UTILIZATION OF CONSTRAINTS IN CONTROL DENSIFICATION

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ABSTRACT

The major costs incurred in control densification by photogrammetric techniques is the surveying of the initial control. This could be reduced if existing control was available. In many localities, there is a wealth of cadastral survey information that could be used in a photogrammetric adjustment through constraints. This information is often in the form of distances, angles, and/or azimuths/bearings. The net result should be a reduction of new terrestrial surveying for the photogrammetric process while maintaining the desired accuracy of the densification for cadastral purposes since local scale and orientation become a part of the adjustment.

BACKGROUND

The United States is thought of as one of the leading developed countries in the world. Despite this, glaring inadequacies dealing with knowledge of the land are becoming more apparent every day. This has led for a call for the development of a multipurpose land information system. The foundation of this system must be a cadastre built upon a dense network of geodetic control [Panel on a Multipurpose Cadastre, 1983]. Yet, out of the over three thousand counties in the U.S., only a few have in place a sufficiently dense geodetic control network that would support a multipurpose cadastre. In fact, if one looks at the 500 leading counties in terms of economic activity, we find that only 10% have a geodetic network of sufficient density from which densification for support of a cadastre could commence [Barr, 1983].

Many methods for densification of control are now available and they offer significant savings over conventional terrestrial surveying techniques. These include Doppler surveying, GPS Satellite surveying (i.e., satellite interferometric systems), inertial surveying systems, airborne laser ranging systems, and analytical photogrammetry. These can all be used in some form to provide geodetic control necessary to support a multipurpose cadastre. The cost of this control does not come lightly. Barr [1983] points out that 40-70% of the costs in the development of a cadastre (survey control, base mapping, and cadastral surveys and maps) are attributed to survey control. These costs fluctuate because of area and the method of control densification utilized.

Brown [1979] points out that in the future, photogrammetry will not be competative, from an economic point of view, with the new surveying technology. Thus, the need to increase the productivity of photogrammetry and the exploitation of the multipurpose nature of the photograph is necessary. If one looks at the form of the cadastre with the need of a geodetic reference framework, base maps and cadastral boundary maps, and the need to integrate this with other types of data, such as natural resource records, then the necessity to simplify data collection and aggregate diverse data types becomes apparent. It is in this area that the multipurpose nature of the photographic medium can be exploited. Since aerial photography would be necessary to provide the base map, why not utilize it for densification as well?

The accuracy of photogrammetry and it's resultant products is well documented. Unfortunately, for control densification, much of the literature deals with projects requiring new control surveys in order to optimize the photogrammetrically derived ground coordinates. The necessity of ground control is a major encumbrance on the economy of photogrammetry. Therefore, if the acquisition of new survey control can be held to a minimum, costs will correspondingly decrease. This can not be at the expense of the desired accuracy of the ground points. Within many localities there exists a wealth of cadastral survey information that could be utilized in the photogrammetric bundle adjustment. This information is normally in the form of distances and directions although other types of data may also be present. This could be incorporated into the adjustment through constraints, either weight or functional [Case, 1961; Merchant, 1973].

MATHEMATICAL MODELS

One can use the functional form to represent the inclusion of the constraint into the adjustment process as

$$G(X_a) = 0 \tag{1}$$

For horizontal angles, the math model is shown as

$$G(X_a) = D_\ell - [(X_j - X_k)^2 + (Y_j - Y_k)^2]^{\frac{1}{2}} = 0 \tag{2}$$

where D_l is the measured distance and X_j , Y_j , Z_j and X_k , Y_k , and Z_k are the coordinates of points j and k at the ends of the line. If the distances are mark-to-mark, then equation (2) must add, within the radical, the difference in Z squared. For azimuth, the mathematical model is normally shown as

$$G(X_a) = \alpha - \tan^{-1}\left(\frac{X_k - X_j}{Y_k - Y_j}\right) = 0 \tag{3}$$

where α is the measured azimuth of the line. The form of equation (3) would have to be altered if the field data were bearings. This could easily be done after the second term in the right side of (3) is computed by simple data manipulation/testing to insure that the proper angle and quadrant are compiled. Therefore,

$$G(X_a) = \beta - \alpha \qquad (\text{for } 0° < \alpha < 90°) \qquad (4)$$

$$G(X_a) = \beta - (180° - \alpha) \qquad (\text{for } 90° < \alpha < 180°) \qquad (5)$$

$$G(X_a) = \beta - (\alpha - 180°) \qquad (\text{for } 180° < \alpha < 270°) \qquad (6)$$

$$G(X_a) = \beta - (360° - \alpha) \qquad (\text{for } 270° < \alpha < 360°) \qquad (7)$$

