THE INFORMATION CONTENT OF RADAR IMAGES, MODELLED ACCORDING TO CORRELATION PROPERTIES OF THE SIGNAL

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1. INTRODUCTION

By treating radar imagery data extraction as a communication system in which the target information is the source and the communication channel is modelled according to the noise properties of the radar sensing process, the information content of the image can be expressed in terms of the average information rate which can be transmitted through that noisy channel. In this paper the information rate is derived using fundamental theorems of information theory for a commonly used model of the target statistical properties, which allows to gain insight into radar image formation effects like "look" summation and radiometric resolution. It is shown that the information content per spatial resolution element of the image is small for a low number (e.g. four) of looks. As a measure of image quality an alternative definition of radiometric resolution, based on rate distortion theory, is introduced which takes account of image correlation properties. Similar derivations has been performed by Frost et al for uncorrelated imagery (ref. 3).

2. SAR IMAGE STATISTICAL PROPERTIES

2.1 Image Pixel Statistics

The statistical properties of speckled SAR imagery of homogenous areas can be conveniently expressed by writing the observed pixel intensivity (I_i) in the form of:

$$I_i = S_i \frac{n}{2L} \qquad (1)$$

where:

S_i = true (mean) reflectivity of the area
L = number of independent samples of intensity averages (looks) to form I_i
n = random variable describing the speckle.

The propability density function (pdf) of n, assuming power detection is given by:

$$P(n) = \frac{n^{L-1}\, e^{-n/2}}{(L-1)!\; 2^L} \qquad \text{for } n \geq 0 \qquad (2)$$

which is a chi-square distribution. Note that expression (1) separated the observed intensity into:

a) a mean value component and
b) a signal independent multiplicative noise component.

The mean and variance of the component n are respectively:

$$E(n) = 2L \quad (3a)$$
$$Var(n) = \sigma_n^2 = 4L \quad (3b)$$

The variance of I_i is therefore:

$$Var\ I_i = \sigma^2(I_i) = S^2_{i/L} \quad (4)$$

2.2 SAR Image Statistics

From the image pixel statistics derived in par. 2.1 the covariance of the image data samples, identified by their row and column coordinates (i,p) and j,q), is given by:

$$E\{(I_{ij}-m)(I_{pq}-m)\} = E(I_{ij},I_{pq}) - m^2 \quad (5)$$

where,

$$m = \text{image mean value} = E(S_{ij})$$

and

$$E\{I_{ij}\ I_{pq}\} = E\{S_{ij}\ S_{pq}\}\{\frac{1}{4L^2}\ E(n_{ij}\ n_{pq})\} \quad (6)$$

For uncorrelated speckle,

$$\frac{1}{4L^2}\ E(n_{ij}\, n_{pq}) = (1+\frac{1}{L})\delta(i-p)\delta(j-q)$$

Hence with (6),

$$E\{I_{ij}\ I_{pq}\} = E\{S_{ij}\ S_{pq}\} + \frac{1}{L}\ E\{S^2_{ij}\}\ \delta(i-p)\delta(j-q) \quad (7)$$

where $E\{S^2_{ij}\}$ is the unspeckled image variance,

$$E\{S^2_{ij}\} = m^2+\sigma_s^{\ 2} \quad (8)$$

The image variance is thus given by:

$$E\{(I_{ij}-m)^2\} = \sigma_s^{\ 2}+\ \sigma_p^{\ 2}$$

where $\sigma_p^{\ 2} = \frac{m^2+\sigma_s^{\ 2}}{L}$ (9)

is the image variance due to speckle.

A commonly used model describing the statistical properties of an unspeckled (i.e. target) image is given by a two dimensional first order Gauss-Markov process of which the covariance matrix values are given by:

$$\phi_{k,p} = E(S_{i,j}-m)(S_{i+k,j+p}-m) = \sigma_s^{\ 2}\rho_1^{\ k}\rho_2^{\ p} \quad (10)$$

where:

$\sigma_s^{\ 2}$ = image variance = $E(S_{ij}^{\ 2})-m^2$
ρ_1 = correlation coefficient between row neighbouring pixels
ρ_2 = correlation coefficient between column neighbouring pixels.

The covariance matrix values of an image effected by uncorrelated speckle is then given by:

$$R(k,p) = E(I_{i,j}-m)(I_{i+k,j+p}-m) = \sigma_s^2\rho_1^k\rho_2^p+\sigma_p^2\delta(k)\delta(p) \qquad (11)$$

2.3 Image Transform Statistics

Applying a two dimensional Karhunen-Loeve or Hotelling unitary transforamtion of size N^2 to the image defined in the preceeding paragraph, it can be shown (Ref. 1) that the first and second order statistics of the transform coefficients, $Z_{n,\ell}$ are repectively given by:

$$E(Z_{n,\ell}) = \begin{cases} 0 \text{ for } n,\ell \neq 0 \\ mN \text{ for } n = \ell = 0 \end{cases} \qquad (12a)$$

