ESTIMATION FOR THE PRECISION OF THE CALCULATED ATTITUDE OF THE STELLAR CAMERA

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Comission I

Abstract

The estimated formulas for the precision of the attitude of the stellar camera determined by photogrammetry are presented in the paper,and compared with the corresponding empirical formulas in "Manual of Photogrammetry"(the fourth edition) by ASP. The tests show that these formulas in this paper are fit for the practical case.

In the satellite photogrammetry, the attitude of the stellar camera is often used to determine that of the terrain camera. The angular precision of the space direction of the stellar camera will therefore have a direct effect on the precision of the later operation. In order to pre-estimate the precision of the attitude of the stellar camera determined by photogrammetry. A set of practical estimated formulas is presented in this paper.

It is well known that for the photos by stellar camera, the equations of the collinearity condition take the following forms[1]:

$$\left.\begin{aligned} x &= -f\,\frac{a_1A+b_1B+c_1C}{a_3A+b_3B+c_3C} \\ y &= -f\,\frac{a_2A+b_2B+c_2C}{a_3A+b_3B+c_3C} \end{aligned}\right\} \qquad (1)$$

where

(x,y)=photographic coordinates of the star image;
$(a_1,b_1\ldots c_3)$=direction cosines of the image-space coordinate system, as a function of the attitudes;
$A=\cos\alpha\cdot\cos\delta$, $B=\sin\alpha\cdot\cos\delta$, $C=\sin\delta$;
α =right ascension of the star;
δ=declination of star;

After linearizing Eqs.(1) by Taylor series, the general matrix forms of observation equations are:

$$\underline{B}\,\underline{\Delta} - \underline{L} = \underline{V} \qquad \text{weight matrix } \underline{P} \qquad (2)$$

and then the normal equations can be written as

$$\underline{B}^T\underline{P}\,\underline{B}\,\underline{\Delta} - \underline{B}^T\underline{P}\,\underline{L} = \underline{0}$$

Then the least squares solution is

$$\underline{\Delta} = (\underline{B}^T\underline{P}\,\underline{B})^{-1}\underline{B}^T\underline{P}\,\underline{L} \qquad (3)$$

and the standard error of the ith unknown parameter is

$$m_i = \sqrt{Q_{ii}}\cdot m_0 \qquad (4)$$

where

Q_{ii}=the ith diagonal element in matrix $\underline{Q}$
$\underline{Q} = (\underline{B}^T\underline{P}\,\underline{B})^{-1}$, which is variance-covariance matrix;
m_0=the standard error of unit weight.

It is a well known method for computing the theoretical precision. But this classical method can only be used with the computation of attitude simultaneously.

Now we intend to derivate the formulas suitable for pre-estimating the attitude precision with the above method.

At first, we analyse the theoretical precision of attitude in the the case of 4 and 9 stars. Assume that the stars are of symmetric distribution as shown in Fig.1.

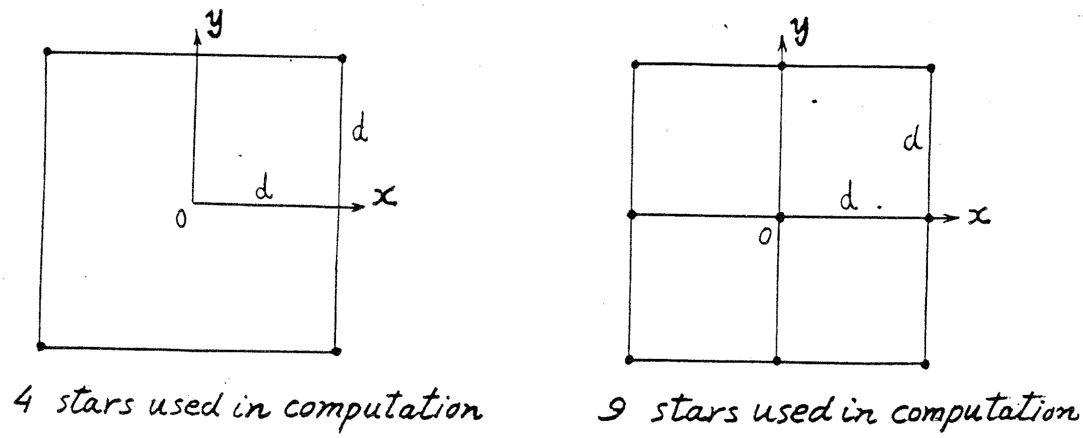

Fig. 1.

