Aerial triangulation by independent models; the covariance matrix of the observations and their influence.

G.H. Ligterink

Department of Geodesy, Delft University of Technology
The Netherlands
Commission III/1.

1. Introduction

In the past considerable investigations have been done to produce programs for fotogrammetric blockadjustments. The method of 3-dimensional independent blockadmustment is nowadays one of the most usual methods. Since a couple of years several studies have been made to gain an insight in the reliability of the adjusted block coordinates, based on the theory of Baarda, Baarda 1968.

The reliability of the testing concerns the magnitude of errors, which just can be detected. The conditions under which errors can be found depends among other things of:

- the condition model, in this case an orthogonal transformation of the modelcoordinates, including the coordinates of the perspective centre.
- the stochastical model, the variance-covariance matrix of the observations, which are the modelcoordinates, the coordinates of the perspective centre and the terrestrial coordinates of control points.
- what types of error will be detected, we will here restrict ourselves to the alternative hypothesis, which means, only one observation has an error and all others are correct.

In this study special attention will be paid to the variance-covariance matrix of the observations. The influence of different variance-covariance matrices of the observations on the internal reliability of photogrammetric bloks will be studied and analysed by means of an artificial block.

2. Theory

a. <u>Condition model and adjustement</u>

The condition model in 3-dimensional blockadjustment is based on the assumption that the photogrammetric model, formed by a stereopair of photographs, is similar to the model of the terrain. This condition can mathematically be described by an orthogonal transformation:

$$\begin{pmatrix} X^k \\ Y^k \\ Z^k \end{pmatrix}_T = \lambda_\rho (R)_\rho \begin{pmatrix} x^k \\ y^k \\ z^k \end{pmatrix}_\rho + \begin{pmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{pmatrix}_\rho \qquad (1)$$

the 7-unknowns of model ρ are:

λ_ρ : scale factor

$(R)_\rho$: orthogonal matrix, 3 rotations

$(\Delta X, \Delta Y, \Delta Z)_\rho$: 3 translations further: $(X^k, Y^k, Z^k)_\rho$:coordinates of modelpoints, including the coordinates of perspective centres

$(X^k, Y^k, Z^k)_T$:coordinates of terrainpoints, which can be unknown or known control points

The adjustment is based on the second standard problem for non-linear equations. The formulae are omitted here.

The covariance matrix (g_{ij}) of the observations.
The observations are:
- machine coordinates of modelpoints;
- machine coordinates of projective centres.

In this study only a short general discription of the covariance matrix will be given. For a more detailed study see Ligterink 1972 and 1970.

Model points.
The random observation errors can be divided in three groups:
1. measuring of a model point: the measuring mark is set at a proper elevation and in a proper planimetric position;
2. relative orientation: the elimination or measuring of the y-parallaxes;
3. inner orientation: the position of the photographs in the plate holders and the setting of the principal distances.

The covariance matrix of the machine coordinates is determined by the standard deviations of these three groupes of observations. These three groups will only shortly be described.

1. Measuring of a model point.
   We write:

$$(\Delta x^i_M) \equiv \begin{pmatrix} \Delta X_M \\ \Delta Y_M \\ \Delta h_M \end{pmatrix}$$: the differentials of the machine coordinates

$$(\Delta M) \equiv \begin{pmatrix} \Delta p_x \\ \Delta x' \\ \Delta y' \end{pmatrix}$$

Δp_x : horizontal parallax

$\Delta x'$: x-setting

$\Delta y'$: y-setting

The relation between the differentials of machine coordinates and the observations, $\Delta p_x, \Delta x'$ and $\Delta y'$, can generally be written as:

$$(\Delta x^i_M) = (A^i_M)(\Delta M) \qquad (2a)$$

The elements of (A^i_M) are made up by model coordinates of the point concerned and the dimension of the model.
The covariance matrix can be computed by applying the law of propagation of errors to (2a):

$$(\sigma_{x^i_M x^j_M}) = (A^i_M)(\sigma^2_M)(A^j_M)^T \qquad (2b)$$

(σ^2_M) : a diagonal matrix with 3 elements, the standard diviations of the observations:

$\sigma^2_{\Delta p_x}$: for the horizontal parallax

$\sigma^2_{\Delta x'}$: for the x-setting

$\sigma^2_{\Delta y'}$: for the y-setting

The covariance matrix $(\sigma_{x^i_M x^j_M})$ in (2b) describes the influence of the random observation errors due to the measuring of a modelpoint. It gives the standard deviations of the coordinates and the correlation of the coordinates of a single point. As a matter of course the correlation between the coordinates of different points is zero.