$n,\ell = 0,\ldots.,N-1$

$$E\{(Z_{n,\ell}-\mu_{n,\ell})(Z_{r,s}-\mu_{r,s})\} = \{\sigma_s^2\lambda_n\beta_\ell+\sigma_p^2\}\ \delta(n-r)\ \delta(\ell-s) \qquad (12b)$$

for $n,\ell = 0,\ldots.N-1$

where:

$$\mu_{n,\ell} = E(Z_{n,\ell})$$

$$\lambda_n = \sum_{\ell=0}^{N-1} E\{Z_{n,\ell}-\mu_{n,\ell}\}^2\ /N\sigma_s^2-\sigma_p^2/\sigma_s^2$$

$$\beta_\ell = \sum_{n=0}^{N-1} E\{Z_{n,\ell}-\mu_{n,\ell}\}^2/N\sigma_s^2-\sigma_p^2/\sigma_s^2$$

What is learned from equation (12) is that:

a) the transform coefficients are uncorrelated and their variances are product separable in row and column indices.

b) the variance of the transform coefficients due to speckle is the same for all coefficients and hence the speckle component can be regarded as an additive uncorrelated random variable to the coefficient value given by the unspeckled image.

The properties of the second order statistics given by (12b) are satisfied for a two dimensional cosine transform of transform size $N \to \infty$ and the variance of the transform coefficients is given by the two dimensional Fourier transform of the image covariance function given by (11). Hence,

$$E\{Z_{n,\ell}-\mu_{n,\ell}\}^2 = \sum_{k=-\infty}^{\infty}\sum_{p=-\infty}^{\infty} R_{k,p}\exp-i\{\frac{k\pi n}{N} + \frac{\ell\pi n}{N}\} \qquad (13)$$

It follows for λ_n and β_ℓ defined in (12b):

$$\lambda_n = \frac{1-\rho_2^2}{1+\rho_2^2-2\rho_2\cos(\frac{\pi n}{N})} \qquad (14a)$$

$$\beta_{\ell} = \frac{1-\rho_1^{2}}{1+\rho_1^{2}-2\rho_1 \cos(\frac{\pi\ell}{N})} \tag{14b}$$

where the coefficient indices n, ℓ are in the range of 0,1,...., N-1 with $N \to \infty$.

3. INFORMATION CONTENT OF SAR IMAGES

Considering the set of independent random variables derived from the unspeckled image by a two dimensional K.L transformation as the information sources and the speckle effect as being introduced by a continuous memoryless communication channel we can derive the amount of information which can be transmitted through these channels by computing its average mutual information measure.

Because of the definition of the image model the distribution of all data sources (the transform coefficients) is Gaussian with a given variance while the aditive speckle noise approaches the Gaussian distribution because of its contribution from many independent noise sources (central limit theorem).

However it should be noted that the closeness of the Gaussian distribution for speckle noise is a function of the number of independent sample averages (L) and the amount of image correlation. Its approximation improves with increasing value of L and increasing values of the correlation coefficients ρ_1 and ρ_2.

Assuming the Gaussian distribution of speckle and data sources, its corresponding mutual information measure yields a lower bound to the mutual information derived from a non-Gaussian speckle distribution.

Because the derivation of the mutual information for the Gaussian channel is relatively straightforward it is pursued first. Thereafter an estimation of the exact value of the mutual information is obtained by interpolation of the differences between the mutual information of the non-Gaussian and the Gaussian channel for the conditions of uncorrelated imagery ($\rho_1=\rho_2=0$) and fully correlated imagery ($\rho_1=\rho_2=1$). For the Gaussian channel the mutual information equalizes its channel capacity given by:

$$C = \frac{1}{2N^2} \sum_{n=0}^{N-1} \sum_{\ell=0}^{N-1} \log_2 \frac{E\{Z_{n,\ell}-E(Z_{n,\ell})\}^2}{\sigma_p^2} \tag{15}$$

With $N \to \infty$, we can apply equation (14) and (15) becomes:

$$C = \frac{1}{2\pi^2} \int_0^{\pi}\int_0^{\pi} \log_2 \left[1+ \frac{\sigma_s^2(1-\rho_1^2)(1-\rho_2^2)}{p^2(1+\rho_1^2-2\rho_1 \cos w_1)(1+\rho_2^2-2\rho_2^2 \cos w_1}\right] dw_1\, dw_2 \tag{16}$$

where $w_1 = \frac{\pi n}{N}$ and $w_2 = \frac{\pi \ell}{N}$

The double integral (16) can be solved analytically for one variable, say for w_1. The result is given by:

$$C = \frac{1}{2} \log_2 \rho_1 - \frac{1}{2\pi} \int_0^{\pi} \log_2 b(w_2) dw_2 \tag{17}$$

where b(w) is defined by:

$$b = a^{-1}\{1-(1-a^2)^{\frac{1}{2}}\}$$

$$a = \frac{2\rho_1(1+r)\ F(w_2)}{(1+\rho_1^2)(1+r)\ F(w_2) + L(1-\rho_1^2)(1-\rho_2^2)}$$

$$r = \frac{m^2}{\sigma_s^2}$$

$$F(w_2) = 1+\rho_2^2-2\rho_2 \cos w_2$$

The channel capacity for a given value of r (image mean square to variance ratio) as a function of image correlation coefficient ($\rho = \rho_1 = \rho_2$) with the number of "looks" (L) as a parameter is shown in figure 1a.

