BIVARIATE SMOOTHING OF DTM DATA

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ABSTRACT :

A method of smoothing DTM data is contrived and applied to various fictitious and actual terrain surfaces. The method combines the direct choice of smoothing parameters with a criteria for evaluating and controlling the degree of smoothing.

Transfer Functions of the smoothing process are determined and an analysis of the fidelity of different smoothing degrees is performed.

INTRODUCTION :

Smoothing is one of the digital processes used to remove errors from observations. These errors may be considered as an added set of data which has different characteristics from the "correct" data. Usually, errors vary randomly at high frequencies, therefore, it is desirable to use a smoothing process that filters out data varies at frequencies higher than the frequencies expected to exist in the phenomenon being observed.

In Digital Terrain Model systems, the smoothing process is one of the Pre-Processes used to convert the row data into a more useful form for further applications,(Makarovic,1976)

Most of the methods used in digital data smoothing are based on the polynomial smoothing technique, (Lafara,1973 & Whittaker,1965). This technique employes least squares to fit successively portions of the data in order to eleminate small variations. Polynomial smoothing is not completely satisfactory because it containes arbitrary elements whose introduction do not appear to be logically necessary,(Scheid,1968).

A method is sought that avoides the drawbacks of polynomials and at the same time establishes a criteria for controlling the smoothing degree.

SMOOTHING WITH VARIABLE WEIGHTS :

The general form of a smoothing formula can be written as :

$$\bar{Z}_P = \sum_{i=1}^{n} W_i \; Z_i \qquad (1)$$

Where :

$\bar{Z}_P$ is the smoothed elevation at point P.
n is the number of points used in smoothing.

Z_i $(i=1,2,\ldots,n)$ are the n observations
W_i $(i=1,2,\ldots,n)$ are a set of weighting constants
The condition of thse weights is :

$$\sum_{i=1}^{n} W_i = 1.0 \qquad (2)$$

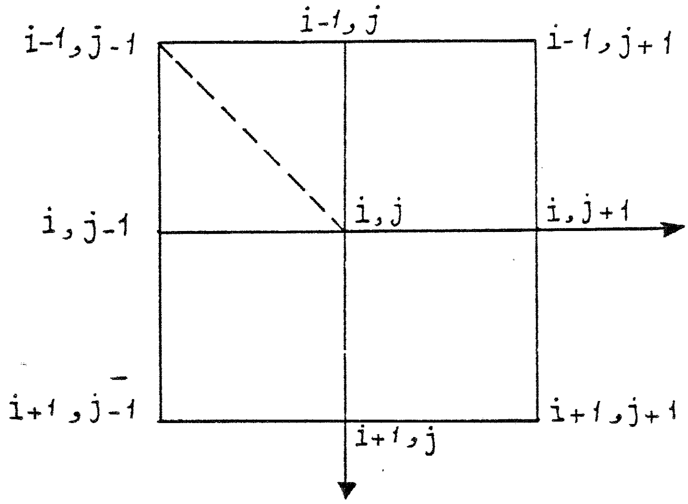

Fig.1 Point pattern and coordinate system

The method is performed in two steps .

1- The smoothing process : in which the elevation of point (i,j), Fig.1 , is smoothed according to :

$$\bar{Z}(i,j)=W_1\,Z(i,j)+W_2(Z(i-1,j)+Z(i+1,j)+Z(i,j-1)+Z(i,j+1)) +W_3(Z(i-1,j-1)+Z(i-1,j+1)+Z(i+1,j-1)+Z(i+1,j+1)) \qquad (3)$$

To relate weights of points on axes to those of points on chords, the following condition is introduced :

$$W_2 \,/\, W_3 = \sqrt{2} \qquad (4)$$

From which :

$$W_3 = (1-W_1)\ /4(1+\sqrt{2}) \qquad (5)$$

$$W_2 = (1-W_1)\ /2(\sqrt{2}+2) \qquad (6)$$

The weight W_1 is assumed to take values equal to, or greater than 0.5 . The initial value of 0.5 gives maximum smoothing .
The smoothed elevations of the boundary points can be computed as follows :

$$\bar{Z}(i,p)=W_1 Z(i,p)+2(W_2+W_3)(Z(i-1,p)+Z(i+1,p)) \qquad (7)$$

$$\bar{Z}(m,j)=W_1 Z(m,j)+2(W_2+W_3)(Z(m,j-1)+Z(m,j+1)) \qquad (8)$$

Where :

