# Testing and internal reliability in the process of analytical abolute orientation.

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**Summary**
In the process of digital mapping the modelcoordinates of digitized terrain objects are transformed to the terrain coordinate system with transformation parameters which are determined in the process of analytical absolute orientation. Errors in the computed transformation parameters can be a consequence of errors in the measured model coordinates and of errors in the elements, which are used in the interior and relative orientation.
In this paper is described in what way (testing) and to what limit (boundary values) the above mentioned errors can be detected.
For the on-line application of the method a microcomputer program has been written. With computed examples it will be shown that the method can be used in practice.

## 1. Introduction

In the process of digital mapping the model coordinates of digitized model objects are transformed to a terrestrial coordinate system, with the help of transformation parameters as determined from an analytical absolute orientation.

Precise and reliable transformation parameters are desired in order to obtain an accurate map.

The model coordinates of the control points, which are used for the computation of the transformation parameters contain errors. These errors are caused by setting errors of the measuring mark on the control points and by errors made in the processes of interior and relative orientation.

Ligterink (1972) describes the method how a variance-covariance matrix for the model coordinates of observed control points can be formed, taking into account <u>small</u> measuring and orientation errors.

<u>Large</u> measuring errors of the size of computed boundary values can be detected by using the method as developed at the University of Technology at Delft. The boundary values indicate the size of the error which just can be detected at a specified level of significance. The method is described in Baarda (1968) and G.C.C. (1982).

By the Section of Photogrammetry of the Department of Geodesy at Delft University of Technology an analytical absolute orientation program for a microcomputer has been developed. This program is part of a digital mapping program package. Integrated in the program are the use of a full variance-covariance matrix for the model coordinates and the above mentioned method of large error detection and boundary value computation.

A special feature of the program is that large error detection can be executed by application of a special alternative hypothesis for the detection of orientation errors. The microcomputer is a Tektronix 4052A and has been coupled on-line with a stereoplotter (Wild A10).

In the following part of this paper the applied method of analytical absolute orientation, large error detection and boundary value computation will be described. Also the results are given of some experiments. These experiments have been executed in order to check if the described method can be used in practice.

## 2. Mathematical model and variance-covariance matrix of the model coordinates.

### 2.1. Mathematical model

In the process of analytical absolute orientation the 7 transformation parameters are solved by least squares adjustment.
The relation between terrain coordinates and model coordinates is given by the transformation formula:

$$\begin{pmatrix} X_T^i \\ Y_T^i \\ Z_T^i \end{pmatrix} = \lambda (R_{\Omega\phi\kappa}) \begin{pmatrix} x_m^i \\ y_m^i \\ z_m^i \end{pmatrix} + \begin{pmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{pmatrix} \quad (1)$$

In (1):

| | |
|---|---|
| $X_T^i$ , $Y_T^i$ , $Z_T^i$ | are the terrain coordinates |
| $x_m^i$ , $y_m^i$ , $z_m^i$ | are the model coordinates |
| $\lambda, \Delta X, \Delta Y, \Delta Z, \kappa, \Omega, \phi$ | are the 7 transformation parameters |
| R | is a rotation matrix |

After linearisation the following type of correction equations are applied:

$$\Delta x^i + \varepsilon^i = f(\Delta\lambda,\ \Delta\phi,\ \Delta\Omega,\ \Delta\kappa,\ \Delta X,\ \Delta Y,\ \Delta Z) \quad (2)$$

After some iterations estimates of the unknown parameters are obtained.

### 2.2. Variance covariance matrix of the model coordinates.

In the process of analytical absolute orientation the following error types can occur:
1. Instrumental errors.
2. Film deformation.
3. Errors in the terrain coordinates of the control points.
4. Errors, made in the process of interior orientation.
5. Errors, made in the process of relative orientation.
6. x-, y-, z-setting errors of the measuring mark on the control points.

The errors mentioned under 4, 5 and 6 are used to form a variance-covariance matrix for the model coordinates. The formula system for this operation is described in Ligerink (1972).

In order to form this matrix the following standard deviations have to be used:

1. The standard deviations of the elements of interior orientation, which are:
   a. the standard deviations in photograph centering: $\sigma_{\Delta XL}, \sigma_{\Delta YL}, \sigma_{\Delta XR}, \sigma_{\Delta YR}$
   b. the standard deviations of the principle distances for the left and the right camera: $\sigma_{CL}$, $\sigma_{CR}$
2. The standard deviation of the measurement of a y-parallax in the procedure of relative orientation: $\sigma_{\Delta PY}$
3. The standard deviations, when measuring a model point: $\sigma_X, \sigma_Y, \sigma_Z$.