From the parametric equations of (20) it can further be derived that,

a) the ratio $\frac{L}{1+r}$ can be taken as a independent variable.

b) the channel capacity for an uncorrelated image is given by:

$$C_{\rho=0} = \tfrac{1}{2} \log_2 \left(1+\frac{L}{1+r}\right) \quad \text{(bits/sample)} \tag{18}$$

The mutual information of an uncorrelated image for the chi-square speckle distribution is derived in the annex, which results yield values which are 30 to 40% higher than those for the Gaussian channel. The notion that the difference reduces monotonically to zero for completely correlated imagery (i.e. $\rho_2 = \rho_2 = 1$), provides a mean to estimate the exact amount of mutual information (dashed lines in fig. 1a).

3.1 Information Content per unit area

The parameter which specifies the information content per unit image area is particularly useful in comparing systems with different number of looks. The information content per unit area is given by:

$$\frac{C}{d_a\ d_r} = \frac{C_o}{d_{oa}\ d_{or}} \frac{\{L_{oa}(1-\gamma_a)+\gamma_a\}\{L_{or}(1-\gamma_r)+\gamma_r\}}{\{L_a(1-\gamma_a)+\gamma_a\}\{L_r(1-\gamma_r)+\gamma_r\}} \tag{19}$$

where:

d_a, d_r = spatial resolution in repectively azimuth and range direction.

d_{oa}, d_r = area of specified "unit area" of dimension d_{oa} in azimuth and d_{or} in range.

C_o = Capacity relvant to "unit area".

L_a, L_r = number of looks in respectively azimuth and range direction relevant to d_a and d_r.

L_{oa}, L_{or} = number of looks in respectively azimuth and range direction relevant to d_{oa} and d_{or}.

γ_a, γ_r = look overlap in respectively azimuth and range direction.

The results of (19) are depicted in figure 1b for the condition $L_{or} = 1$, $L_{oa} = 4$, $\gamma_a = \gamma_r = 1/3$, $L_a = 4L_r$ and the total number of looks $L = L_a L_r = 4L_r^2$ for $\rho = 0.7$ and $r = 3$ (high variance image) and $r = 9$ (low variance image).

Figures 1a and 1b show that the information content per unit area is low for relatively low number of looks. Taking the typical case of the Seasat or ERS-1 radar sensor, when the range and azimuth resolution of the processed data is approximately 25m, it is evident that for a typical correlation coefficient of 0.7, the information content is only 0.2 - 0.35 bits per resolution square area with four look processing. Note that this is only about 50% of the information per unit area provided by an optical sensor using 8 bit pixel quantization with a spatial resolution of 80x80m (like Landsat III). In other words the ratio between the information content per 8 bit sample of an optical and a radar sensor is at least 15 to 1.

4. RADIOMETRIC RESOLUTION

Rate distortion theory learns that for an information source with variance σ^2, transmitting through a communication channel with C bits/sample capacity, the minimum mean square error of the reconstructed signal is given by:

$$D(C) = \sigma^2 \, 2^{-2C} \qquad (20)$$

Applying (17) to each of the channels with the transform coefficient $Z(n,\ell)$ as information source, the channel capacity is, using (15) and (12b):

$$C(n,\ell) \geq \tfrac{1}{2} \log_2 \left[\frac{\sigma^2(n,\ell) + \sigma_p^2}{\sigma_p^2} \right] \qquad (21)$$

where: $\sigma^2(n,\ell) = \sigma_s^2 \lambda_n \beta_\ell$

The minimum error per channel is then upperbounded by:

$$D(n,\ell) = \frac{\sigma^2(n,\ell)\,\sigma_p^2}{\sigma^2(n,\ell) + \sigma_p^2} \qquad (22)$$

The total distortion can be derived for transform size $N \to \infty$, for which (14) applies. It follows:

$$D_T = \frac{\sigma_p^2 \sigma_s^2 (1-\rho_1^2)(1-\rho_2)^2}{\pi^2} \int_0^\pi \int_0^\pi \frac{dw_1\, dw_2}{\sigma_s^2(1-\rho_1^2)(1-\rho_1^2) + \sigma_p^2 F_1(w_1) F_2(w_2)} \quad (23)$$

where:

$$F_i(w_i) = 1+\rho_i^2 - 2\rho_i \cos w_i$$

The double integral (20) can be solved analytically for one variable say w_1. The result is given by:

$$\frac{D_T}{m^2} = \frac{(1+r)(1-\rho_1^2)(1-\rho_2^2)}{\pi r L} \int_0^\pi \frac{dw_2}{\{aF_2^2(w_2) + bF_2(w_2) + c\}^{\frac{1}{2}}} \quad (24)$$

where:

$$a = \left(\frac{1+r}{L}\right)^2 (1-\rho_1^2)^2$$

$$b = \left(\frac{1+r}{L}\right)(1-\rho_1^4)(1-\rho_2^2)$$

$$c = (1-\rho_1^2)^2 (1-\rho_2^2)^2$$

$$d = m^2/\sigma_s^2$$

A commonly used measure of radiometric resolution is given by (expressed in dB)

$$r_d = 10 \log_{10} \left(1+\frac{\sqrt{D_T}}{m}\right) \quad (25)$$

Using (24), the value of r_d has been numerically evaluated as a function of image correlation coefficient $\rho = \rho_1 = \rho_2$ with (r) and (L) as parameters (see figure 2).