$i=2,3,\ldots,(n-1)$
$j=2,3,\ldots,(n-1)$
p is either 1 or n
m is either 1 or n

Corner elevations can be smoothed as follows :

$$\bar{Z}(1,1)=W_1 Z(1,1)+2(W_2+W_3)(Z(1,2)+Z(2,1)) \qquad (9)$$

Similarly the smoothed values of the other three corners can be computed .

2- The control of the smoothing degree :
Three alternatives of control criteria are employed :

i) The first law of conservation states that the graph of the smoothed data and that of the unsmoothed data should have the same area(Whittaker,1965) .

$$\sum_{i,j=1}^{n} \bar{Z}_{i,j} = \sum_{i,j=1}^{n} Z_{i,j} \qquad (10)$$

To test whether or not this law is satisfied the two sums are compared and the parameter W_1 is increased if the difference exceeds a pre-determined value .

ii) The root mean squre of the differences between the smoothed and unsmoothed elevations is computed and, according to its value, the smoothing parameter W_1 is either increased(to decrease the smoothing effect) or the process is terminated,which means that the achieved smoothing degree is adiquate .

iii) The Transfer Functions graph can be used to determine the values of the smoothing parameter that give the required smoothing degree .

TRANSFER FUNCTIONS OF THE SMOOTHING PROCESS :

The presented smoothing method can be considered as a low-pass filter which smooth out local fluctuations and estimate a local weighted mean for each elevation .

Filters are usually desined to produce outputs with emphasis on variations at particular frequencies, therefore, they are often studied in the frequency domain using frequency response functions which are known as Transfer Functions(Tempfli,1978) .

Transfer Functions show how a linear system (filter) responds to sinusoids at different frequencies . It is usually defined as the Fourier transform of the impulse response function .

Throughout the present investigation, Transfer Functions of the smoothing process are determined for W_1=0.1,0.2,....,1.0 using the following parameters :

- Sampling interval =π/20 , homogenious square grid .
- Frequency of the double Fourier wave = 2,3,...,20 ,because Transfer Functions are only defined for frequencies up to the Nyquist frequency which is equal to π/sampling interval
- Number of reference points in one direction = 20

RESULTS :

The method is applied on fictitious as well as actual data. The fictitious data is generated using the basic element of a double Fourier series which can be written as :

$$Z(i,j)=A \sin(\omega_1 \Delta x\, X(i,j) + \omega_2 \Delta y\, Y(i,j)) \quad (11)$$

Where:

ω_1 & ω_2 are the frequencies in the directions X & Y
Δx & Δy are the sampling intervals in the two directions
$X(i,j)$ & $Y(i,j)$ are the planimetric coordinates of point(i,j)
A is the amplitude of the double Fourier wave .
$Z(i,j)$ is the computed elevation of point(i,j) .

Different types of terrain surfaces are generated by giving different values to the frequencies ω_1 & ω_2 .

Blocks from the Qattara depression DEM (Egypt) are used as actual data as shown in Figure 7 .

The increasing effect of smoothing on rough terrain is shown in figure 2 which illustrates the filtering properties of the process as a low-pass filter.

Transfer Functions of different smoothing degrees are shown in figure 3 which can be interpreted as follows :

- For the frequency where the Transfer Ratio τ is unity, the original data are passed without change, i.e. No filtering is performed.
- For frequencies where τ is less than unity, the original data are attenuated by the smoothing process. The degree of attenuation of high frequencies increases as the value of W_1 decreases . With $W_1 = 0.5$, 50 to 70 % attenuation is observed for frequencies higher than 10 .
- The value $(1-W_1)$ represents the degree of attenuation at $\omega = 10$, e.g. at $W_1 = 0.7$, 30 % of the data which have frequency = 10 are absorbed by the process .
- The Transfer Functions of all smoothing degrees takes the shape of Gaussian curves, which always intersect with the vertical from $\omega = 10$, at the value of W_1 .