When measuring a model point an error can be made in the elimination of the x-parallax. The consequence is that also x- and y-setting errors are made. These x- and y-setting errors are not the same for marked and natural points (see, also Figure 1, page 4). In the procedure for setting up the variance covariance matrix marked and natural points are treated therefore in a different way.

## 3. Testing and internal reliability.

### 3.1. Large error testing

When testing the model coordinates on large errors so called c-vectors are used. The size of the c-vector is determined by the number of observations. The convential alternative hypothesis is used, if in the testing procedure it is assumed that one observation is wrong and all the other observations are correct. In this case the element in the c-vector corresponding to the observation which is assumed to be wrong, is equal to one. All the other elements are zero.

For the detection of a large orientation or height setting error a special alternative hypothesis is used. More observations are assumed to be wrong and the formed c-vector contains more then one element unequal to zero.

The applied c-vector when making a height setting error can be deduced from Figure 1. In Figure 1, P is a model point, which has to be measured. The error setting in height is $\Delta h_m$. When P is a natural point then the measuring mark is set in M. If P is a marked point in the left photograph then the mark is set in ML. For the point marked in the right photograph in MR.

The errors in the x, y, z-model coordinates of point P due to a height setting error $\Delta h_m$ can be deduced from Figure 1 and can be expressed by:

$$\begin{pmatrix} \Delta x^i_M \\ \Delta x^i_{ML} \\ \Delta x^i_{MR} \\ \Delta y^i_M \\ \Delta h^i_M \end{pmatrix} = \begin{pmatrix} -\frac{x^i}{z} \\ -\frac{1}{2z}(2x^i+b) \\ -\frac{1}{2z}(2x^i-b) \\ \frac{y^i}{z} \\ 1 \end{pmatrix} \begin{pmatrix} \Delta h^i \end{pmatrix} \qquad (3)$$

When P is a marked point in the left photograph then the height error testing for point P can be executed with a c-vector, which contains zero's and according to (3) the values: $-\frac{1}{2z}\ (2x^P + b),\ \frac{-y^P}{z},\ 1$

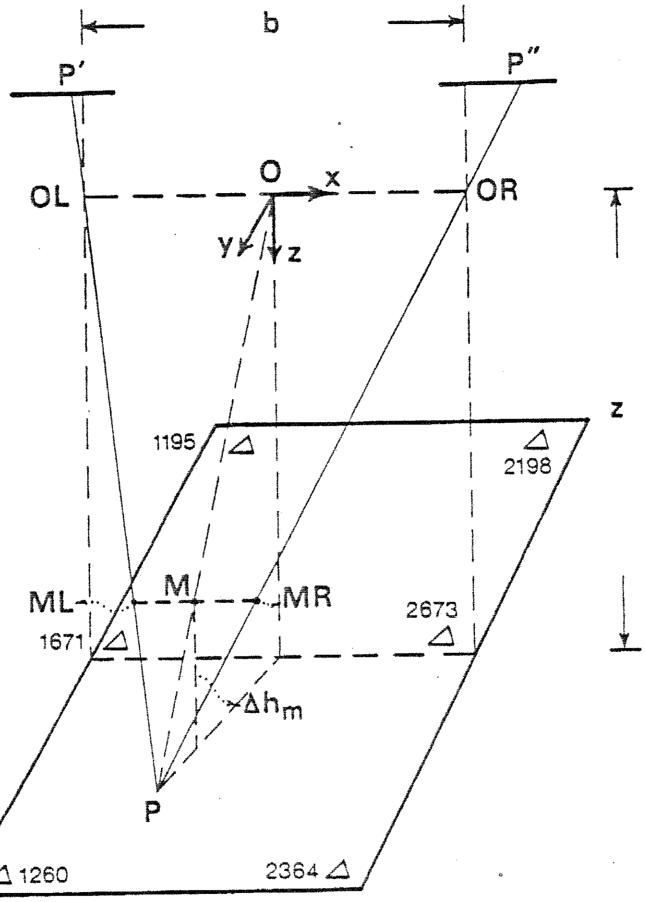


Figure 1. The influence of a height setting error and the position of the control points

In Ligterink (1972) the formulas are given, which describe the effect of a small change in one of the orientation elements on the x, y, z model coordinates. These formulas are used for the formation of c-vectors in case of large error testing of one of the orientation elements.

