Analysis of Remaining Systematic Image Errors

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1. Introduction

The photogrammetry, using metric cameras, is based on the mathematical model of a central perspective. But the geometry of the photos is a little different from this model. The systematic differences are usually named as systematic image errors although they are errors of the mathematical model.
In the bundle block adjustment the main systematic image errors can be determined by self calibration, using additional parameters. The additional parameters can fit only the main systematics and they are not able to fit systematic errors with slight changes from photo to photo.
The remaining systematic image errors after precise bundle block adjustment with additional parameters are analysed.

2. Investigated Material

The following bundle blocks are analysed for remaining systematic image errors. The photos are measured with precise comparators or analytical plotters. The object points are signalised.

1 Eggebeck
camera: Zeiss RMK A 15/23 Pleogon A 2
sigma 0: 4,0 microns, endlap: 70 %, sidelap: 35 %
202 photos, mean photo scale 1 : 7000

2 Hordorf 1
camera: Wild RC 10 Universal-Aviogon I
sigma 0: 4,8 microns, endlap: 60 %, sidelap: 20 %
276 photos, mean photo scale 1 : 5800

3 Hordorf 34
camera: Zeiss RMK A 30/23 Topar A
sigma 0: 4,8 microns, endlap: 60 %, sidelap: 60 %
524 photos, mean photo scale 1 : 6100

4 Husum

camera: Zeiss RMK A 15/23 Pleogon A 2
sigma 0: 4,2 microns, endlap: 80 - 90 %, sidelap: 35 %
236 photos, mean photo scale 1 : 7100

5 Jaemijaervi

camera: Zeiss RMK A 15/23 Pleogon A 2
sigma 0: 3,2 microns, 2 * (endlap: 60 %, sidelap: 60 %)
96 photos, mean photo scale 1 : 4000

6 Koenigshuegel

camera: Zeiss RMK A 15/23 Pleogon A
sigma 0: 4,0 microns, endlap: 60 %, sidelap: 40 %
42 photos, mean photo scale 1 : 7700

7 Schwienkuhlen

camera: Hasselblad MK 70, Biogon and Pleogon
sigma 0: 4,5 microns, endlap: 86 %, sidelap: 68 %
146 photos, mean photo scale 1 : 13500, 1 : 8200

8 Willunga

camera: Zeiss RMK AR 15/23 Pleogon A R
sigma 0: 4,0 microns, endlap: 60 %, sidelap: 60 %
48 photos, mean photo scale 1 : 12600

3. Self Calibration and Methods for Analysis of Remaining Systematic Image Errors

The analysis of the remaining systematic image errors was made with the program system for bundle block adjustment of the University Hannover BLUH. In this program system a set of 24 additional parameters is used for fitting the systematic image errors of a group of photos (see 2.). The basic formulas of this set are similar to a Fourier analysis in polar coordinates. That means, the correlations between the additional parameters are small, if the photo points are regular distributed. Nevertheless, for getting optimal results of the bundle block adjustment, the computation should be done only with the additional parameters which must be used (see 1. and 2.). The additional parameters, which should not be included in the adjustment, are detected with 4 different statistical tests:

a) Correlation between additional parameters

$$r_{ij} = \frac{q_{ij}}{\sqrt{q_{ii} \cdot q_{jj}}}$$

r = correlation coefficiant

q = element of empirical cofactor matrix of unknowns

b) Total correlation
(information of the fitting of the influence of one additional parameter by the set of all other used parameters)

$$\underline{B}_i = \underline{I} - [(\text{diag } \underline{N}) \cdot (\text{diag } N^{-1})]^{-1}$$

$\underline{B}_i$ = coefficiant of total correlation (diagonal matrix)