Figure 4 shows another way of representing the influence of the smoothing process on different terrain types . The root mean square residuals E, which is the norm of the residual function e , is plotted against the frequency for five degrees of smoothing, where :

$$e = \sum (\bar{Z} - Z) \quad (12)$$

$$E = \left(\sum_{i,j=1}^{n} (\bar{Z}(i,j) - Z(i,j)^2 / n \right)^{1/2} \quad (13)$$

From the resulting five curves, the following remarks can be observed :

- At frequencies near 20, the value of E tends to be equal to $(1-W_1)$.
- Linear relationship occures between E and W_1 at a constant frequency , i.e. at the same terrain type .
- At very low frequencies(flat terrain) the norm E tends to deminish .

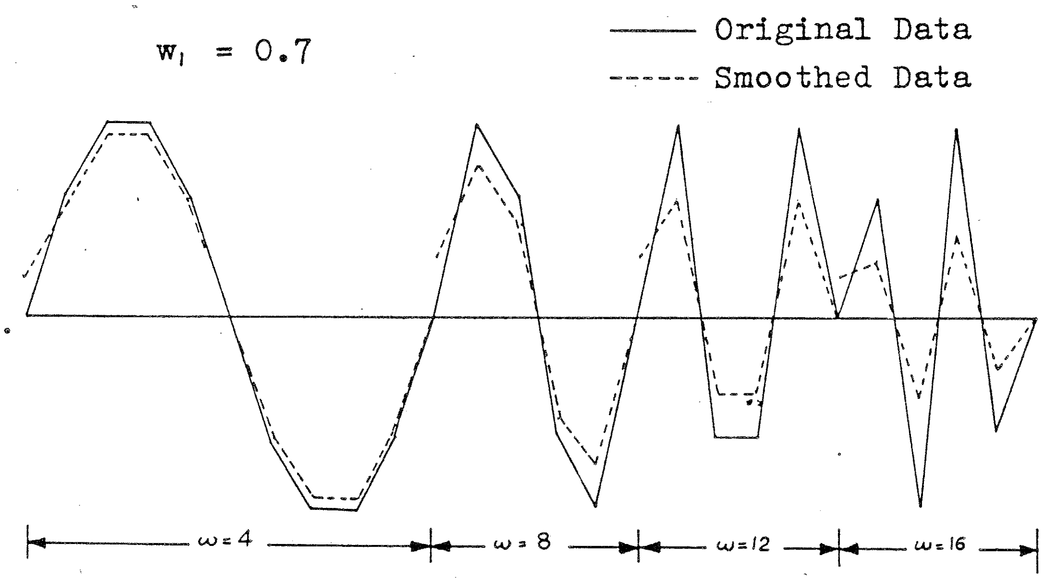

Fig.2 Effect of applying the same smoothing degree on different terrain types (Fourier Data) .