By a lot of derivation, according to the above classical method, we can get:

in the case of 4 stars

$$\left.\begin{aligned} Q_{11} &= \frac{m^2_{\alpha_0}}{m^2_0} = \frac{1}{\cos^2\delta_0}\cdot Q_{22} \\ Q_{22} &= \frac{m^2_{\delta_0}}{m^2_0} = \frac{f^2}{4(2d^4+f^4+2f^2d^2)} \\ Q_{33} &= \frac{m^2_{K_0}}{m^2_0} = \frac{1}{8d^2} + tg^2\delta_0\cdot Q_{22} \end{aligned}\right\} \qquad (5)$$

in the case of 9 stars

$$\left.\begin{aligned} Q_{11} &= \frac{m^2_{\alpha_0}}{m^2_0} = \frac{1}{\cos^2\delta_0}\cdot Q_{22} \\ Q_{22} &= \frac{m^2_{\delta_0}}{m^2_0} = \frac{f^2}{10d^4+9f^4+12f^2d^2} \\ Q_{33} &= \frac{m^2_{K_0}}{m^2_0} = \frac{1}{12d^2} + tg^2\delta_0\cdot Q_{22} \end{aligned}\right\} \qquad (6)$$

The varying rule of precision for the calculated attitude in Eqs.(5) and (6) is different from that shown in the corresponding empirical formulas in "Manual of Photogrammetry"[3]. These empirical formulas are:

$$\left.\begin{aligned} \delta_\varphi &= \frac{\sqrt{2}}{2n-3}\cdot\frac{m_x}{f} \\ \delta_K &= \frac{\sqrt{2}}{2n-3}\cdot\frac{m_x}{L} \end{aligned}\right\} \qquad (7)$$

It will be seen from this that Eqs.(7) seem to be not so ideal to provide estimated precision and show its varying rule.

But the given formulas (5) and (6) are only suitable for the above case. In practice, there need be a set of general formulas that can be suitable for all cases. In fact, it's impossible to derive the rigorous formulas, we can only get approximate formulas. Assume that n stars are distributed as shown in Fig.2, ahd then derive the estimaled formulas for n stars. These formulas can be used instead of the general ones approximately.

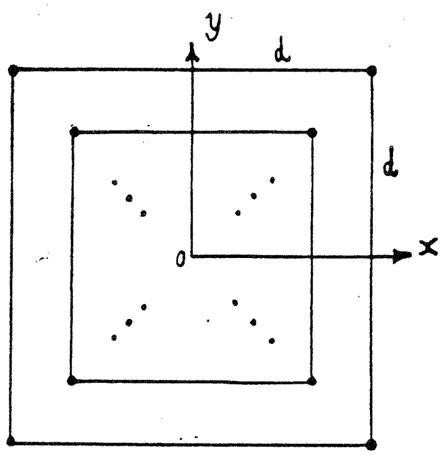

Fig. 2.

In Fig.2, the locations of n stars takes the style of Fig.1, with different values d, we get

$$\left.\begin{aligned} Q_{11} &= \frac{m^2_{\alpha_0}}{m^2_0} = \frac{1}{\cos^2\delta_0}\cdot Q_{22} \\ Q_{22} &= \frac{m^2_{\delta_0}}{m^2_0} = \frac{15n^3f^2}{2d^2(n+2)(n+4)[(3n^2+12n-16)d^2+5n^2f^2]+15n^4f^4} \\ Q_{33} &= \frac{m^2_{K_0}}{m^2_0} = \frac{3n}{2(n+2)(n+4)d^2} + tg^2\delta_0\cdot Q_{22} \end{aligned}\right\} \quad (8)$$

If we don't consider the principal distance is much longer than d, we may get

$$Q_{22} = \frac{3n}{2(n+2)(n+4)d^2+3n^2f^2} \qquad (9)$$

In order to check the exactness of these formulas, some tests are made in Electronic Computer DJS-220. The results are listed in the following table.

