>
<td>(8)</td>
<td>2.970</td>
<td>2.692</td>
<td>15.134</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>theoretic value</td>
<td>1.981</td>
<td>1.797</td>
<td>10.560</td>
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<tr>
<td>(7)</td>
<td>0.370</td>
<td>0.370</td>
<td>1.109</td>
<td></td>
</tr>
<tr>
<td>(8)</td>
<td>1.985</td>
<td>1.799</td>
<td>11.180</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>theoretic value</td>
<td>1.705</td>
<td>1.545</td>
<td>9.188</td>
</tr>
<tr>
<td>(7)</td>
<td>0.285</td>
<td>0.285</td>
<td>0.855</td>
<td></td>
</tr>
<tr>
<td>(8)</td>
<td>1.708</td>
<td>1.548</td>
<td>9.886</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>theoretic value</td>
<td>1.626</td>
<td>1.474</td>
<td>9.169</td>
</tr>
<tr>
<td>(7)</td>
<td>0.255</td>
<td>0.255</td>
<td>0.766</td>
<td></td>
</tr>
<tr>
<td>(8)</td>
<td>1.628</td>
<td>1.475</td>
<td>9.552</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>theoretic value</td>
<td>1.403</td>
<td>1.272</td>
<td>8.331</td>
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<tr>
<td>(7)</td>
<td>0.194</td>
<td>0.194</td>
<td>0.581</td>
<td></td>
</tr>
<tr>
<td>(8)</td>
<td>1.404</td>
<td>1.272</td>
<td>8.452</td>
<td></td>
</tr>
</tbody>
</table>

* $\alpha_o = 120^\circ$, $\delta_o = 25^\circ$, $\kappa_o = 3^\circ$, $m_o = 5\mu m$.

The theoretic value is obtained by Eqs.(4).
Conclusion

1. The formulas of estimated precision presented in this paper are exact and can be used in practice. It seems to be not so ideal to estimate attitude precision with Eqs. (7).

2. The precision of $\alpha$, $\delta$, $\kappa$ is different from each other in the computation of attitude of the stellar camera, where the precision of $\delta$ is the best, and that of $\kappa$ is the worst.

3. The precision of $\delta$ depends on the location of the star images and the principal distance, and has nothing to do with the value of $\alpha$, $\delta$, $\kappa$.

4. The precision of $\alpha$ has something to do with $\delta$ and is inversely proportional to $\cos \delta$. The greater the value of $\delta$ is, the lower the precision of $\alpha$ is, but the $m_{\alpha} \cos \delta$ is constant.

5. The precision of $\kappa$ primarily depends on the size of the field where the star images distribute, and it is also under the influence of $\delta$. The greater the value of $\delta$ is, the greater the influence of it is, and the lower the precision of $\kappa$ is.

6. The relation of the precision and the number of the stars to be used in computation is shown in Fig. 3. From Fig. 3, it may be seen that when the number of the used stars which are of uniform distribution is more than about 20 stars, to increase the number of the used stars again is impossible to raise the precision of the result obviously.

Part of the test is done by E LIXUN and HUANG WENHIN.
Fig. 3. Precision vs Number of Stars

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