Note that for $\rho = 0$, (25) becomes:

$$(r_d)_{\rho=0} = 10 \log_{10} \left[1+ \{ r(1+\frac{L}{1+r})\}^{-\frac{1}{2}}\right] \quad (26)$$

Applied on a sample per sample basis (25)becomes with (3):

$$r_d^1 = 10 \log_{10} (1+{}^1/_{\sqrt{L}}) \quad (27)$$

which value is often used as a measure of image quality. The minimum distortion value however, which is a function of look number and the image characterisation expressed by its covariance matrix, provides a more realistic measure of image quality. This is because image interpretability obviously does depend on (local) image first and second order statistics. The results of figure 2 show a dramatic improvement of radiometric resolution over the sample radiometric resolution for low numbers of looks. The distortion bound ranges from ~ 1.1 dB at r=3 (high variance image) to ~ 0.9 dB for r=9 (low variance image) for four looks (L=4) and a typical correlation coefficient (ρ) of 0.7.

The distortion upperbound can be readily achieved by performing an image processing operation consisting of, 2D-KL transform, transform coefficient weighting and 2D-reverse K.L. transform. The transform coefficient weighting factor W(n,) is derived from (22), i.e.

$$W(n,\ell) = \frac{\sigma^2(n,\ell)}{\sigma^2(n,\ell)+\sigma^2_p} \qquad (28)$$

where $\sigma^2(n,\ell) = \sigma^2{}_s\lambda_n\beta_\ell$ is the transform coefficint variance of the unspeckled image which can be determined from the transform coefficients $Z(n,\ell)$ according to (12b).

The selection of transform size is a compromise between adaptability to statistical variations from block to block and the accuracy of weighting factor estimation. Adequate block size values are 8x8 or 16x16 image pixels, while the transform can efficiently be carried out by the two-dimensional cosine transform which performs very closely to a KL transform.

5. SUMMARY

The results of the analysis on the information content of SAR imagery in a homogeneous area show that there is little information per spatial resolution element as compared to e.g. imagery generated by optical sensors with the same spatial resolution. This means in particular that terrain classification procedures, which operate on a sample basis are unworkable for SAR imagery (unless the number of looks is very large). The dependence of information content per unit area on the number of looks is consistent with experience in SAR image interpretation (e.g. image interpretability of homogeneous area is about equal in the range of say 1 to 8 looks with a slight preference for low number of looks) (Ref. 2).

However, using the estimates of the spatial sample correlation within a relatively small area (e.g. 64 or 256 samples) the radiometric accuracy of the image sample can be dramatically improved as compared to the single sample accuracy. For example 4 look SAR imagery of moderate activity (mean square to variance ratio of four) gives 1 dB radiometric accuracy using the spatial correlation information, while at least 16 looks are needed to achieve the same radiometric acuracy measured on a sample basis. A two-dimensional cosine transformation followed by adaptive coefficient weighting can be applied to achieve this performance.

6. REFERENCES

1) S.M. Melzer: "An Image Transform Coding Algorithm based on a Generalised Correlation Model.
SPIE Vol. 149 Applications of Digital Image Processing (1978).

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3) V.S. Frost et al: "The Information Content of SAR Images of Terrain". IEEE Transact. on Aerospace and Electonic Systems AES-16, No. 5, Sept. 1983 (768-774).

ANNEX

Information content for uncorrelated image samples

The average information content per sample, in case of uncorrelated samples, is given by the mutual information measure:

$$I(X,Y) = H(Y) - H(Y/X) \tag{A1}$$

where:

$$H(Y) = -\int_0^\infty f_y(y) \log_2 f_y(y)\, dy \tag{A2}$$

$$H(Y/X) = -\int_0^\infty\int_0^\infty f_p(y/x)\, f_x(x) \log_2 f_p(y/x)\, dxdy \tag{A3}$$

and

$$f_y(y) = \int_0^\infty f_p(y/x)\, f_x(x)\, dx$$

where f_x, f_p, f_y are respectively the probability density functions of the unspeckled image, the speckle and the speckled image. It can be shown that for the conditional pdf of the speckle component derived from (2),

$$H(Y/X) = \log_2(L-1)! - \log_2 L - \left(\frac{L-1}{\ln 2}\right)\left[\sum_{j=1}^{L-1} \frac{1}{j} - C\right] + \int_0^\infty f_x(x) \log_2 x\, dx \tag{A4}$$

where C = 0.5772... is Euler's constant.