The following formulas are applied for the computation of the w-test values in order to test the residuals of the observations:

$$w_P = \frac{1}{\sigma_o \sqrt{N_P}} \ (c_P^j)^* (g_{ji}) (-\varepsilon^i) \qquad (4)$$

In (4): $N_P = (c_P^j)^* (g_{ji}) (g^{ij} - G^{ij}) (g_{ji}) (c_P^i)$ (5)

In (4) and (5):

$(g^{ij})$ =matrix of weight coefficients of the model coordinates

$(g_{ji}) = (g^{ij})^{-1}$

$(G^{ij})$ =weight coefficient matrix of the adjusted observations

$\sigma_o$ =square root of the variance factor

$\varepsilon^i$ = corrections to the observations.

The test is: $|w_p| \leq \sqrt{F_{1-\alpha_o}}\ ; 1, \infty$ (6)

For: $\alpha_o = 0.001$ and $\beta_o = 80\%$ : $|w_p| \leq 3.29$

3.2. Internal reliability (boundary values)

The errors in the observations $\nabla x^i$, which just can be detected with a probability $\beta_o$ are called boundary values or marginal detectable errors.
The boundary values are computed as follows:

$$\nabla_p x^i = (c_p^i) \sqrt{\frac{\lambda_o}{N_p}}\ \sigma_o$$

For: $\alpha_o = 0.001$ and $\beta_o = 80\%$ : $\lambda_o = 17$

## 4. Computer program

The program language used is BASIC.

When using the program, three types of variance covariance matrices for the observed model coordinates can be used. They are defined as follows:

Type A: Diagonal matrix. This matrix is formed with the standard deviations: $\sigma_x$, $\sigma_y$, $\sigma_z$ .

Type B: Autocorrelation is present. This means the x, y, z model-coordinates per point are correlated. The matrix is formed as described in Ligterink (1972).
To form this matrix, values for $\sigma_x$, $\sigma_y$ and $\sigma_z$ have to be introduced.

Type C: Correlation is present between all model coordinates. This correlation is indicated as cross correlation. The method to form this matrix is described in Ligterink (1972). The standard deviations to be introduced are mentioned already in paragraph 2.2.

With the program the following computations can be executed:

1. Computation of the 7 transformation parameters.
2. Computation of the residual errors after analytical absolute orientation.
3. Computation of w-test values of:
   a. the single x, y and z model coordinates, by applying the conventional alternative hypothesis. These values are indicated as: $w_x$, $w_y$, $w_z$.
   b. the height setting of the measuring mark by application of a special alternative hypothesis. This w-test value is indicated as $(w_z)$.
   c. the elements of relative orientation.
   d. the elements of interior orientation.
4. Computation of the boundary values of the tested quantities as mentioned in point 3.
5. Computation of the external reliability for the quantities mentioned in 3a.

For the photogrammetric operator only the computations of the seven transformation parameters and the w-test values are important.

The computation of the internal and the external reliability can be used for instruction or for reconnaissance with simulated photography, different types of control point location and varying standard deviations.

## 5. Experiments

### 5.1. Introduction

To determine if the programmed method of large error detection can be used in practice some analytical absolute orientations have been executed with observations, which contained large errors. Each absolute orientation has been followed by the computaton of the w-test values of the observations.

Then it has been checked if the observation with the largest w-test value corresponds with the observation in which the large error has been made.

The type of observational errors used in this case are some orientation errors and the height setting of the measuring mark. Large setting errors of the measuring mark in x- and y-direction are easily detected and are not mentioned in the experiments.

At first the boundary values of the observations have been computed, in order to determine how large the errors of the observations should be in the experiments.

In order to determine the influence of the three described v.c.m.'s for the model coordinates in the process of large error detection, each computation has been executed three times.

### 5.2. Experiments

The experiments are executed with real observations. The stereoplotter used is a Wild A10.

The photography data are: photo scale 1:6.000, principle distance = 210 mm, overlap 60%.
The model scale is 1:3.000. The measured control points are marked points.

The control point numbers and their positions are given in Figure 1.