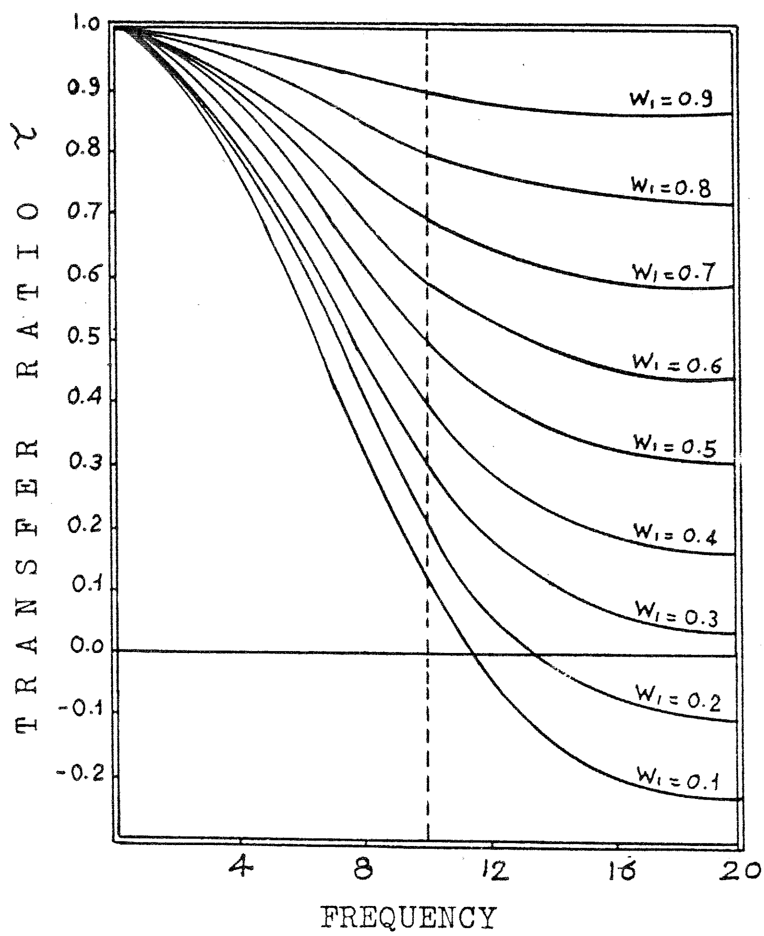

Fig.3 Transfer Functions for different smoothing degrees

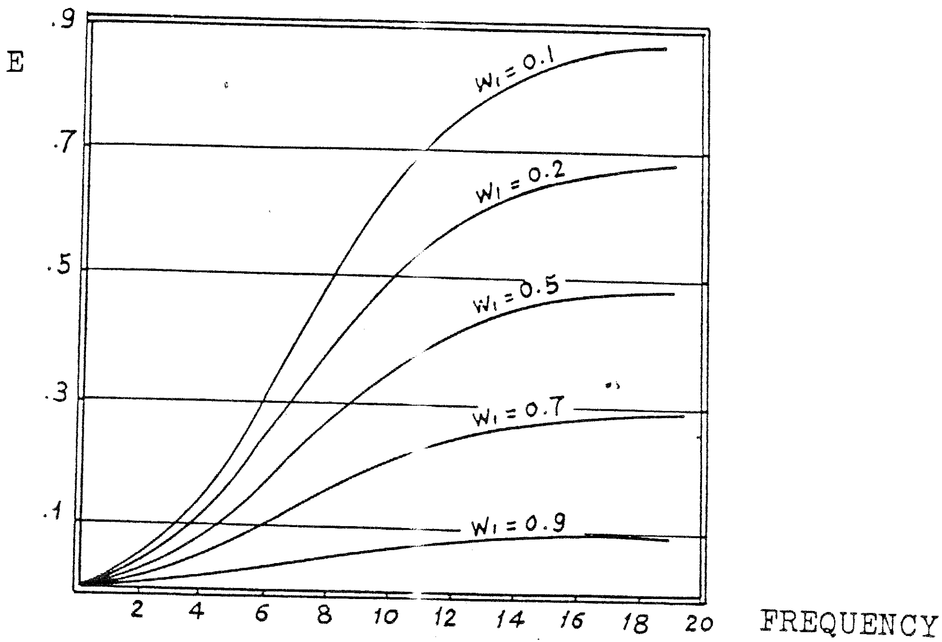