The entropy values of (A2) and (A4) has been numerically evaluated for the source intensity distribution given by,

$$f_x(x) = f_g(x) + f_g(-x) \quad \text{for } x \geq 0$$

where $f_g(x)$ is the Gaussian distribution with mean μ and variance σ_s^2. The comparison with the computed lower bound of I(X,Y) is performed with the same value of

$$r = \frac{m^2}{\sigma_s^2}, \text{ where } m=E\{f_x(x)\} \text{ and } \sigma_s^2 = E\{f_x(x)\}^2 = E\{f_g(x)\}^2$$

The results are given in the table below.

$r=m^2/\sigma_s^2$	L	I(X,Y)	$C_{\rho=0}$
3	1	0.22	0.161
	5	0.74	0.585
	10	1.27	0.904
9	1	0.09	0.069
	5	0.37	0.292
	10	0.64	0.500

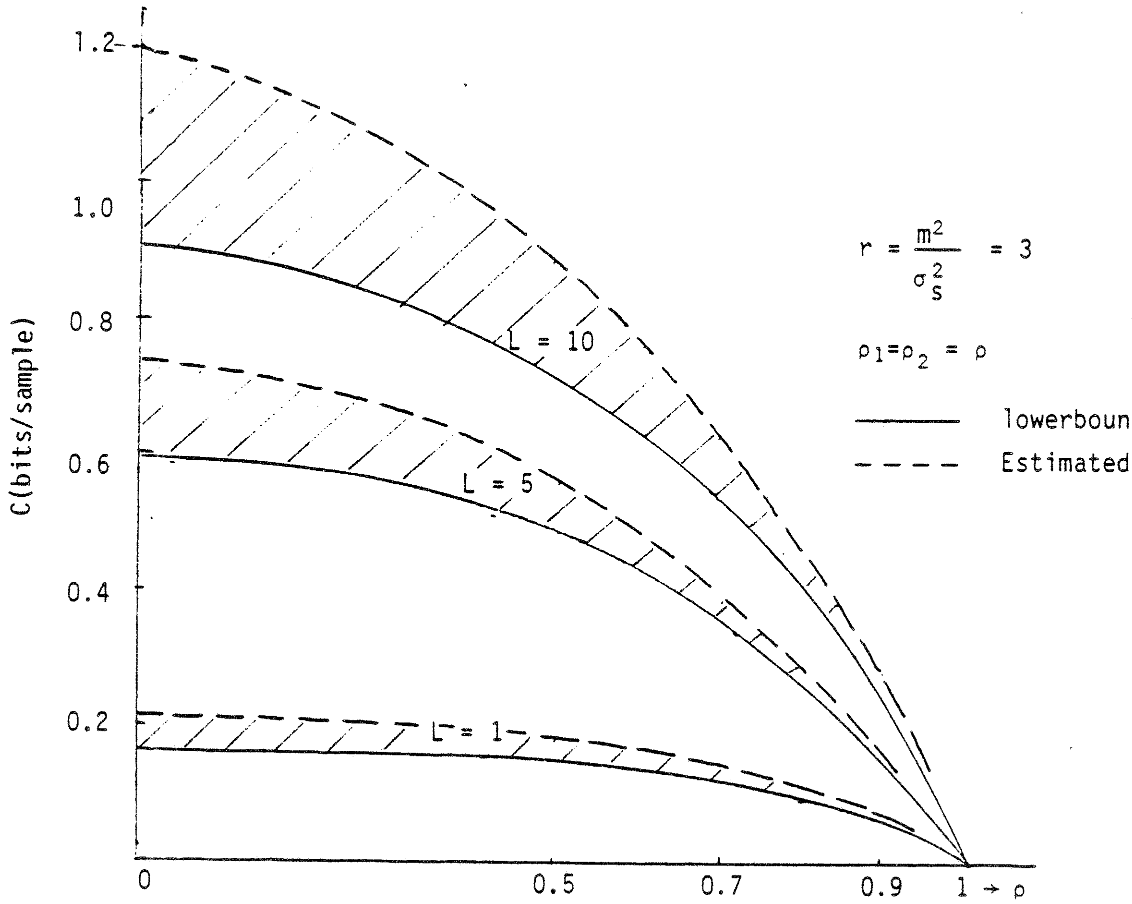

Figure 1a Information content per sample.

