"CIP"
A PROGRAM PACKAGE FOR INTERPOLATION AND PLOTTING OF DIGITAL HEIGHT MODELS
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#### **ABSTRACT**

A program package for interpolation of digital height models and their applications, together with plotting software, has been developed by WILD. The input comprises arbitrarily distributed reference points captured by photogrammetric or geodetic devices. The interpolation is based upon a triangular grid using the method of finite elements (Zienkiewicz functions). From these triangles, a rectangular grid and/or contour lines are derived. Two different breakline types (sharp and round) as well as structure lines are considered. Additional application programs, for example for the computation of profiles, volumes and perspectives, will be available. The programs are written in FORTRAN and need very short computation times. A very sophisticated user-interface menu together with "HELP" facilities provide an easy use of the program package even for inexperienced users. The programs are implemented on the NOVA 4/X and VAX computers.

### INTRODUCTION

The program package called CIP (Contour Interpolation Program) computes a high-quality Digital Height Model (DHM). It is the result of continuous development and the experience of many years as well as of tests of several computing and interpolation techniques (Zumofen, Leoni 1977). The following technique is used here: A given pointfield, within which the distribution of points may be arbitrary, is divided into triangles. This technique was chosen in order to achieve a continuous surface reproducing the greatest possible number of fine details. Geomorphological breaklines, which can be of different types ("sharp" for artificial, "round" for natural breaks), are considered as well as structure lines (Breaklines define discontinuities of the surface). The triangular sub-division has the advantage that the original data are used directly and so approximate the ground surface in a better way than does square-grid interpolation. Also the computing time is much lower than in those square-grid methods which take breaklines into account.

Each triangle is subdivided into sixteen sub-triangles. The method of finite elements is used here, together with the Zienkiewicz function. In each point of a main triangle the normal to the surface is computed from all surrounding points, a procedure which can be interpreted as the generation of a curved surface. Filtering can be used in two different ways, firstly for smoothing the surface within the limits defined by the predetermined standard deviation of the reference points and, secondly, for the smoothing of the contour lines, where a bandwidth can be entered. Additionally, Akima interpolation may be used during plotting.

Many applications are possible, e.g. the computation of a rectangular grid, the computation of contour lines with many different features or the interpolation of single points (compute Z from a given pair of X, Y coordinates). Also, one can compute the volume between a given horizontal reference plane and a part of the surface enclosed by a polygon (option).

The program is written in FORTRAN, has approximately 22 thousand statements and is divided into eight independent parts, which are controlled automatically by a very user-friendly human interface. Of course it is also possible to run the single parts independently. The computation time is very short because the latest computing techniques are used. CIP can be executed on VAX computers and on the NOVA 4/X, which requires a minimum storage capacity of 128K bytes (1 byte of 8 bits).

### DATA ACOUISITION

Data for the CIP program are arbitrarily distributed reference points. They can be measured on analytical plotters such as the WILD Aviolyts AC1 or BC1, in grid or profile mode. A new option for data capturing is a new WILD program for PROGRESSIVE SAMPLING, which already considers the interpolation with triangles. Here in a very sophisticated way as little data as possible are measured which describe the terrain optimally. This program will be described separately. The data may also come from analog instruments or from digitized contours or from tacheometric surveys. The data must be in files with a maximum length of 32'000 records, where one record contains an arbitrary 8-character feature code, X, Y, Z and an arbitrary 2-digit control code. The total number of reference points is dependent only on the available disk space. Feature and control codes may be omitted. Separate files are prepared for breakline and structure-line points. A special file can also be prepared for "dead areas" (inner windows) in which contours are not to be drawn (e.g., lakes, houses etc.) The dead areas are described by polygons.

# THE INTERPOLATION ALGORITHM

The interpolation is subdivided into two parts, first for computing the terrain surface (smoothing included), second for deriving and smoothing the contours. The interpolation is only performed within the area where points are measured, so no special polygon must be entered.

a) Interpolation of the surface: The interpolation method is based on a triangular network in which the reference points are the vertices. The procedure proposed by Delauney (explained e.g. in Gottschalk 1981) is applied for the construction of the triangular mesh, i.e. three points form a triangle in the network if the circle they define contains no other reference points. Breaklines and structure lines always form the edges of triangles. (Fig. 1). If two points on a break- or structureline are not closer than the mean length of the triangles, they are subdivided automatically.