I = identity matrix

diag $\underline{N}$ = diagonal of normal equation system

c) Student tèst of each additional parameter

$$\underline{t}_i = \frac{\hat{z}_i}{\sigma_o \cdot \sqrt{q_{ii}}}$$

$\hat{z}_i$ = value of the additional parameter

σ_o = standard deviation of unit weight

d) Student test of the a posteriori orthogonalised additional parameter

$$w_i = \frac{t_i}{\sigma_o \cdot \sqrt{\lambda_i}}$$

$$\underline{t} = \underline{S}^T \cdot \hat{\underline{z}}$$

λ_i = eigenvalues of $\underline{Q}_{pp}$

t_i = a posteriori orthogonalised additional parameter

S = modal matrix, columns = eigen vectors of $\underline{Q}_{pp}$

The additional parameters, which should not be included in the adjustment, are detected by the program itself on the basis of these tests (see 2.). If a part of the systematic image errors has a geometry, which cannot be fitted by the used formulas of the additional parameters or the systematic errors are not stable, these residual effects will have a negative influence to the block adjustment, but they cannot be detected without additional tests.

A simple, but effective method of the detection of remaining systematics is the computation of the mean of the residuals after bundle block adjustment. Every residual corresponds to a photo coordinate. Depending upon the photo coordinate, the residuals are meaned in photo sub areas. The value for each sub area indicates remaining systematics. Residuals of points, measured only in 2 photos are not used for this, because the partial redundance is too small.

With this test, the used set of additional parameters can be checked for completeness. Significant mean values are showing, the used formulas for additional parameters are not able to fit the systematics sufficiant. Some more parameters have to be added. The set used in the program system BLUH was able to fit the mean systematic image errors of every computed bundle block. But it is not possible to check changing systematics with this test.

Some blocks have been tested with additional parameters with influences limited to a group of photos, like strips or subblocks. In no case, the block accuracy, determined with independent check points, has been raised by using more than one group of additional parameters. Sigma 0 has been reduced and the block was able to fit better to not accurate control points, but the differences at independent check points have been raised. Only in the case of different outer conditions, like photo flight not in one day, the use of more than one group of additional parameters has been justified. These investigations have been made under the condition of the

MEAN VALUES OF RESIDUALS IN PHOTO SECTIONS. CAMERA 1. TOP:X, MIDDLE:Y, BOTTOM:NO. OF PHOTO PTS.

9 PHOTO SECTIONS

	-7	0	7 cm in the photo
7	-1.4 -2.3 211	-.1 1.6 130	.8 -.7 224
0	.7 -.9 147	2.2 .4 65	1.8 -.0 145
-7	-2.1 1.4 248	.5 -.6 142	.6 1.2 241

[microns]

25 PHOTO SECTIONS

	-9	-4	0	4	9 cm
9	-4.1 -4.8 45	-2.2 -.9 41	-.5 1.2 59	1.4 -.2 50	.9 -2.4 44
4	-.2 -3.7 56	-.0 -.4 69	.1 1.9 71	1.8 .9 71	-1.1 -1.9 59
0	1.2 -1.5 61	.4 -.4 86	2.2 .4 65	2.7 .6 80	.7 -.8 65
-4	-.4 1.7 66	-.8 -.4 71	1.0 -1.2 78	1.5 .3 66	-.3 1.1 65
-9	-4.9 4.3 48	-3.4 .7 63	-.2 .0 64	.2 1.4 64	1.0 2.7 46

9 PHOTO SECTIONS

	-7	[illegible]	7
7	-.1 .0 211	[illegible] .0 [illegible]	[illegible] -.[illegible] [illegible]
0	.2 -.2 147	.4 .2 65	.2 .1 145
-7	-.1 -.0 [illegible]	[illegible] .0 142	-.3 -.1 [illegible]