The standard deviations used for the formation of the v.c.m.'s are:

| | | |
|---|---|---|
| a. | $\sigma_x = \sigma_y$ | = 4 microns at photo scale (2.4 cm in the terrain) |
| | $\sigma_z$ | = 0.06 % of the flying height (7.6 cm in the terrain) |
| b. | $\sigma_{\Delta XL}, \sigma_{\Delta YL}, \sigma_{\Delta XR}, \Delta\sigma_{\Delta YR}$ | = 20 microns at photo scale |
| | $\sigma_{CL} = \sigma_{CR}$ | = 3 microns |
| | $\sigma_{\Delta PY}$ | = 8 microns at model scale |

The values of these standard deviations are the same as used in Ligterink (1972).
The used variance covariance matrices are indicated as type A, B and C. The standard deviations mentioned in a. are used to form the v.c.m.'s of type A and B. The v.c.m. of type C is formed, using the standard deviations mentioned in a. and b.

The boundary values of the x, y, z model coordinates of the four observed control points in the model corners are computed, using the variance covariance matrices of type A, B and C. The computed boundary values of point number 2198 are given in table 1. The values are given in meters in the terrain.

Table 1: Boundary values of the x, y, z model coordinates of point 2198.

| Point | $\nabla_x$(m) | $\nabla_y$(m) | $\nabla_z$(m) | v.c.m. type |
|---|---|---|---|---|
| 2198 | 0.14 | 0.14 | 0.52 | A |
| 2198 | 0.15 | 0.16 | 0.40 | B |
| 2198 | 0.17 | 0.18 | 0.44 | C |

The computed boundary values of the x, y, z coordinates of the other control points are about as large as those of point 2198. Therefore they are not listed in table 1.

The v.c.m. of type B can be obtained, when in the formation procedure of the v.c.m. of type C the standard deviation of the y-parallax measurement and the standard deviatons of the interior orientation elements are set equal to zero. Also the v.c.m. of type C is a simplified version of the v.c.m. of type B. Hence it can be expected when in the computational procedure successively the v.c.m.'s of type A, B and C are used and assuming no correlation values in the v.c.m.'s, that the computed boundary values of the x, y, z coordinates will become larger.
Table 1 shows that the boundary values of the z-coordinates using the v.c.m.'s of type B and C are considerably smaller than the boundary values when using a v.c.m. of type A. For the x, y coordinates the effect is opposite and corresponds with the expectation as stated above.

The conclusion is that with this control point configuration smaller errors in the z-coordinates can be detected when using a v.c.m. of type B and C instead of a v.c.m. of type A. Apparently this is caused by the correlation in the v.c.m.'s of type B and C. Further that the correlation in the v.c.m.'s of type B and C does not influence the detection of large errors of the x, y coordinates.

The computed boundary values of the elements of relative orientation (by" excluded), using the variance covariance matrix of type C are:

$\nabla_{\kappa''} = 0.014$ gr., $\nabla_{\omega''} = 0.005$ gr., $\nabla_{bz'} = 0.10$ mm, $\nabla_{\phi''} = 0.05$ gr.

Experiment 1: Dectection of a large height setting error
Four control points.

An analytical absolute orientation is executed, using the four control points in the corners of the model in Figure 1.

When measuring the marked control point 1195 a height setting error has been made of 0.20 mm. This error corresponds with 0.60 m in the terrain and is larger than the boundary value of the z-coordinate in Table 1. Due to the height setting error a setting error in y-direction has been made of 0.08 mm. The measuring of the other three control points has been executed as good as possible.

In the procedure of analytical absolute orientation using the v.c.m. of type A the residual errors are computed. These errors are listed in Table 2.

Table 2: Residual errors. V.c.m. type A.

| point | dx(m) | dy(m) | dz(m) |
|---|---|---|---|
| 2198 | 0.08 | 0.08 | -0.25 |
| 1195 | -0.04 | -0.15 | 0.25 |
| 1260 | -0.06 | 0.06 | -0.20 |
| 2264 | 0.03 | 0.00 | 0.20 |

The computed w-test values, using the v.c.m.'s of type A and C are listed in Table 3.

