

A PROCEDURE FOR ON-LINE CORRECTION OF SYSTEMATIC ERRORS

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Bundesrepublik Deutschland

Kommission III

Abstract

Using the latest development in analytical and numerical photogrammetry a procedure for on-line correction of systematic and local-systematic errors in Analytical Plotters has been elaborated. Inspired by the finite element method for height interpolation, the proven strategy - to correct within local grid squares - has led to a modified and more flexible form. Using this method local-systematic errors of the photogrammetric system cannot only be discovered, but also corrected on-line after model formation. An example for extended calibration of the analytical plotter Planicomp C100 demonstrates the efficiency of the chosen method.

Introduction

The development in the field of analytical and numerical photogrammetry in the last two decades has caused an enormous increase in the accuracy of the results gained by photogrammetric means.

Refinement of the mathematical model permits a proper description of systematic and local-systematic errors caused by image deformations or instrumental deficiency. Using computer assisted calculations these errors can be corrected more easily in comparison to the methods of analogue photogrammetry. In many cases the use of calibrated grid plates has proven to be suitable for detecting systematic errors as mentioned above. The analysis and processing of information derived from grid plate measurements is the main theme of this paper.

Concerning réseau images the efficiency of different mathematical setups has been tested several times with practical examples /Ziemann 1971, Jacobsen 1980, Wester-Ebbinghaus 1983/. Their use solely refers to off-line procedures in the field of photogrammetric point determination.

Furthermore, Analytical Plotters offer the possibility to correct local-systematic effects directly at the stage of model formation as well as at the following stage of stereo plotting.[†] A falsification of orientation parameters can be suppressed and a higher degree of accuracy can be achieved.

The development of a procedure for on-line correction and actual publications about the application of the finite element method concerning height interpolation /Ebner 1979, Ebner 1983/ led to the realization of a suitable and effective model for systematic error correction.

Mathematical aspects

Starting-point for the following considerations is the presence of reference point measurements which contain information about systematic errors. This situation is given, for example, when grid plate measurements have been made for calibration of photogrammetric measuring equipment or when réseau images are to be corrected. In such cases it has proven to be effective to correct measurements using local functions derived from surrounding reference points /Ziemann 1971, Kratky 1972/, instead of determining a general correction function from all reference points -i.e. a high-grade polynomial- for the whole measuring range. Considering four neighbouring reference points the bilinear or pseudo-affine transformation in the general form

$$\begin{aligned}x' &= a_1 + a_2x + a_3y + a_4xy \\ y' &= a_5 + a_6x + a_7y + a_8xy\end{aligned}\tag{1}$$

is suitable for a correction within this local mesh.

If the reference point distribution represents a regular quadratic or rectangular grid, this transformation system, which has no redundancy, effects correspondence between the measurements made and their given nominal values, without further contradictions. Furthermore, it avoids gaps or overlaps along the edges of the meshes. This model is not unobjectionable seen from the

[†]Possibilities for on-line correction of radial-symmetric lens distortion, atmospheric refraction and general affine effects are commonly provided in Analytical Plotters ; therefore, they will not be taken into consideration here.

statistical point of view, because random errors which are not filtered influence the transformation parameter. Nevertheless the application is acceptable, if the systematic errors exceed the level of noise significantly /Kupfer 1971/. Following this strategy each grid mesh gets its own set of transformation parameters (according to eq. (1)) and local corrections are made available for measurements within a mesh. For the purpose of on-line correction this procedure has to be realized for the position of the left and the right photo carriage relative to the corresponding floating mark. This simple solution becomes somewhat more complicated for on-line processing, should the regular grid structure get lost (i.e. réseau crosses cannot be measured because of missing contrast).

However, the fundamental idea mentioned above can be maintained independent of the grid point disposition, if the residuals of irregularly distributed reference points are used to predict the deformations of grid points of an adequate regular grid. Subsequently an interpolation within the grid meshes follows. To realize these steps a procedure for interpolation with finite elements developed by H.Ebner which has been applied successfully to generate digital height models can be adopted /Ebner 1979, Ebner 1983, Ebner et al. 1984/. In this case, it has been assumed that the residual components of the reference points in the x- and the y-direction do not correlate with each other. Hence follows - in analogy to /Ebner 1983/- in a $m \times n$ unit grid for s reference points with the residual components δx_k in x-direction (see fig. 1) the resulting bilinear relation:

$$\begin{aligned}
 v(\delta x)_k &= (1 - \Delta x_k)(1 - \Delta y_k) \delta x_{i,j} + \Delta x_k(1 - \Delta y_k) \delta x_{i+1,j} \\
 &+ (1 - \Delta x_k) \Delta y_k \delta x_{i,j+1} + \Delta x_k \Delta y_k \delta x_{i+1,j+1} \\
 &- \delta x_k \\
 &k = 1, \dots, s
 \end{aligned} \tag{2}$$