25 PHOTO SECTIONS

	-9	-4	0	4	9
9	.2 .3 [illegible]	-.1 -.2 [illegible]	[illegible] -.1 [illegible]	.2 -.3 [illegible]	-.5 .0 [illegible]
4	.1 -.2 [illegible]	-.4 .2 69	-.5 .1 71	.3 .1 [illegible]	-.5 -.1 59
0	.2 -.4 [illegible]	.2 -.1 [illegible]	.4 .2 [illegible]	-.3 .5 [illegible]	.7 -.4 [illegible]
-4	.2 -.1 [illegible]	-.2 -.2 71	.3 -.4 [illegible]	-.2 .1 [illegible]	-.2 -.1 65
-9	-.3 .3 [illegible]	-.2 -.0 [illegible]	.5 .5 [illegible]	-.4 .2 [illegible]	-.4 -.2 46

mean residuals in photo sub aereas

fig. 1: Block Jaemijaervi [microns]

without additional parameters | with additional parameters

orientation of the additional parameters to the camera. That means, if the flight direction was changed, also the orientation of the photo coordinate system was following.
The achieved results indicate no general changing of the systematic image errors, which can be determined with the used set of additional parameters.

A general analysis of remaining systematic image errors can be made by covariance analysis. But also the usual covariance analysis is not able to detect changing systematics. By this reason, the cross covariance is computed separate for photo combinations with the same difference in the photo number.

normed covariance

$$c_x = \frac{\Sigma r_{x1} \cdot r_{x2}}{n \cdot \sigma_0^2}$$

$$d = \sqrt{(x_1 - x_2)^2 - (y_1 - y_2)^2}$$

n = number of combinations

σ_0 = standard deviation of unit weight

r_{x1} = residual 1

r_{x2} = residual 2

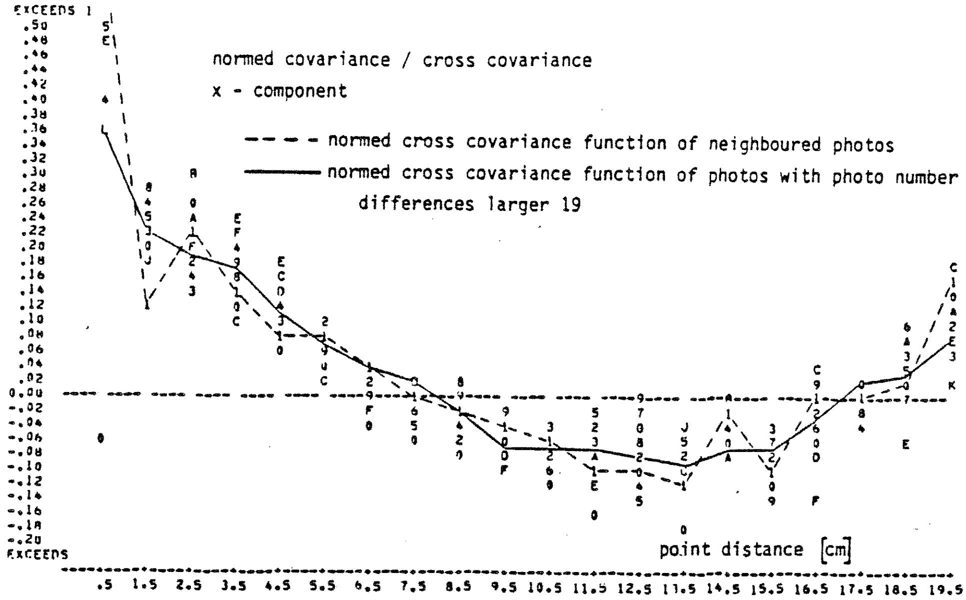
d_l = lower limit of point distance in the photo

d_u = upper limit

x,y = photo coordinates

separate computation of c_x for residuals r_{x1} from photos with same difference in photo number

Residuals of points, measured only in 2 photos are not respected, because the partial redundance is too small.