Comparison of the Estimated Precision*

number of used stars	mode	m''_{α_0}	m''_{δ_0}	m''_{κ_0}
5	theoretic value	4.″263	3.″820	17.″496
	(7)	1.766	1.766	5.297
	(8)	4.213	3.818	18.093
9	theoretic value	2.964	2.686	14.274
	(7)	0.774	0.774	2.322
	(8)	2.970	2.692	15.134
15	theoretic value	1.981	1.797	10.560
	(7)	0.370	0.370	1.109
	(8)	1.985	1.799	11.180
18	theoretic value	1.705	1.545	9.188
	(7)	0.285	0.285	0.855
	(8)	1.708	1.548	9.886
20	theoretic value	1.626	1.474	9.169
	(7)	0.255	0.255	0.766
	(8)	1.628	1.475	9.552
25	theoretic value	1.403	1.272	8.331
	(7)	0.194	0.194	0.581
	(8)	1.404	1.272	8.452

* $\alpha_0 = 120°$, $\delta_0 = 25°$, $\kappa_0 = 3°$, $m_0 = 5\,\mu m$.
The theoretic value is obtained by Eqs.(4) .

Conclusion

1. The formulas of estimated precision presented in this paper are exact and can be used in practice. It seems to be not so ideal to estimate attitude precision with Eqs.(7).

2. The precision of α_o , δ_o , κ_o is different from each other in the computation of attitude of the stellar camera, where the precision of δ_o is the best, and that of κ_o is the worst.

3. The precision of δ_o depends on the location of the star images and the principal distance, and has nothing to do with the value of α_o, δ_o, κ_o .

4. The precision of α_o has something to do with δ_o and is inversely proportional to $\cos\delta_o$. The greater the value of δ_o is, the lower the precision of α_o is. But the $m_{\alpha_o} \cdot \cos\delta_o$ is constant.

5. The precision of κ_o primarily depends on the size of thefield where the star images distribute, and it is also under the influence of δ_o . The greater the value of δ_o is, the greater the influence of it is, and the lower the precision of κ_o is.

6. The relation of the precision and the number of the stars to be used in computation is shown in Fig.3. From Fig.3, it may be seen that when the number of the used stars which are of uniform distribution is more than about 20 stars, to increase the number of the used stars again is impossible to raise the precision of the result obviously.

Part of the test is bone by E LIXUN and HUANG WENBIN.

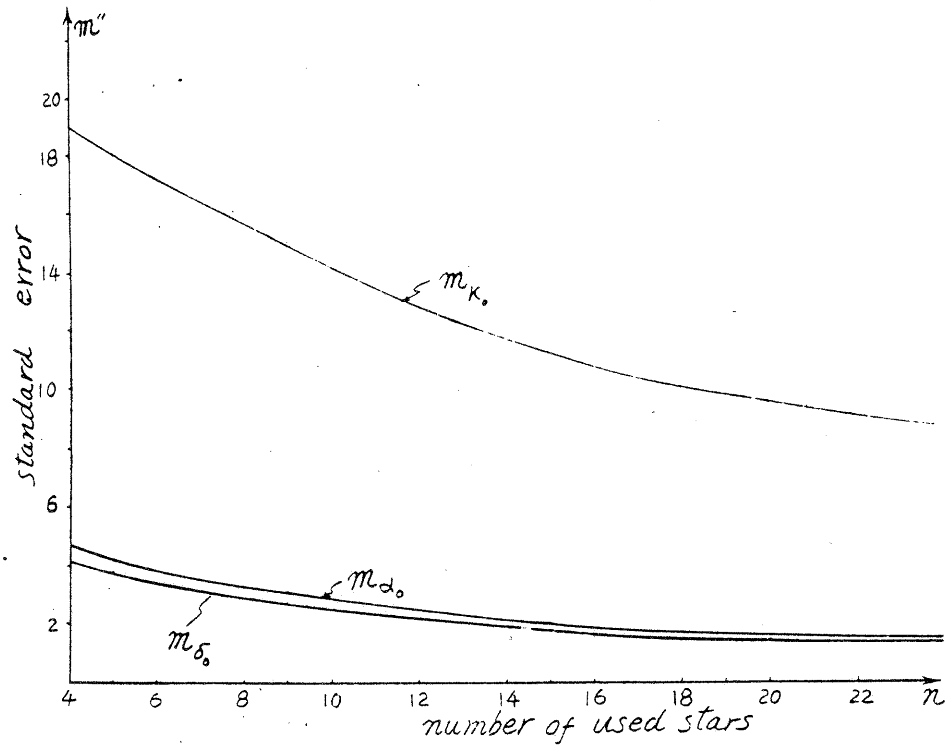

Fig.3. Precision vs Number of Stars

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【3】 American Society of Photogrammetry,
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