Fig.4 Relationship between the norm E and the frequency

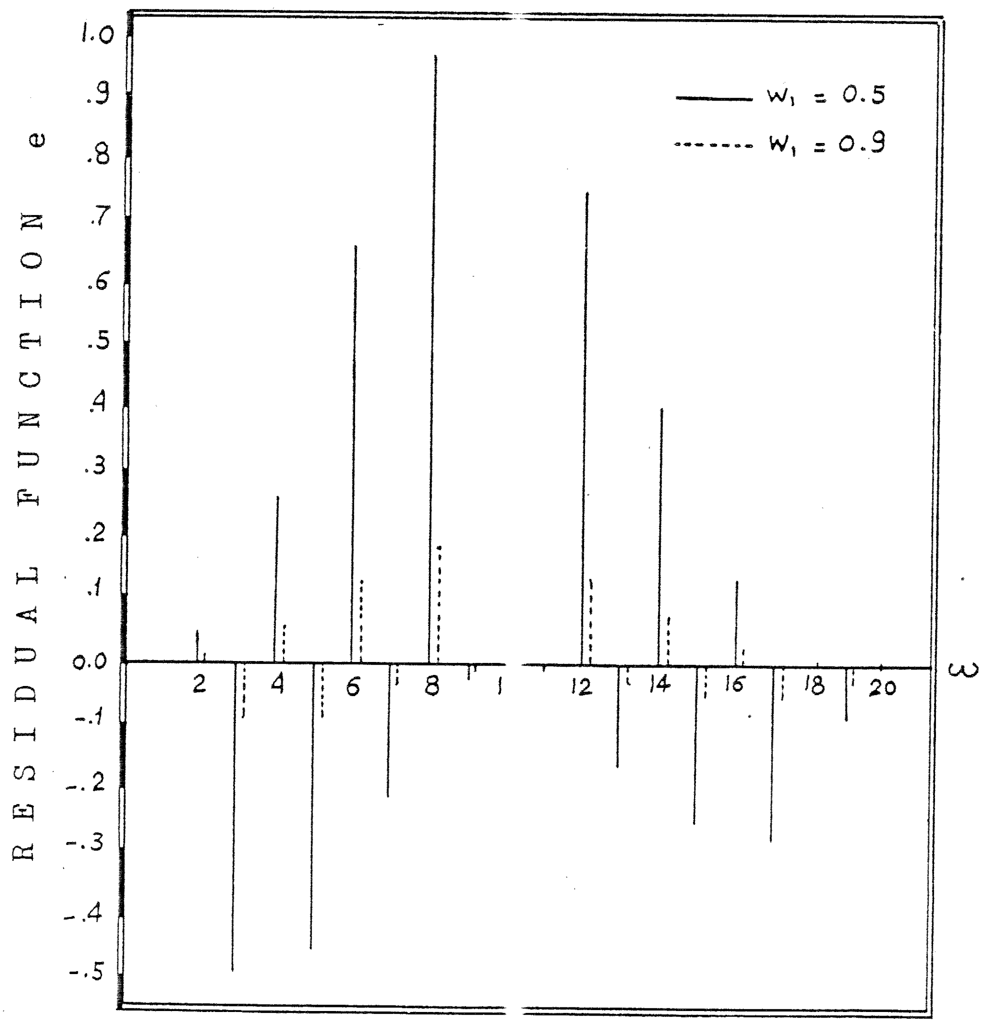

Fig.5 The fluctuations of e t different frequencies .