2. Relative orientation

The second group of observations are the y-parallaxes on behalf of the relative orientation.

The relation of the differentials of machine coordinates and orientation elements can generally be written as follows.

We write:

$$(\Delta x^i_O) \equiv \begin{pmatrix} \Delta x_O \\ \Delta y_O \\ \Delta h_O \end{pmatrix}$$: the differentials of the machine coordinates

$$(\Delta O) \equiv \begin{pmatrix} \Delta\omega_2 \\ \Delta\phi_2 \\ \Delta\kappa_2 \\ \Delta bz_2 \\ \Delta by_2 \end{pmatrix}$$: the differentials of the orientation elements

The relation is generally:

$$(\Delta x^i_O) = (A^i_O)\,(\Delta O) \qquad (3a)$$

Applying the law of propogation of errors gives:

$$(\sigma_{x^i_O x^j_O}) = (A^i_O)\,(\sigma_{OO})\,(A^j_O)^T \qquad (3b)$$

with the covariance matrix of the orientation elements:

$$(\sigma_{OO}) = \sigma^2_{p_y}\ \overline{(\Delta O), (\Delta O)}^T$$

$\sigma^2_{p_y}$: standard deviation of y-parallax observation

(σ_{OO}) : a matrix of 5 by 5, the covariance matrix of the relative orientation elements

The covariance matrix $(\sigma_{x^i_O x^j_O})$ in (3b) describes the observation errors due to measuring or eleminating the y-parallaxes. The standard deviation of the coordinates and the correlation between coordinates of single points and between the coordinates of different point are both determined.

3. Inner orientation

The third group of observations which influence the precision of the machine coordinates are the elements of inner orientation. These elements can be discribed by three variables of each camera:

$\Delta x', \Delta y', \Delta c'$: left camera

$\Delta x'', \Delta y'', \Delta c''$: right camera

The relation of the differentials of machine coordinates and inner orientation elements can generally be written as follows:
We write:

$$(\Delta x_I^i) \equiv \begin{pmatrix} \Delta x_I \\ \Delta y_I \\ \Delta h_I \end{pmatrix}$$: the differentials of the machine coordinates

$$(\Delta I) \equiv \begin{pmatrix} \Delta x' \\ \Delta x'' \\ \Delta y' \\ \Delta y'' \\ \Delta c' \\ \Delta c'' \end{pmatrix}$$: the differntials of the inner orientation elements.

The relation is:

$$(\Delta x_I^i) = (A_I^i)(\Delta I) \tag{4a}$$

Applying the law of propagation of errors gives:

$$(\sigma_{x_I^i x_I^j}) = (A_I^i)(\sigma_I^2)(A_I^j)^T \tag{4b}$$

(σ_I^2) : a diagonal matrix with 6 elements, the standard deviations of the observations:

$\sigma^2_{x'} = \sigma^2_{x''} = \sigma^2_{y'} = \sigma^2_{y''}$: the standard deviations of centering the photo

$\sigma^2_{c'} = \sigma^2_{c''}$: the standard deviations of principal distance setting

The convariance matrix of machine coordinate can be computed by addition of the influence of the observation errors resulting from:

$(\sigma_{x_M^i x_M^j})$: measuring of a model point, see (2b)

$(\sigma_{x_O^i x_O^j})$: relative orientation, see (3b)

$(\sigma_{x_I^i x_I^j})$: inner orientation, see (4b)

Simple addition can in fact be applied as the three groups of observations are mutually free of correlation.

$$(\sigma_{x^i x^j}) = (\sigma_{x_M^i x_M^j}) + (\sigma_{x_O^i x_O^j}) + (\sigma_{x_I^i x_I^j}) \quad (5)$$

Perspective centres.
The coordinates of the perspective centre can be determined by spatial resection or intersection. The method of intersection from two or more levels in model space has several advantages. The optimum is 6 well defined points in two levels; see Ligterink 1970.