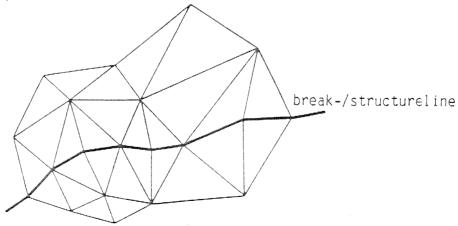


Fig. 1: Example of triangle intermeshing including a break- or structure-line.

The interpolation involves a piecewise complete bicubic approximation of the surface:

$$Z(X,Y) = a_1 + a_2X + a_3Y + a_4X^2 + a_5XY + a_6Y^2 + a_7X^3 + a_8X^2Y + a_9XY^2 + a_{10}Y^3$$
 (1)

For the numerical realization Zienkiewicz shape functions (Bazeley [1965]) are used:

$$Z(X,Y) = \sum_{i=1}^{9} u_i N_i$$
 (2)

where  $u_i$  are parameters and  $N_i$  are the shape functions which are written in natural triangular coordinates  $L_i$ , e.g.:

$$N_1 = L_1 (3-2L_1) + L_1L_2L_3 \tag{3}$$

Heights and tangent planes at the corners of the triangles serve as input to the parameters  $\mathbf{u}_i$ . The tangent planes are computed from least squares fits:

$$\sum_{j \in k_i} (x_j - x_i) + c_i (Y_j - Y_i) - (Z_j - Z_i) ]^2 = \min$$
(4)

where the sum is indicated to run over the nearest neighbours of node i in the network and  $b_i$ ,  $c_i$  are the unknowns. For smoothing purposes (considering here only the interpolation of the surface) an alternative formula for the least squares fit is applied:

$$\sum_{j \in K_i} (x_j - x_i) + c_i (y_j - y_i) - (z_j - z_i - v_{z_i})]^2 = \min$$
 (5)

with  $Vz_i$  a correction to the height Z. The corrections are limited by a parameter  $\delta$  chosen by the user:

$$\bar{Z}_{i} = Z_{i} + sign(V_{Z_{i}}) \cdot min(\delta, V_{Z_{i}})$$
 (6)

The generated surface is smooth within the triangles and continuous from triangle to triangle except across breaklines. Segments of breaklines are assumed to be straight lines in space with discontinuities in slope perpendicular to them whereas structure lines are considered as straight but continuous across them. The procedure allows the computation of the height of any point of given X, Y lying within the area covered by the network.

<u>b) Interpolation of the contours:</u> In order to compute the contour lines each triangle is divided into 16 sub-triangles; the surface of an individual subtriangle is approximated by a plane. The rather detailed mesh of sub-triangles represents a very close approximation to the surface in terms of FINITE ELEMENTS; it guarantees accurate contours and also the correct course of contours within a main triangle. In order to keep the numerical effort low not all computed contour points are stored. The following inequality has been taken as guide for the elimination of redundant contour points: si+1

$$\int ds \left[ \left( \frac{\partial x}{\partial s} \right)^2 + \left( \frac{\partial y}{\partial s} \right)^2 \right]^{1/2} \Rightarrow f \cdot \overline{r}$$
 (7)

Here i, i+1 denote the intervals of stored points on the contours,  $\bar{r}$  is a critical length calculated from the triangles and f is a density factor chosen by the user. In the extreme case of very dense data sets (i.e. small main triangles) only the contour points on the edges of the main triangles are stored. Finally, the contour lines are brought into an optimum shape by interpolation and, if necessary, by smoothing.

Following Figure 2 clarifies the densification of a main triangle into subtriangles.

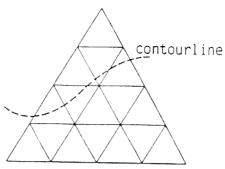


Fig. 2: Densification of a main triangle into 16 subtriangles.