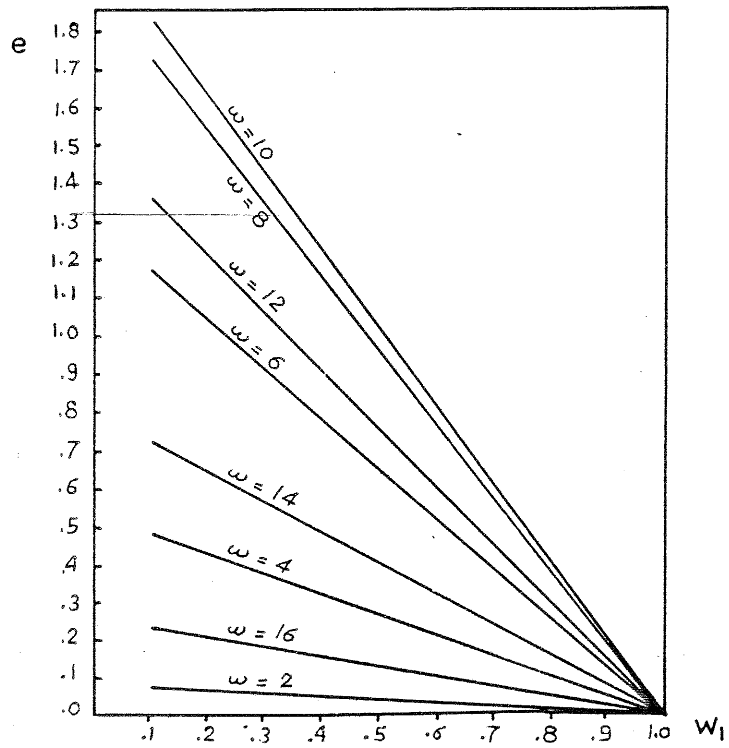

Fig.6 Linear relations between e and w_1 at even frequencies

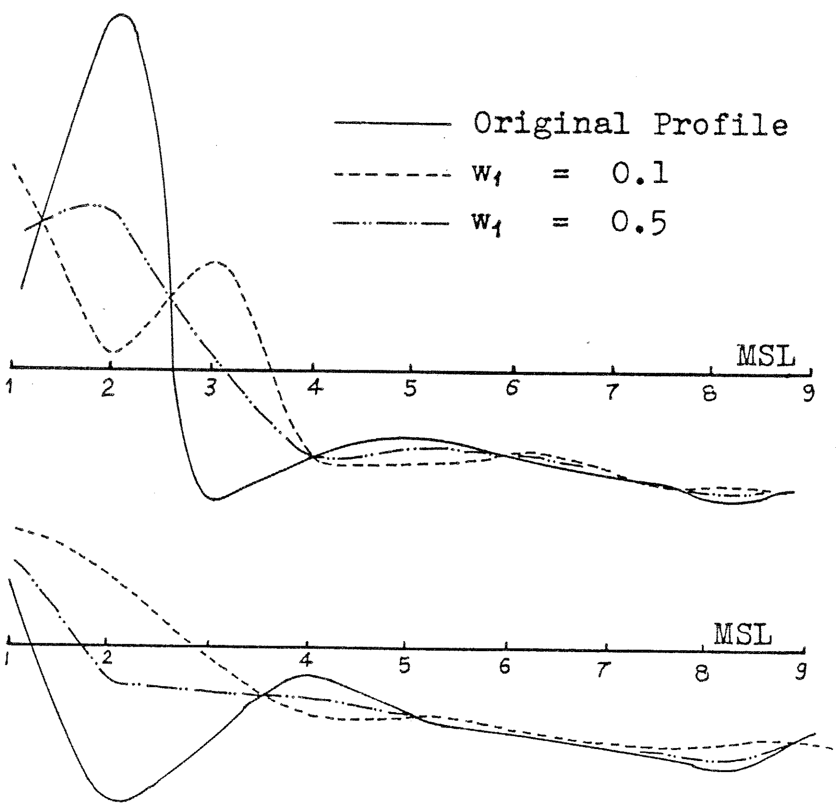

Fig.7 Examples from the Qattara DEM .

CONCLUSIONS AND RECOMMENDATIONS :

The bivariate smoothing process presented in this paper proved to be efficient and economic. The idea of employing only one parameter to change the smoothing degree makes the method flexible and easy to control. The use of the moving coordinate system results in small storage capacity which is independent on the block size or the number of points.

The analysis of the results of the Transfer Function program shows linear relationship between the norm E and the degree of smoothing at each frequency, this relationship is valid for all terrain types. Similar relationship also exists between the residual function e and the parameter W_1 at constant frequency, Fig.5 , which means that for a given terrain type, the increase in the value of W_1 reduces e linearly, Fig.6 .

These relationships facilitate the pre-determination of the smoothing degree which corresponds to a specified value of the residual function.

When applying the presented smoothing process on real DEM blocks, oversmoothing may occure in rough areas, Fig.7, this over smoothing can be controlled to filter out the extreme values which may be regarded as gross errors. However, this filtering effect is not unlimited as can be seen from figure 4. The maximum smoothing that can be achieved is equal to $(1-W_1)$ which is independent on the terrain type.

It is recommended to smooth DEM data before using them in a collocation surface fitting technique. The maximum smoothing degree is determined according to terrain type and the amount of noise expected in the data.

REFERENCES :

1. Lafara, R.L., 1973, "Computer Methods for Science and Engineering", Intertext, USA .
2. Makarovic, B., 1976, "A Digital Terrain Model System", ITC Journal-1, p:57-82 .
3. Scheid, F., 1968, "Theory and problems of numerical analysis", Schaum's outline series, McGraw Hill .
4. Tempfli, K., and Makarovic, B., 1978, "Transfer Functions of Interpolation Methods", ITC Journal-1, p:50-80 .
5. Whittaker, E., and Robinson, G., 1965, "The Calculus of observations", Blackie & son , London .