In equations (4) - (7), β is the measured bearing of the line. For an angle measured from a third point i between points j and k, the conventional form of the mathematical model is

$$G(X_a) = \theta - \left\{ \tan^{-1} \left[\frac{X_k - X_i}{Y_k - Y_i} \right] - \tan^{-1} \left[\frac{X_j - X_i}{Y_j - Y_i} \right] \right\} = 0 \qquad (8)$$

where θ is the measured field angle.

In the U.S., a large portion of land was surveyed under the Rectangular Survey System by the Government Land Office (now Bureau of Land Management). Figure 1 shows the subdivision of the land into townships consisting of six square miles numbered from a designated meridian and base line. Figure 2 depicts the subivision of these townships into one mile square sections while figure 3 shows the breakdown of the section into aliquot parts. Circles indicate government corners set in the field. One can see that this configuration will cause problems with equations (3) to (8). Therefore, it is recommended that the tangent function be replaced by the sine function for values up to 45-degrees and the cosine function for angles from 45- to 90-degrees. Thus, azimuths in a North/South direction would be represented by

$$G(X_a) = \alpha - \sin^{-1} \left[\frac{X_k - X_j}{D} \right] = 0 \qquad (9)$$

while azimuths in an East/West direction can be shown as

$$G(X_a) = \alpha - \cos^{-1} \left[\frac{Y_k - Y_j}{D} \right] = 0 \qquad (10)$$

where D is the distance between points j and k. Similarly, equation (8) can take the following form for section corners

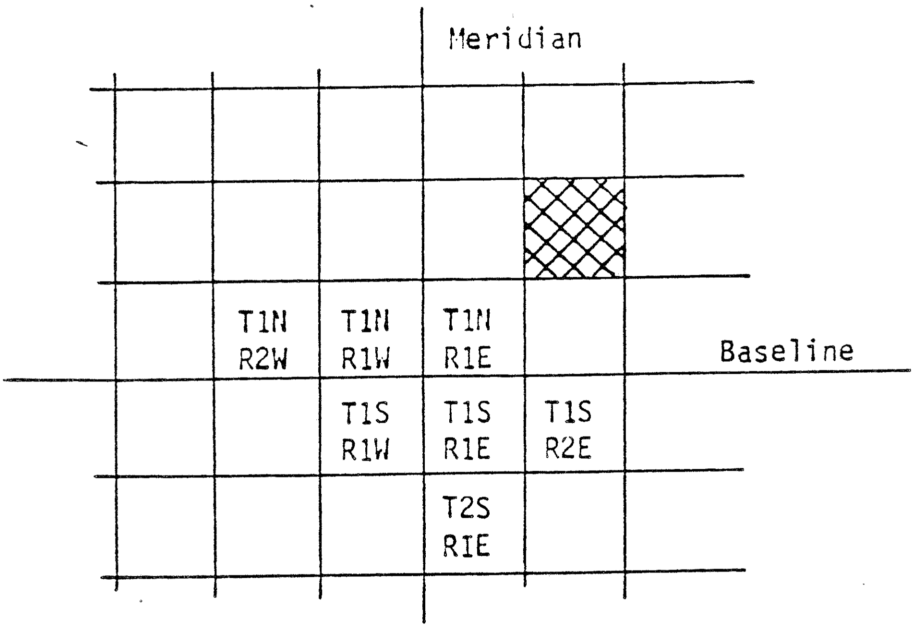


Figure 1. Subdivision of land into townships 6 miles square.