b. Internal reliability

The reliability of the observations is studied according the "B method of testing" of Baarda 1968. The internal reliability is described by the boundary values of the observations, according:

$$(\nabla x^i) = \sigma(c^i) \sqrt{\lambda_o / N} \quad (6)$$

(∇x^i) : boundary value of the observation

σ : the variance factor

c^i : the vector defining an error in the observation

λ_o : the level of λ_o is determined by:

$$\lambda_o = \lambda(\alpha_o, \beta_o, 1, \infty)$$

β_o : probability value (= 0.8)
α_o : significance level (= 0.001)

$$N = (c^j)(\bar{g}_{ij})(g^{ij} - G^{ij})(\bar{g}_{ji})(c^i)$$

(g^{ij}) : covariance matrix of the observations

$$(\bar{g}^{ij}) = (g^{ij})^{-1}$$

(G^{ij}) : covariance matrix of the adjusted observations

3. Articficial block

In this paragraph the influence of the covariance matrix, (g^{ij}) as given in (5b) on the internal reliability, e.q. the boundary values of the observations will be demonstrated.
In order to examine this phenomenon we take a 3-dimensional artificial block measured with an analogue stereo-instrument with the following data:

size of the photo's	:	23 x 23 cm
principal distance	:	150 mm
photoscale	:	1:4000
modelscale	:	1:2000
forward overlap	:	60%
side lap		20%
number of strips	:	3
models/strip	:	4
height differences	:	0

Two cases are distinguished for tie points which are well defined points in the threefold overlap:

a. only one point in each corner of the model;
b. double points in the corners of the model;

In order to study the influence of the covariance matrix of the observations (g^{ij}) we distinguish three cases:

A: (g^{ij}) is a diagonal matrix with elements for model points and projective centres:

$$\sigma_x = \sigma_y = \sigma_z = 10\mu$$

B: (g^{ij}) is a diagonal matrix with the following elements:
model points:

$$\sigma_x = \sigma_y = 10\mu, \ \sigma_z = 20\mu$$

projective centres:

$$\sigma_x = 11\mu, \ \sigma_y = 12\mu, \ \sigma_z = 24\mu$$

C: (g^{ij}) is a full covariance matrix for both modelpoints and projective centres.

Projection centres, the full covariance matrix.
According to Ligterink 1970, the coordinates of the projection centres are determined by spatial inter section in two levels in model space; six well defined points are measured in the highest and lowest position of the XY-corriage of the analogue instrument which gives the following covariance matrix for the projection centres expressed in μ^2:

	ΔX	ΔY	ΔZ
ΔX	$+(11.1)^2$	0	$+(11.0)^2$
ΔY		$+(11.5)^2$	
ΔZ	$+(11.0)^2$	0	$+(24.1)^2$

Remark: the correlation between the X- en Z-coordinate is positive for the right camera and negative for the left camera; see Ligterink 1970, page 14.

Model points, the full covariance matrix.
The covariance matrix of the model points is computed according the formulae fiven in paragraph 2. The following standard deviations in photo scale are introduced.

Measuring of a model point, assuming a well defined point, see (2b) and Ligterink 1972, page 9:

$$\sigma_{\Delta p_x} = 4.9\mu; \ \sigma_{x'} = 4.6\mu \ \text{and} \ \sigma_{y'} = 6.0\mu$$

Relative orientation, see (3b) and Ligterink 1972, page 24, assuming numerical relative orientation in the well-known six points with:

$$\sigma_{p_y} = 9\mu$$

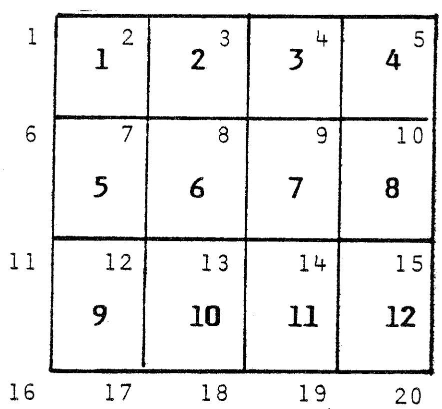
Inner orientation, see (4b) and Ligterink 1972, page 45:

$$\sigma_{x'} = \sigma_{x''} = \sigma_{y'} = \sigma_{y''} = 20\mu$$

$$\sigma_{c'} = \sigma_{c''} = 3\mu$$

For complete description of the determination of the covariance matrices of model points is referred to the literature already mentioned.
The results are presented in the following way.

Block: numbering of models and points:



1.......12: number of models

1.......20: number of points

Remark:
two blocks will be studied.

a. tie points in the corner of the models are singel points, number of points without accent, tables 1, 2 and 3.
b. tie points in the corner of the models are double points, number of points with accent tables 4, 5 and 6.