Furthermore, error equations are formulated as in /Ebner 1983, Ebner et al. 1978/ resulting from the second residual component differences of neighbouring grid points:

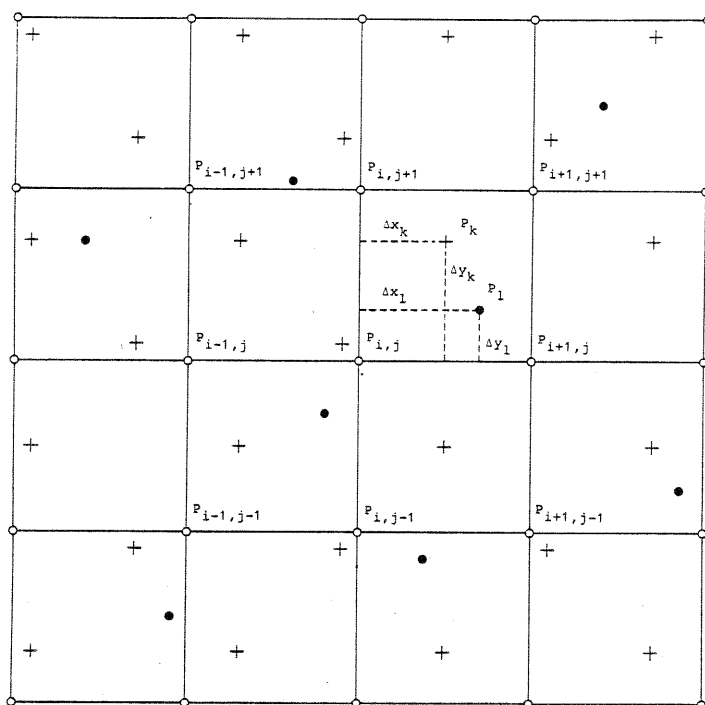
in x-direction:

$$v(\delta x)_{xx}_{i,j} = \delta x_{i-1,j} - 2 \delta x_{i,j} + \delta x_{i+1,j} - 0, \quad \begin{matrix} i=1, \dots, m-1 \\ j=1, \dots, n \end{matrix}$$

(3)

in y-direction:

$$v(\delta x)_{yy}_{i,j} = \delta x_{i,j-1} - 2 \delta x_{i,j} + \delta x_{i,j+1} - 0, \quad \begin{matrix} i=1, \dots, m \\ j=2, \dots, n-1 \end{matrix}$$



- - reference point
- - grid point
- + - image point

fig.1: reference points and image points
within a regular interpolation grid

A further equation system corresponding to eq.(2) and (3) can be set up independently for the residual components δy_k . The reference point observations as well as the second residual component differences of neighbouring grid points can be evaluated individually. Nevertheless, effective filtering of random errors require an optimal choice of the a priori accuracy of the observations. A fairly good variance estimation can be obtained for the reference point observations from multiple measurements, whereas the estimation of a reliable stochastic model for the second residual component differences of neighbouring grid points may be problematic (see /Ebner 1983/).

The residuals of the grid points can be estimated by adjustment of intermediate observations. All the mentioned equations are linear by definition, thereby avoiding iterations and rendering a direct solution.

As a result this setup does not guarantee continuous differentiability when passing from one mesh to another; however, it allows an exact linkage between the meshes and permits a fast prediction of the unknowns /Stark et al. 1982/.

Moreover, with this procedure reference point measurements can be limited to the particular area of concern and the grid constants can be chosen optimally depending on the density of the reference points.

When the measuring range of interest is covered by grid points with calculated deformation components $\delta x_{i,j}$, $\delta y_{i,j}$, then to achieve the desired "ideal" sys-

tem, the t image points within their grid meshes can be corrected in x -direction according to

$$\begin{aligned} \delta x_l = & (1 - \Delta x_l)(1 - \Delta y_l) \delta x_{i,j} + \Delta x_l(1 - \Delta y_l) \delta x_{i+1,j} \\ & + (1 - \Delta x_l) \Delta y_l \delta x_{i,j+1} + \Delta x_l \Delta y_l \delta x_{i+1,j+1} \end{aligned} \quad (4)$$

$$l=1, \dots, t$$

and in the y -direction by analogy (see fig.1).