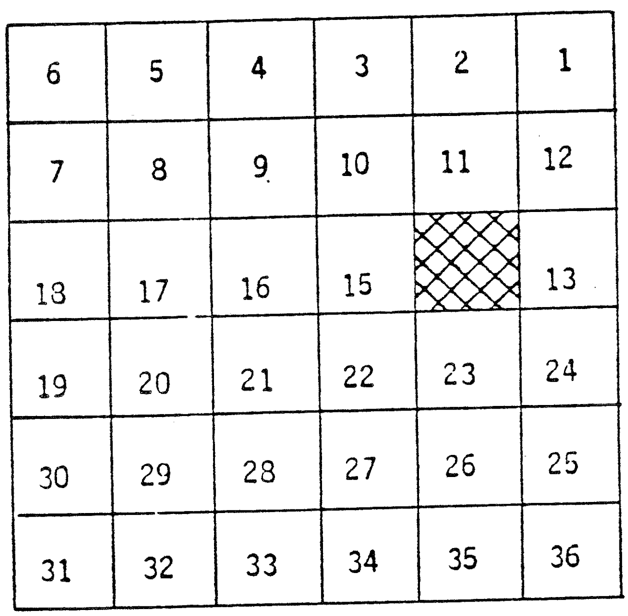
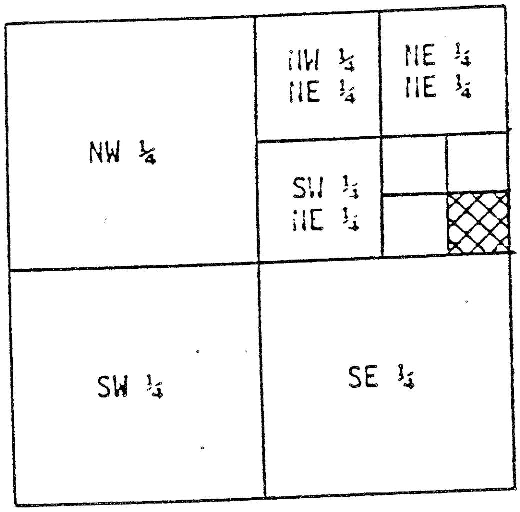


Figure 2. Subdivision of a township into sections 1 mile square.



igure 3. Subdivision of a section into aliquot parts.

$$G(X_a) = \theta - \left\{ \sin^{-1} \left[\frac{X_k - X_i}{D_{ik}} \right] - \cos^{-1} \left[\frac{Y_j - Y_i}{D_{ij}} \right] \right\} = 0 \qquad (11)$$

quation (11) can be rearranged for the different points such hat the second part of the equation on the right side will ield a positive value related to the measured angle (θ).

nother type of data that could also be used in a constrained olution would be geodetic values. These can be shown by the ollowing relationships

$$\left. \begin{aligned} X &= (N + H)\cos\phi\cos\lambda \\ Y &= (N + H)\cos\phi\sin\lambda \\ Z &= [N(1 - e^2) + H]\sin\phi \end{aligned} \right\} \qquad (12)$$

here: N = radius of curvature in the prime vertical
H = geodetic height
Φ = geodetic latitude
λ = geodetic longitude
a = semi-major axis of the ellipsoid, and
e = first eccentricity

FUNCTIONAL CONSTRAINTS

he use of functional constraints within the adjustment offers a significant advantage over weight constraints [Burtch, 1983] and this is through the sequential inclusion of the constraint within the ajustment. The observation equations for the functional constraint can be shown as

$$C\bar{\bar{\Delta}} + \overset{c}{\varepsilon} = 0 \qquad (13)$$

This could be added to the combined form of the observation equations for the bundle adjustment shown as

$$\bar{V} + \bar{B}\bar{\bar{\Delta}} + \bar{\varepsilon} = 0 \qquad (14)$$

where: C is the design matrix for the functional constraint
$\overset{c}{\varepsilon}$ is the discrepancy vector for the functional constraint
$\bar{\bar{\Delta}}$ is the alteration vector
$\bar{V}$ is the vector of residuals
$\bar{B}$ is the design matrix for the observation equations of the bundle adjustment, and
$\bar{\varepsilon}$ is the discrepancy vector of the functions evaluated within the bundle adjustment.