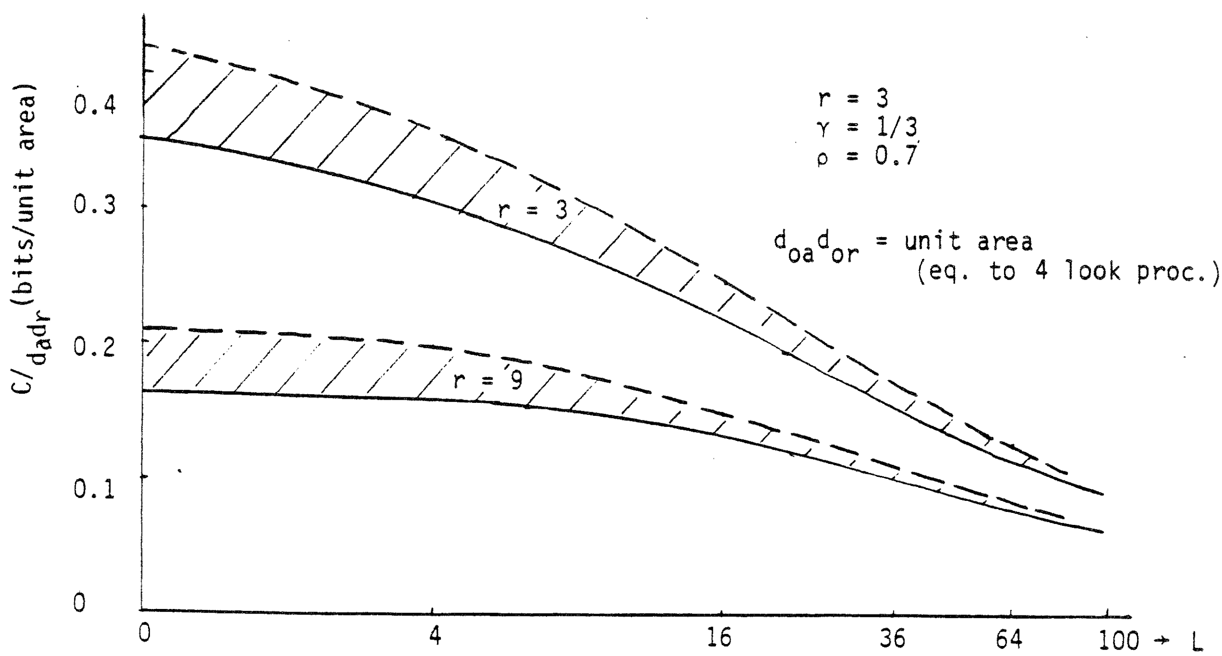

Figure 1b Information content per unit area

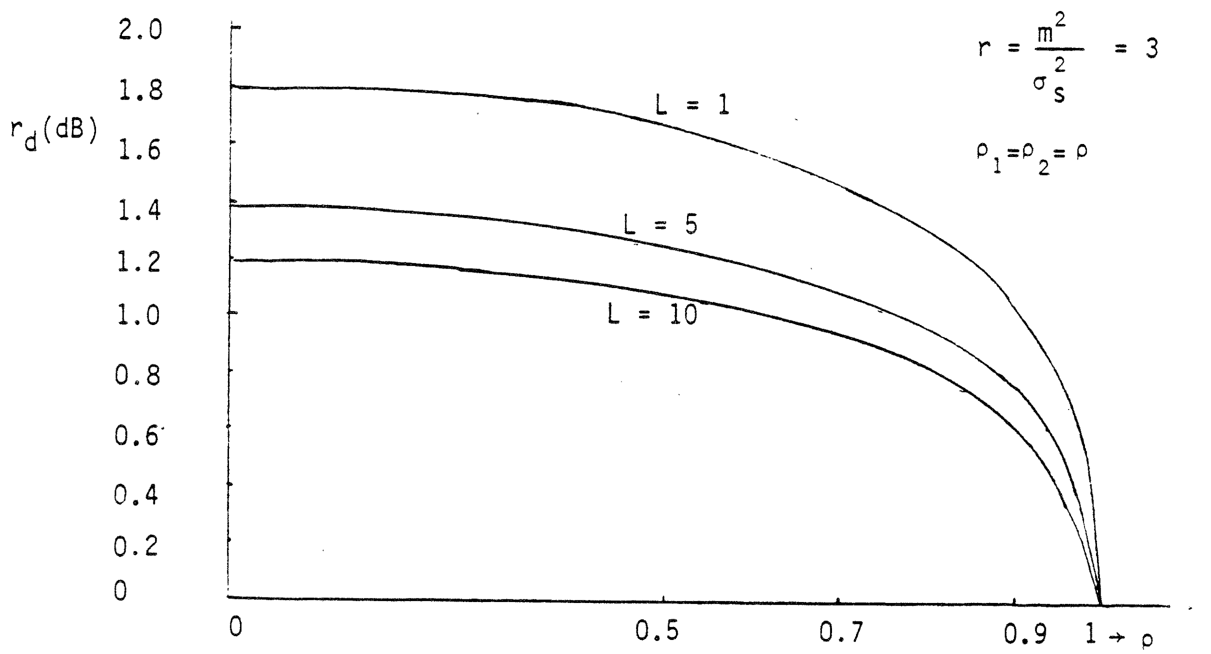

Figure 2a Radiometric distortion vs correlation coefficient (ρ)