Table 3: w-test values of x, y, z coordinates

| point | $w_x$ | $w_y$ | $w_z$ | $(w_z)$ | v.c.m. type |
|---|---|---|---|---|---|
| 2198 | -4.62 | -5.09 | 6.02 | 7.47 | A |
| 1195 | 2.38 | 8.61 | -5.96 | -10.61 | |
| 1260 | 3.90 | -3.78 | 5.94 | 0.40 | |
| 2264 | -1.63 | -0.08 | -6.02 | -2.58 | |
| 2198 | -1.84 | 1.61 | 4.41 | 3.41 | C |
| 1195 | 2.87 | 0.95 | -4.40 | -6.80 | |
| 1260 | 1.52 | -3.58 | 4.40 | 1.71 | |
| 2264 | -2.61 | -0.04 | -4.41 | -4.63 | |

In this example there is only one redundant observation in height for the determination of the absolute values of phi and omega.
The consequence is that equal values are found for dx in Table 2 and for $w_z$ in Table 3, and that error detection with these values is impossible. However the error in height in point 1195 is detected in the colum of the $(w_z)$ values. Table 3 shows that when using a v.c.m. of type A and C the computed $(w_z)$ values of point 1195 are -10.61 and -6.80. In both cases these values are larger than the values of the other tested quantities, and this means that the height setting error of the measuring mark in point 1195 has been detected.

Expriment 2: Detection of a large $\phi$"error.
Six control points.

An analytical absolute orientation is executed using the six control points in Figure 1.

After the procedure of interior and relative orientation an error of 0.10 grades is introduced in the orientation element phi of the right camera. This values is equal to two times the computed boundary value. After the measuring of the control points an analytical absolute orientation has been executed using a v.c.m. of type C. The results of large error testing are listed in the tables 4, 5 and 6.

Table 4: w-test values of x, y, z coordinates

| point | $w_x$ | $w_y$ | $w_z$ | $(w_z)$ | v.c.m. type |
|---|---|---|---|---|---|
| 2673 | -1.88 | -2.58 | -2.10 | -2.00 | C |
| 2198 | -3.22 | 0.49 | 1.66 | 1.35 | |
| 1195 | 1.53 | 5.27 | 0.79 | -3.99 | |
| 1671 | 1.61 | 1.97 | -2.42 | -2.38 | |
| 1260 | 4.59 | -4.29 | 1.64 | -1.44 | |
| 2264 | -2.43 | -2.18 | 0.69 | -1.44 | |

Table 5: w-test values of the elements of relative orientation (by'' excluded)

| $w_{\phi''}$ | $w_{bz'}$ | $w_{\kappa''}$ | $w_{\omega''}$ | v.c.m.type |
|---|---|---|---|---|
| -8.65 | -3.28 | 0.51 | -3.08 | C |

Table 6: w-test values of the interior or. elements.

| $w_{\Delta XL}$ | $w_{\Delta YL}$ | $w_{CL}$ | $w_{\Delta XR}$ | $w_{\Delta YR}$ | $w_{CR}$ | v.c.m. type |
|---|---|---|---|---|---|---|
| -2.48 | -7.49 | -0.10 | 2.94 | -1.06 | 0.16 | C |

The largest value in the tables 4, 5 and 6 is equal to -8.65 and belongs to the w-test quantity of the orientation element phi of the right camera.
The same computations are executed by using the v.c.m.'s of type A and B. Also in these cases the large error in the relative orientation element phi right has been detected.

Experiment 3: Large observational error of ω'' and by''.

First an observational error of 0.05 grades is made for ω''. Then the y-parallax in orientation point 1 is eliminated with by''. The result of the large error test is that the observation of κ'' is rejected. The w-test value for kappa right is -7.50 (critical value: 3.29). The explanation for this result is that in the six marked control points the model deformation caused by a combined ω'' - by'' error is the same as the model deformation caused by κ''. Conclusion:the relative orientation is rejected but the large error test rejects the wrong orientation element.

Remark: In all experiments the standard deviations of the terrestrial X, Y, Z coordinates of the control points have been $\sigma_X = \sigma_Y = 3$ microns at photo scale and $\sigma_Z = 0.04\ ^o/_{oo}$ Z.

## 6. Conclusions and recommendations

The results of experiments show that large observational errors in the process of analytical absolute orientation can be detected better when using both the conventional alternative hypothesis and the special alternative hypothesis in the procedure of large error testing. The special alternative hypothesis can be applied very well for the detection of large height setting errors of the mea-

suring mark and for the detection of large orientation error. A striking example is the detection of a large height setting error of the measuring mark in case of absolute orientation with four control points.
It is recommended to treat marked and natural control points in a different way, both for setting up a variance covariance matrix (v.c.m.) for the model coordinates, and for forming c-vectors in the procedure of large error testing. The boundary values, when using a full v.c.m. of the model coordinates are more realistic than in case of using a diagonal v.c.m.. However when using a full v.c.m. much computer memory is needed. It can be expected, that a good alternative is the use of a v.c.m. of the model coordinates in which x, y, z coordinates are correlated per point.

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