For implementation in analytical plotters all homologous image points participating in the orientation must be corrected acc. to eq. (4) in order to suppress the influence of the detected systematic errors on the orientation parameters. At the stage of stereo plotting the actual coordinates of the left and the right photo carriage are calculated in real time from the model coordinates which have been set up by the hand and the foot wheels. This transformation leads to an "ideal" coordinate system in which homologous image points do not coincide with the floating marks unless corresponding deformation components are added. These deformation components can easily be gained by defining the regular grid in the "ideal" instead of in the "real" system and proceeding as mentioned above.

For on-line processing the regular grid structure proves to be advantageous, because the needed parameters for interpolation which depend on the actual position of the floating marks within the grid can be made available very quickly.

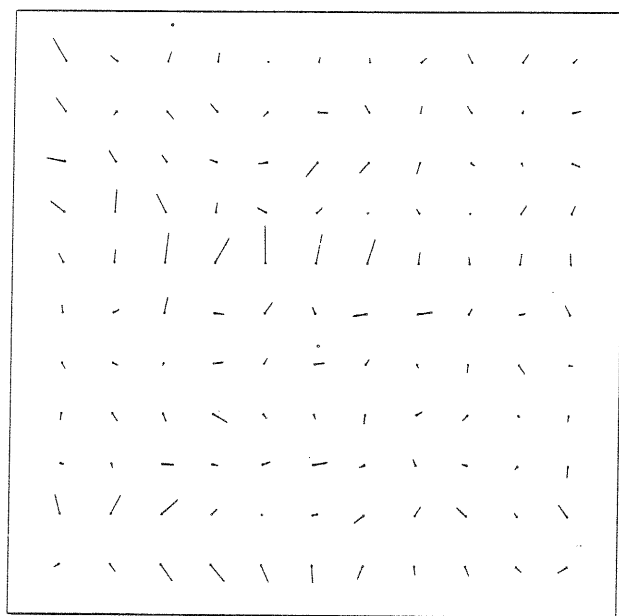
This setup for meshwise interpolation has been realized for the Planicomp C100 as an integrated part of a photogrammetric software package /i.e. Wester-Ebbinghaus 1983/.

An example for the efficiency

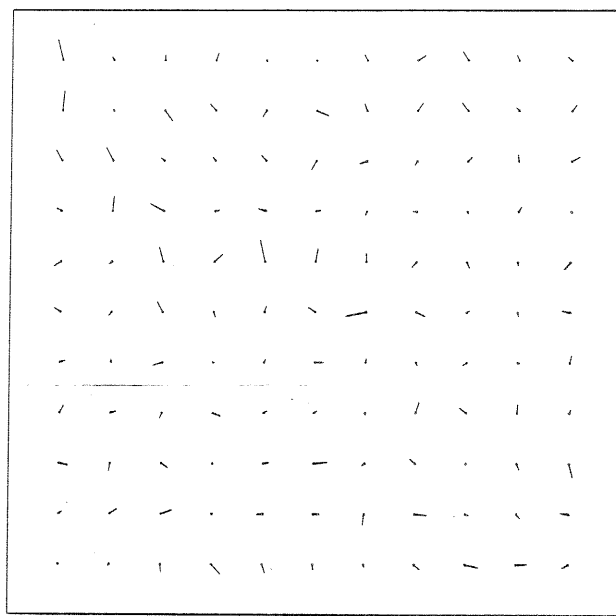
The example deals with an extended calibration of the measuring device, Planicomp C100. For this purpose, calibrated grid plates⁺ with $23 * 23$ grid points and a grid constant of 1 cm were measured. After a general affine transformation for the whole measuring range eliminating global trend, local transformation parameters were calculated as described in the previous paragraph. In the next step the control points with well-known nominal values which were not involved in the determination of the transformation parameters were submitted to a meshwise interpolation. The remaining residuals give information about the efficiency of the method when compared with the results of a standard calibration. Standard calibration for a Planicomp C100 means an affine transformation with $3 * 3$ grid points well distributed over the area of concern /Hobbie 1980/.