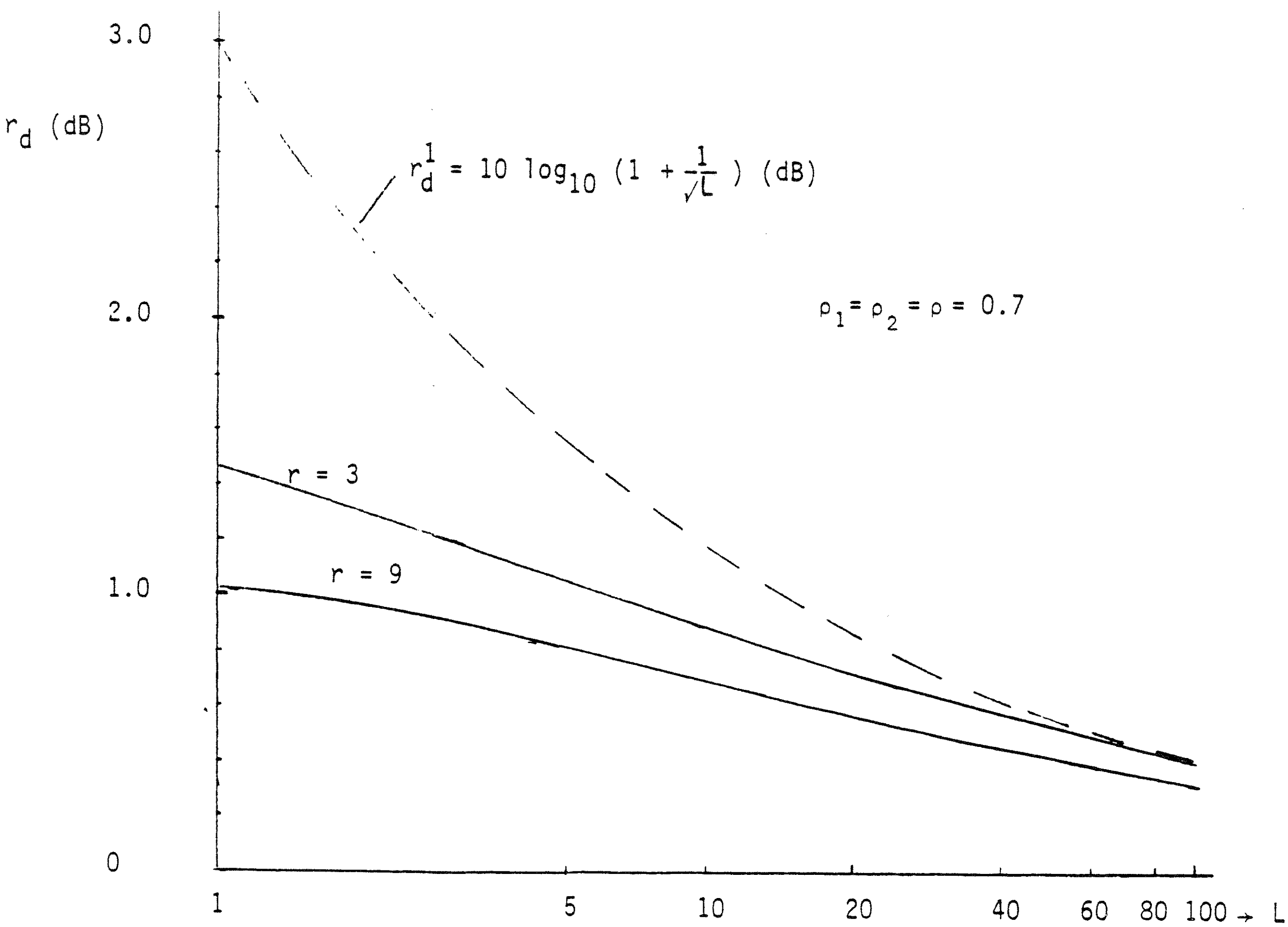

Figure 2b Radiometric distortion vs n$^{\underline{o}}$ of looks (L)

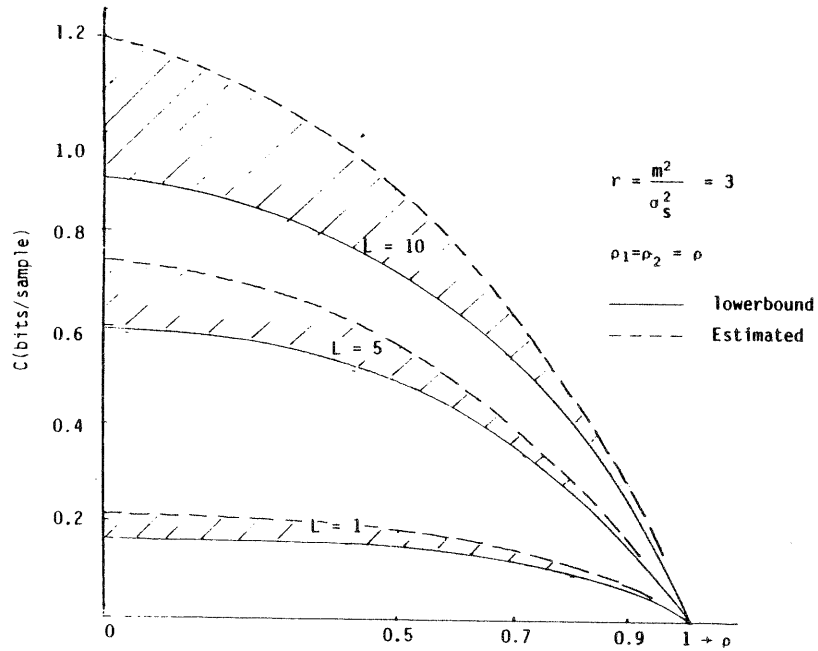

Figure 1a Information content per sample.