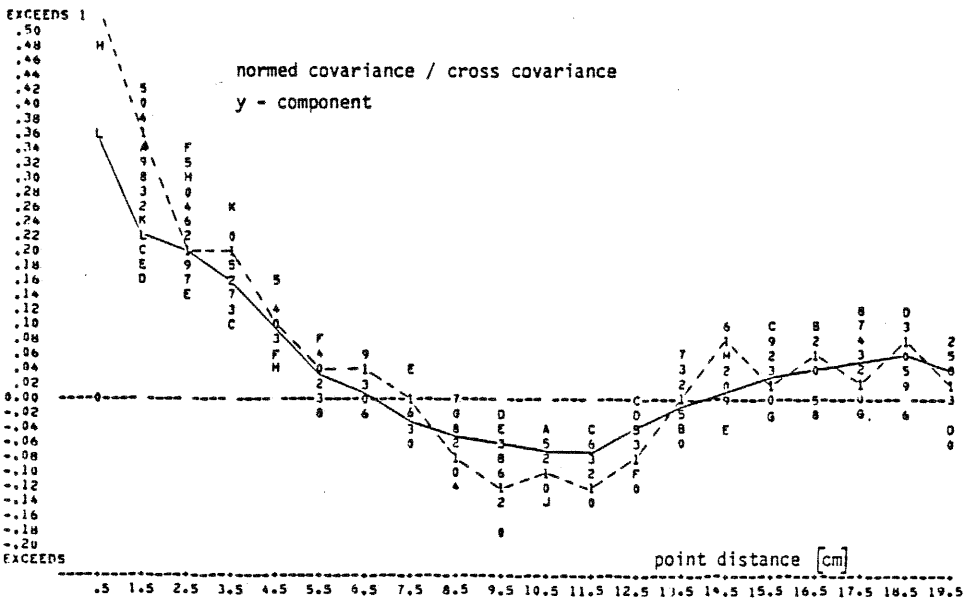


fig. 2: normed covariance function block Jaemijaervi
without additional parameters

0 = auto covariance (combination of the residuals in the same photo)

1 = cross covariance of neighboured photo (difference of photo numbers = 1)

2 = cross covariance - difference of photo numbers = 2

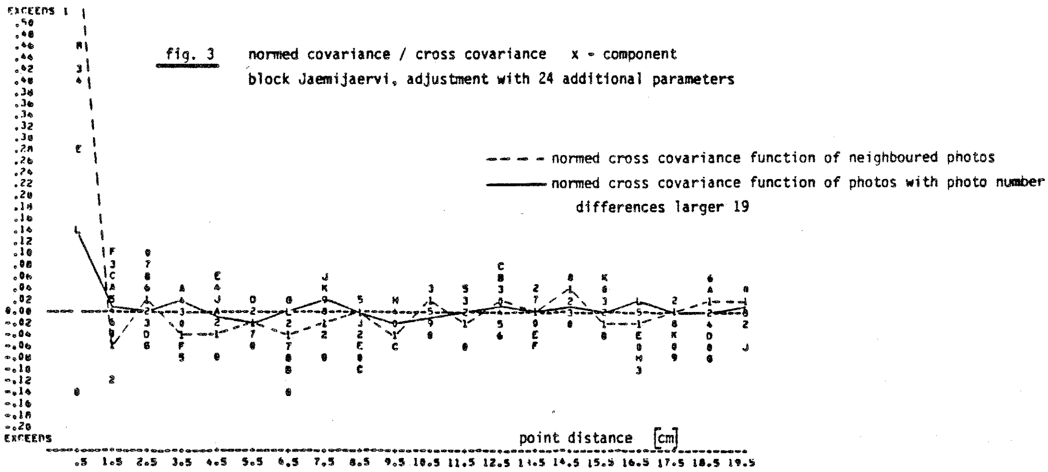
A corresponds to 10, B corresponds to 11