For reason of symmetry the boundary values of only a limited number of points will be given here:

three points of the border of the blocks:
- in model **2** point 2 and 2'
- in model **1** point 6 and 6'
- in model **2** projection centre left (p.c. left)

two points in the centre of the blocks:
- in model **6** point 8 and 8'
- in model **6** projection centre right (p.c. right)

Aa	∇ x	∇y	∇ z
model 2 point 2	0.202	0.100	0.101
model 1 point 6	0.107	0.117	0.115
model 6 point 8	0.085	0.073	0.078
model 2 p.c.left	0.188	0.097	0.094
model 6 p.c.right	0.157	0.103	0.087

Table 1: the boundary values in m in terrain scale;
(g^{ij}) of the observations is a diagonal matrix as defined under A:
model points and projection centres:

$$\sigma_x = \sigma_y = \sigma_z = 10\mu$$

tie points are singel points.

Ba	∇x	∇y	∇ z
model 2 point 2	0.237	0.147	0.167
model 1 point 6	0.113	0.125	0.196
model 6 point 8	0.088	0.085	0.135
model 2 p.c.left	0.269	0.154	0.174
model 6 p.c. right	0.245	0.159	0.169

Table 2: the boundary values in m in tarrain scale;
(g^{ij}) of the observations is a diagonal matrix as defined under B:
model points:

$$\sigma_x = \sigma_y = 10\mu, \ \sigma_z = 20\mu$$

projection centres:

$$\sigma_x = 11\mu, \ \sigma_y = 12\mu, \ \sigma_z = 24\mu$$

tie points are single points

Ca	∇ x	∇y	∇ z
model 2 point 2	0.431	0.193	0.299
model 1 point 6	0.155	0.230	0.363
model 6 point 8	0.122	0.132	0.211
model 2 p.c. left	0.591	0.440	0.175
model 6 p.c. right	0.507	0.393	0.172

Table 3: the boundary values in m in terrain scale;
(g^{ij}) is a full matrix as defined under C.
tiepoints are single points.

Ab	∇x	∇y	∇z
model 2 point 2'	0.081	0.074	0.075
model 1 point 6'	0.072	0.072	0.078
model 6 point 8'	0.060	0.058	0.061
model 2 p.c. left	0.143	0.080	0.078
model 6 p.c. right	0.109	0.079	0.074

Table 4: the boundary values in m in terrain scale;
(g^{ij}) of the observationsis a diagonal matrix as defined under A:
model points and projection centres:

$$\sigma_x = \sigma_y = \sigma_z = 10\mu$$

tie points are double points.

Bb	∇x	∇y	∇z
model 2 point 2'	0.082	0.079	0.142
model 1 point 6'	0.072	0.072	0.150
model 6 point 8'	0.060	0.060	0.116
model 2 p.c. left	0.236	0.120	0.157
model 6 p.c. right	0.161	0.117	0.155

Table 5: the boundary values in m in terrain scale;
(g^{ij}) of the observations is a diagonal matrix as defiend under B:
model points:

$$\sigma_x = \sigma_y = 10\mu, \quad \sigma_z = 20\mu$$

projective centres:

$$\sigma_x = 11\mu, \quad \sigma_y = 12\mu, \quad \sigma_z = 24\mu$$

tiepoints are double points.

Cb	∇x	∇y	∇z
model 2 point 2'	0.090	0.099	0.105
model 1 point 6'	0.073	0.117	0.121
model 6 point 8'	0.065	0.082	0.089
model 2 p.c.left	0.511	0.399	0.164
model 6 p.c. right	0.407	0.348	0.161

Table 6: the boundary values in m in terrain scale;
(g^{ij}) is a full matrix as defined under C:
tie points are double points.

4. Conclusions and remarks

Concerning the internal reliability of a photogrammetric block in relation to the variance-covariance matrix of the observations the following conclusions and remarks can be made.

- Boundary values are strongly influenced by both the variance-covariance matrix and the number and position of tiepoints.
- Double points give smaller and more regular boundary values for the model points than single points. Here the assumption is made that the observations of double points are uncorrelated. This remarks corresponds with other investigations; Neleman 1980.
- The variance-covariance matrix of the observations influences the boundary values particular if only single points in the corner of the model are used. For double points the influence is less, not including perspective centres.
- Still other remarks can be made, e.q. the differences in x- and y-coordinates, the differences of points in the side lap of strips compared with other points of the block, etc. However the question is as to how far these differences are realistic.

Generally can be said a simple variance matrix can deceive the conlusions and for that carefulness should be taken.
Here only the internal reliability is computed for a relative small block. More investigation is necessary to study the precision and the external reliability in relation to covariance matrix of the observations. Special care should be taken if the quality of photogrammetric measurements will be compared with other methods of point determination, e.q. terrestrial method of point determination.

Literature

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