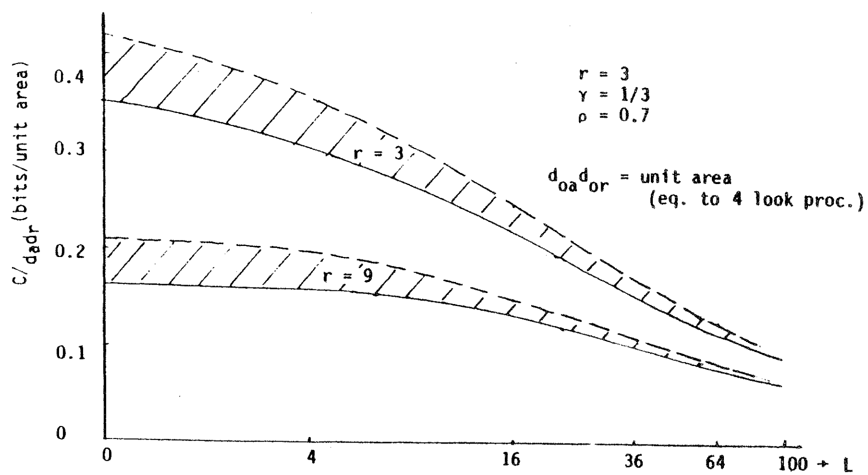

Figure 1b Information content per unit area

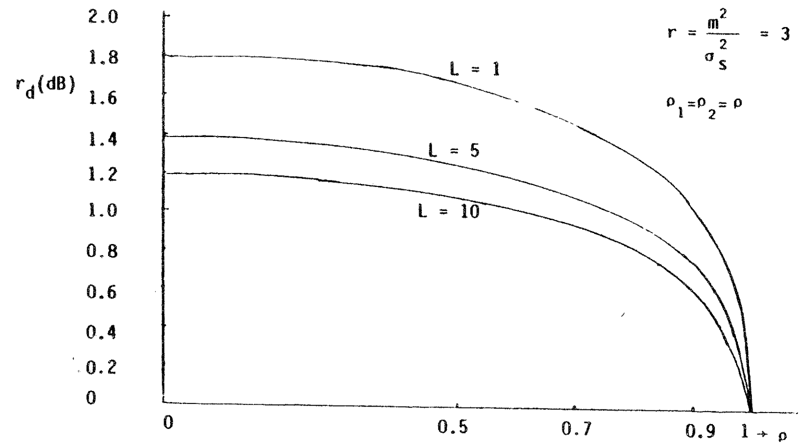

Figure 2a Radiometric distortion vs correlation coefficient (ρ)

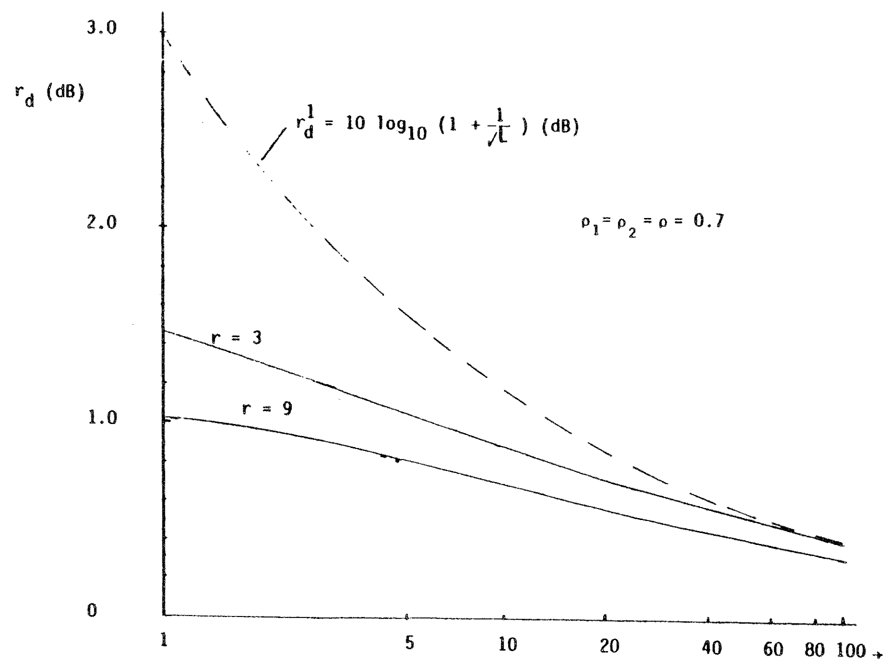

Figure 2b Radiometric distortion vs n° of looks (L)