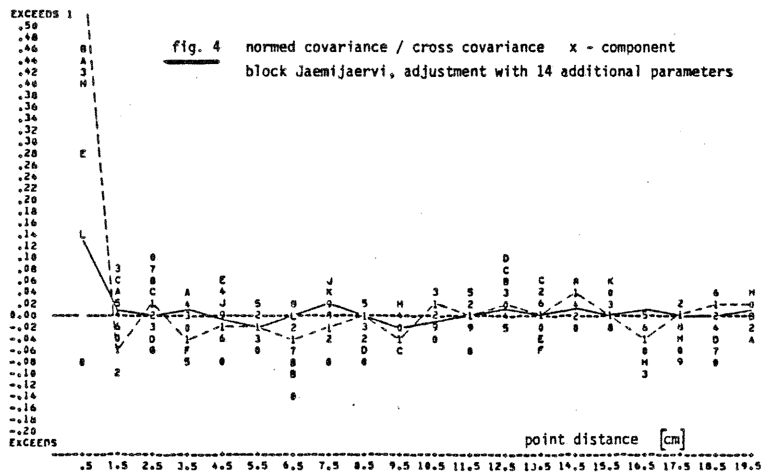
L = cross covariance of all photos with photo number differences larger 19

If more than one value is located in the same field, only the lowest alphanumeric is printed.

4. Covariance Analysis

The covariance functions are quite different between bundle block adjustments with or without additional parameters. But there is no significant difference between adjustments with the whole set of additional parameters (24) or the set, reduced with the tests described above (see fig. 2 and 3).





The auto covariance cannot be used for the analysis, because the residuals in the same photo are not independent. Neighboured points in the same photo usual have a negative correlation coefficiant, caused by the determistic and this is quite different from the cross covariance. The normed cross covariance of the residuals of a block adjustment with additional parameters to which the following explanations are limited, usual is small - in the case of Jaemijaervi it is between + 0.1 and - 0.1, in the mean 0.03. But there is the exception of the normed cross covariance of points in different photos, but with approximately the same location in the photo. The normed cross covariance for distances up to 1 cm is reaching a value of 0.68 in the block Jaemijaervi. This cross covarance is depending upon the neighbourhood of the photos - it is a little smaller if the difference of the photo numbers is larger.

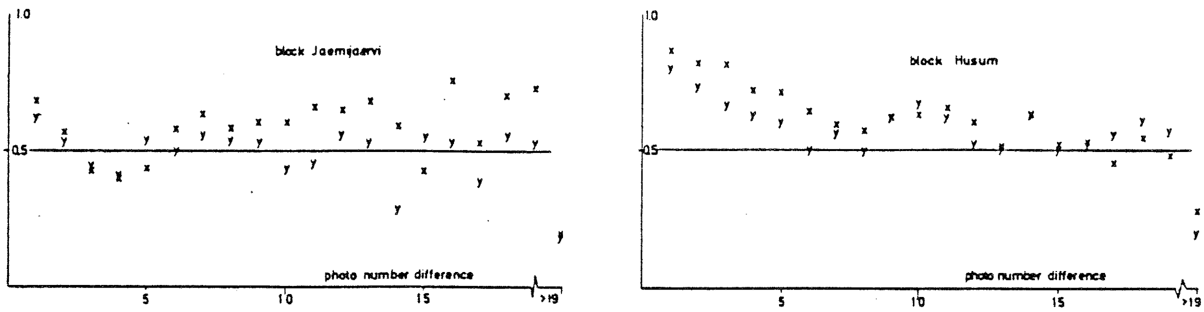


fig. 5: Jaemijaervi Husum

normed cross covariance for distances up to 1 cm as function of the difference of the photo numbers

The reduction of the cross covariance for distances up to 1 cm (fig. 5) with growing photo number differences is only small. In the block Jaemijaervi there is no difference between the direct neighboured photos and photos with photo number differences of 19 - for both the mean of cx and cy reaches 0.65, but in the case of the block Husum the mean of cx and cy is reduced from 0.83 to 0.53. The reduction can be seen very clear at the cross covariance of the residuals with photo number differences larger 19. This mean value is in both blocks identical to 0.17.
The normed cross covariance functions of the blocks Eggebeck and Koenigshuegel are similar but for Hordorf 1 and Hordorf 34 the value for distances up to 1 cm reaches only 0.45 and 0.32. In the case of Willunga and Schwienkuhlen, this effect was not existing. For both blocks reseau cameras have been used.