The following equation is used for smoothing:
$$\int_{i}^{\infty} ds \, \rho^{2}(s) + \sum_{i}^{\infty} g_{i} \left[ (\tilde{x}(s_{i}) - x_{i})^{2} + (\tilde{y}(s_{i}) - y_{i})^{2} \right] = \min$$
where  $\rho$  is the curvature of the actual contours represented by  $\tilde{x}(s)$ ,  $\tilde{y}(s)$  and  $x$ ,  $y$  are the contour points given from the tornain

and x, y are the contour points given from the terrain.  $g_i$  is a weight factor proportional to the slope of the terrain and proportional to a parameter chosen by the user, i.e. the smoothing is more efficient where the terrain is flat.

### THE PROGRAM PACKAGE " C I P "

CIP (Contour Interpolation Program) is a combination of eight different programs. They are called one from another, but can also be called independently. A very important part for the user is the handling of the input parameters and this part is designed to be very user-friendly. All informmation is entered via menus. These can easily be used by an operator with little experience in photogrammetry or the use of computers. The menus have sub-menus and the descriptions of the parameters are either self-explanatory or they are supported by HELP functions. Many sophisticated features such as blinking, ringing a bell, dimming, and the checking of parameter and input files assist the user as guides, hints or warnings. Once all menus are completed, all parts can be started together, so that the operator need not wait for intermediate results. Fig. 3 shows the flowchart of the program.

### **APPLICATIONS**

a) Contours: The main application of the CIP program package is the derivation of contour lines. The generation of a contour map from a digital elevation model (DEM) can be interpreted as an information transfer from the surface to the map (Makarovic, 1972). Two factors largely govern the final quality of the contour lines - first, the sampling of the data and, second, the method of interpolation. For a morphologically high-quality map both sampling and interpolation must be optimal. The sampling is responsible for the information which reaches the user. For sampling the WILD PROGRESSIVE SAMPLING program is recommended. The interpolation is responsible for the conversion of the digital data into a graphic representation of the surface. Obviously, breakline and structure-line data are necessary to describe the surface satisfactorily (Figures 6 and 7). Breaklines may cause sharp bends for artificial surfaces, e.g. kerbs, embankments, or they may cause rounded bends for natural discontinuities. Structure lines lead to round bends, and the contour lines intersect the structure lines at approximately 90 degrees. The contours in dead areas must be suppressed.