In fig. 2(a) and 2(b) the results of these two methods are presented as a typical example. The accuracy expressed by the mean square of the residual vectors increases by nearly 30% from $\pm 2.8 \mu\text{m}$ to $\pm 2.1 \mu\text{m}$, when using extended calibration. Concerning visual systematic effects a great decrease in row- and columnwise appearing systematic errors (i.e. resulting from electronic or mechanical backlash) can be observed.

⁺The calibration report guarantees an accuracy of $\pm 1 \mu\text{m}$ for the grid points.



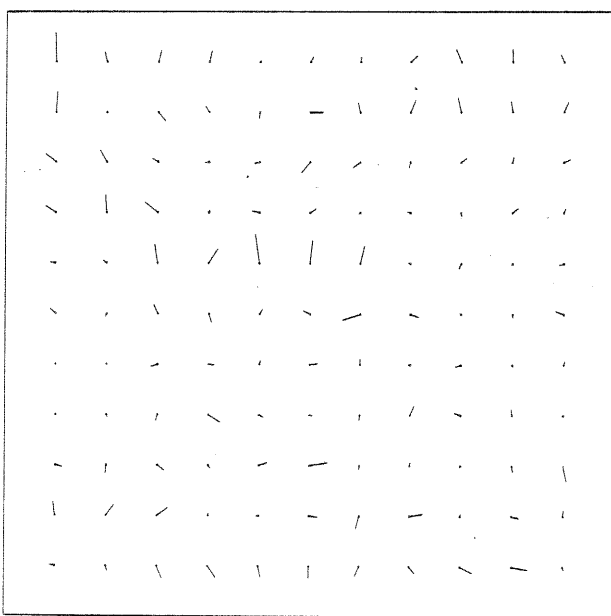
QUADRATIC MEAN: 2.9 MICRONS
 SCALE FOR RESIDUALS: ——— 10 MICRONS
 IMAGE SCALE: ——— 50 MILLIMETER



QUADRATIC MEAN: 2.1 MICRONS
 SCALE FOR RESIDUALS: ——— 10 MICRONS
 IMAGE SCALE: ——— 50 MILLIMETER

fig. 2(a): residuals of control points
 after standard calibration

fig. 2(b): residuals of control points
 after extended calibration
 (grid constant = 1 cm)



QUADRATIC MEAN: 2.5 MICRONS
 SCALE FOR RESIDUALS: ——— 10 MICRONS
 IMAGE SCALE: ——— 50 MILLIMETER

fig. 2(c): residuals of control points
 after extended calibration
 (grid constant = 2 cm)

But it must be noted that the density of reference points has to be sufficient for detecting local-systematic errors. Fig. 2(c) shows the situation after an extended calibration with a grid constant of 2 cm (144 grid points). The accuracy decreases to $\pm 2.5 \mu\text{m}$, but even more important, local-systematic errors can hardly be corrected.

A comparison of the results of standard and extended calibration from measurements made by three other Planicomp measuring devices with different operators showed a similar increase of accuracy, although, the absolute level of accuracy varied in some cases. At the moment it is being examined, to what degree this improvement effect the orientation parameters and the model coordinates.

Another suitable application for the presented procedure is the detection and on-line correction of local-systematic image deformations by *réseau* images. In this case the greater efficiency of meshwise interpolation compared to other current transformations has been confirmed as well /Kotowski 1984/.

Conclusions and prospects

First experiences with the analytical plotter Planicomp C100 indicate that the presented procedure can lead to a remarkably high level of accuracy in the field of analytical and numerical photogrammetry, provided that the distribution of the reference points shows sufficient density. However, the application of this procedure is, of course, not limited to the mentioned possibilities.

Specifically, given or calculated systematic errors in the photogrammetric surveying and evaluating field can, for example, also be corrected on-line. Concerning the surveying field following deviations from the collinearity condition can be mentioned:

- In the optical system (i.e. radial-asymmetric and decentering lens distortion or ray deflection caused by fish-eye lenses)
- In the object space (i.e. refraction when making multi-media photographs /Masry et al. 1970/ or projection distortions of the ground survey data when using photographs made by satellite).

Concerning the evaluating field a correction of systematic alterations of the optical path in an Analytical Plotter caused by separate zoom-optics and anamorphote lenses is conceivable, when these are placed between the image plane and the floating mark. For a stereoscopic pair of photographs with different photo scales and for a high convergent pair of photographs an increase in accuracy could be possible.

Acknowledgement

For the helpful suggestions in translation we are grateful to Miss Atessa Sadrieh.

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