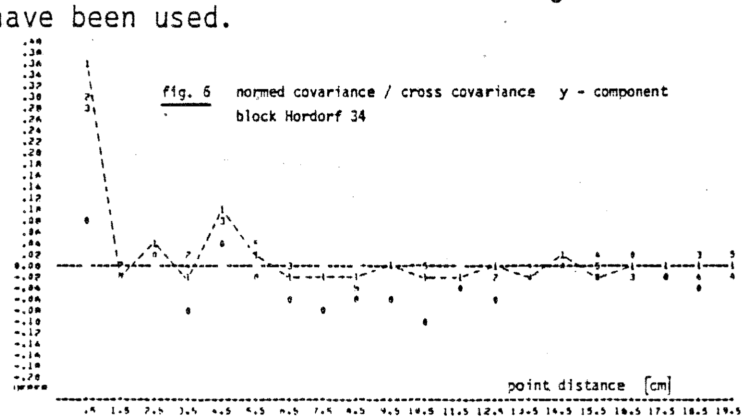


fig. 6 normed covariance / cross covariance y - component
block Hordorf 34

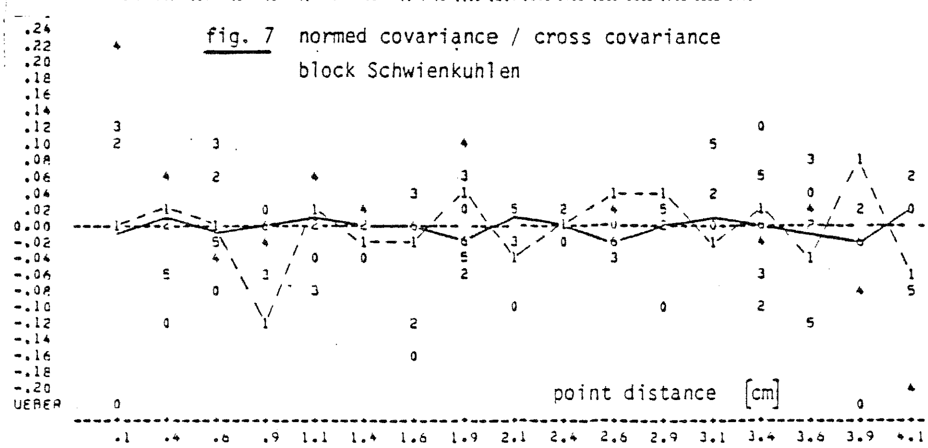


fig. 7 normed covariance / cross covariance
block Schwienkuhlen

In some blocks, the normed cross covariances for distances from 1 cm to 4 cm are larger 0 (see fig. 6). But there was no dependency upon the photo number difference. That means, the used additional parameters have not been able, to fit more or less local systematic errors.