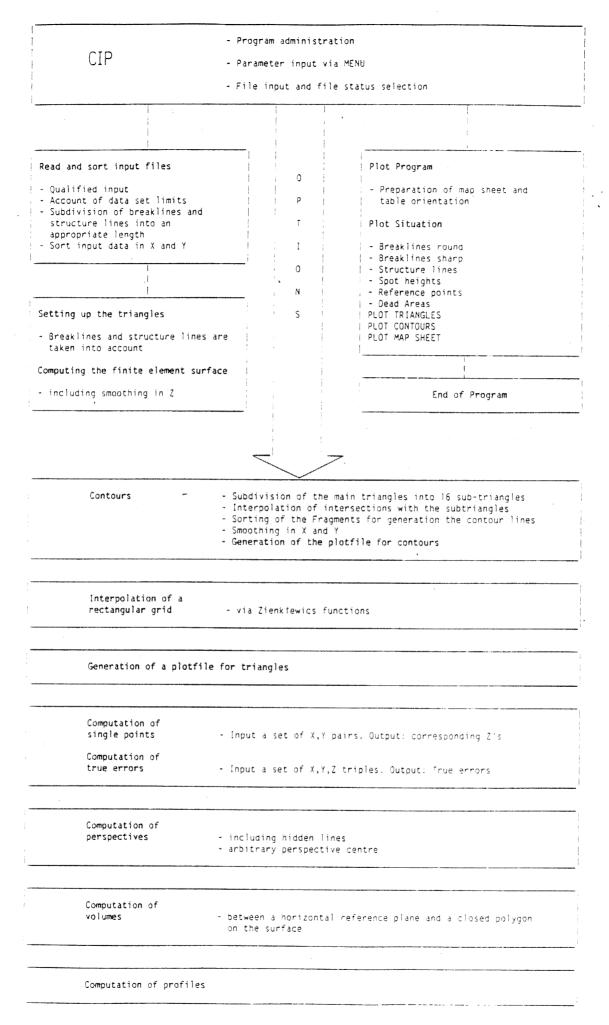


Figure 3: Flowchart of the program

Also, spot heights contribute to the morphological quality. A further very important point is the application of filtering or smoothing techniques, where a certain bandwidth can be specified. Beside the annotation of the contours (height and interval between two annotations are variable) all contour lines without annotation can be marked by a small arrow, which points downhill.

All the points mentioned above are realized in the CIP package. In addition, the user has many possibilities for the plotting of the contours. He can leave out parts of the contours if the surface is too steep to give a pleasing representation. Different pens (up to four in the WILD TA2 and TA10) and plotting tools and the possibility of many combinations all make for excellent graphic quality with economic effort.

- b) Interpolation of a rectangular grid: A rectangular, regular grid may be required for a variety of reasons (for example, for user programs which need a regular grid as input). Such a grid, with points of known X and Y, can be computed from the finite-element surface. The result is a file of the X, Y, Z, coordinates.
- c) Generation of a plotfile for triangles: A plot of the triangles may be interesting for several purposes, the main one being a check of the input data (gaps in the data can be recognised, for example).
- d) Computation of single points and/or true errors: A field of arbitrarily spaced points of known X, Y, Z, coordinates may be as important to some users as a regular grid. Such a field of points can be interpolated from the computed ground surface in the same way as a regular grid, with the difference that the individual X, Y coordinates for each required point must be entered into a file before computation. On the other hand for an analysis of the accuracy of the DTM, points can be abstracted from the input data with a preprogram and stored in a special file. From this data set the differences of the observed and interpolated heights are computed. If the difference is larger than 3 times the mean true error the points are printed out.

Plot program: It consists of a part for map sheet preparation, where up to five different map frames can be specified, a part for table orientation with the possibility of digitzing points on an existing map to which the plotfile has to be transformed (least squares). It pilotes all plotting, such as situation plot, triangle plot, contour plot etc.). The texts and the annotation of the coordinates, as well as the coordinate, can be located arbitrarily.

e) Perspective Program: The program is designed with hidden lines and the perspective center can be selected arbitrarily, even in the terrain.

#### THE HUMAN INTERFACE

The human interface has been designed to be as user-friendly as possible and the relatively complex program is easy to use even by inexperienced operators. The following features contribute to this result.

<u>Clear, simple menus:</u> All communication with the program is made via two main menus, one for the computational part, the other for the plotting part. The menus are calling submenus on request. In a menu the user can assign values to parameters, define data files to be used or execute one or more program steps.

<u>Default values:</u> For a new job, the system provides reasonable default values for the parameters. Some of these default values are computed by the program itself and the user can then decide if he wants to change them or not. For each project a parameter file is generated, and if the project has to be repeated the old parameters are default.

File handling: Up to 20 files each can be specified for mass points, breaklines, structure lines and spot heights and one file for dead areas. The files are catalogued. Each file has a certain status, e.g. File Missing, Ready for Use, Not to be Used, Breaklines Sharp or Round, Structureline. The file names appear in the lists on the screen normally, dimmed or underlined, according to their status.

Help functions: At all places, where input is required explanatory text can be called by entering "?". This is a very convenient feature for the parameter input, especially for non experienced users.

Indication of already computed steps: An indicator shows if a program part has already been computed.

<u>Starting Computation:</u> Either a particular program part may be started alone (e.g. just only that part which defines borders or mean lengths of triangles) or pre-specified parts of the program may be started in arbitrary combination.

On-line checks: All input is instantaneously checked for blunders, orthographic and logical errors and plausibility.

Debugging tools: There are several ways to analyse a data set. It is possible to draw the triangular mesh to discover possible errors in the data set. An interface program reads a plotfile and interprets the contents. In all program parts error messages and warnings are provided to detect gross errors during computation.

Matrix for simple pen assignment in the plotting program: The user defines in a matrix which plotting tools are to be used with line symbols etc.

<u>Output protocol:</u> A long and a short protocol can be specified, which gives much information on the computed parts. Also the parameters can be printed out separately.

#### **EXAMPLES**

EXAMPLE 1: The photographs were taken with a commonly-used Polaroid camera, the same camera for both pictures. The photo scale was 1:3.5 and the base-height ratio 1:5.

The mass points were measured in the Aviolyt BC1 in a regular grid. The grid width was 6 mm. Breaklines were digitized around the eyes, nose and mouth. A total of 900 points was measured.

Computation: Contour interval = 1 mm. The standard deviation of the "heights" for the triangulation network was set to 0.02 mm. Only the intersections with the main triangles were used. The smoothing criterion for contour lines was set to 0 (this criterion is defined in height and applied according to the surface slope). A total numer of 9'600 points was interpolated and the computation time was 13 minutes on the Nova 4/X. Plotting time on TA2 table was 6 minutes.

 $EXAMPLE\ 2:$  Photo scale = 1:10'000. The mass points were measured in a regular grid with a grid width of 20 m in the ground. A total of 4'300 mass points and breakline points was measured.

Computation: Contour interval =  $2\,\text{m}$ . The standard deviation in height for the triangulation network was set to  $0\,\text{so}$  as to produce a characterful plot. During the interpolation the intersections with the main triangle sides and some additional sub-triangle sides were used. The smoothing criterion for the contour lines was set to  $10\,\text{cm}$ . A total number of  $18'000\,\text{points}$  was interpolated. The computation time on the Nova 4/X was  $38\,\text{minutes}$ . Plotting time on TA2 table was  $9\,\text{minutes}$ .



Figure 4: Example 1. Human Face

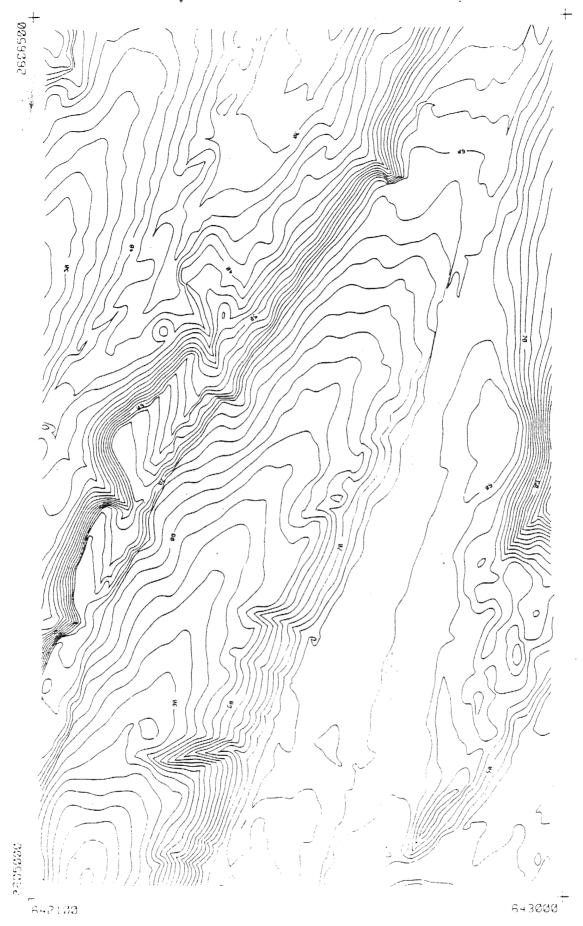


Figure 5: Example 2. Topographic Application

# EXAMPLE 4: Comparison between structure lines and breaklines:

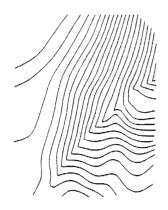


Figure 6
Edge declared as a break line

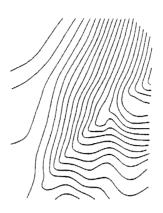


Figure 7

Edge declared as a structure line

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