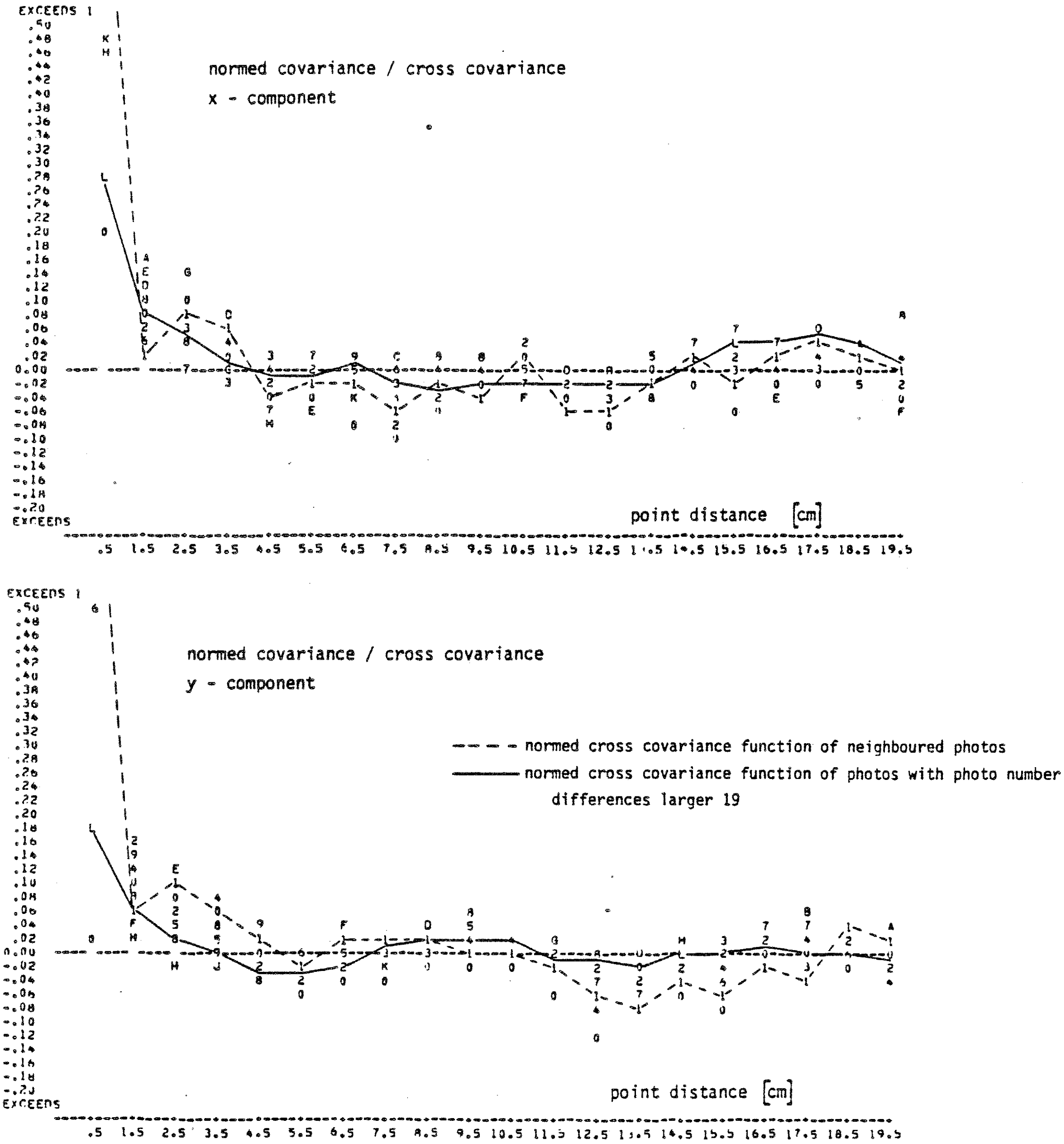


fig. 8: block Husum, adjustment with 15 additional parameters

No general difference between the normed cross covariance function for the x- and the y-component was existing (see fig. 6).

5. Summary

Local, not stable systematic image errors have been detected by normed cross covariance coefficiants. These effects cannot be fitted by the usual additional parameters. The size of the local systematics is different from camera to camera and it could not been detected at reseau cameras.

References

1. Jacobsen, K.: Vorschläge zur Konzeption und Bearbeitung von Bündelblockausgleichungen, Doctoral thesis, Hannover 1980

2. Jacobsen, K.: Selection of Additional Parameters by Program, ISP Com. III, Helsinki 1982