The function to be minimized is

$$\phi = \bar{V}'\bar{W}\bar{V} - 2\lambda'(\bar{V} + \bar{B}\bar{\bar{\Delta}} + \bar{\varepsilon}) - 2\lambda_c'(C\bar{\bar{\Delta}} + \overset{c}{\varepsilon}) \qquad (15)$$

Differentiating (15) with respect to $\bar{V}$ and $\bar{\bar{\Delta}}$ and including the observation equations results in four equations

$$\left.\begin{array}{l} \bar{W}\,\bar{V} - \lambda = 0 \\ -\bar{B}'\,\lambda - C'\,\lambda_c = 0 \\ \bar{V} + \bar{B}\,\bar{\bar{\Delta}} + \bar{\varepsilon} = 0 \\ C\,\bar{\bar{\Delta}} + \overset{c}{\varepsilon} = 0 \end{array}\right\} \qquad (16)$$

It can be shown that the solution can be computed by the following equations [Uotila, 1973]:

$$\bar{\bar{\Delta}} = -\bar{N}^{-1}\bar{U} + \{\bar{N}^{-1}C'(C\bar{N}^{-1}C')^{-1}[C\bar{N}^{-1}\bar{U} - \overset{c}{\varepsilon}]\} \qquad (17)$$

or more concisely as

$$\bar{\bar{\Delta}} = \bar{\Delta} + \Delta \qquad (18)$$

where:

$$\bar{\Delta} = -\bar{N}^{-1}\,\bar{U} \qquad (19)$$

$$\Delta = \bar{N}^{-} C'(C\bar{N}^{-1}C')^{-1}[C\bar{N}^{-1}\bar{U} - \overset{c}{\varepsilon}] \tag{20}$$

Δ is the influence of the constraint on the solution. From (17) or (18), one can see the sequential nature of the adjustment.

If this approach is to be useful, then equation (20) must be evaluated to see if the computational burden negates any economic savings that led to the selection of aerial photography in the first place. Let

$$\bar{N}^{-1} = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ & M_{22} & M_{23} \\ & & M_{33} \end{bmatrix} \tag{21}$$

The design matrix for the functional constraint (C) can be shown to be

$$C = [\overset{e}{C} \quad \overset{s}{C} \quad \overset{i}{C}]' \tag{22}$$

where the superscript e, s, and i relate to exterior orientation, survey, and interior orientation parameters respectively. Since the parameters only involve the survey points

$$\overset{e}{C} = \overset{i}{C} = 0$$

Therefore,

$$\bar{N}^{-1}C' = \begin{bmatrix} M_{12}\overset{s}{C}' \\ M_{22}\overset{s}{C}' \\ M_{32}\overset{s}{C}' \end{bmatrix}$$

and

$$(C\bar{N}^{-1}C')^{-1} = (CM_{22}C')^{-1} \tag{23}$$

where M_{22} is block diagonal composed of 3x3 submatrices. Thus, one can see that only a small portion of the normal coefficient matrix is needed for computations. If only a few points would be included within the constrained solution, then it would be necessary to extract from M_{22} only that part which relates to points upon which the constraint is involved. The influence of the constraint can be computed on a point by point basis. Under the case where multiple observations were made on the same point, i.e., distances and angles to a number of adjacent stations, then correlation could be present and equation (23) would be formed completely. Again, because of the sparsity nature, no significant computational burden would be required. Sparsity is preserved because measurements normally involve adjacent points and seldom go across sections of land. This is not true for geodetic values though. In cases such as this, proper ordering could lead to the creation of a banded/bordered matrix in which methods such as recursive partitioning can be employed.

KILLSBUCK TEST AREA

A test adjustment was performed on the Killsbuck Area located in Holmes County, Ohio [Burtch, 1983]. The area encompasses approximately 30 square kilometers. It was flown with a Zeiss RMKAR aerial camera with a focal length of 152.02 mm at a height of approximately 2100 meters from datum. The camera utilizes a 23x23 cm reseau pattern. The block was flown in two strips with a total of 32 photographs with 60% overlap and sidelap. A total of 81 ground points were used of which 38 were full control points, 31 were only vertical control points, 3 were only horizontal control points, and 9 were unknown survey points.

Three different ajustments utilizing Case III theory [Merchant, 1973] were performed on the data. The third adjustment involved fifteen distance constraints. Eight control points associated with some of the constraints were treated as non-control points. The standard deviations of the points which were changed from control to non-control status are predominantly below the 5-cm level after the adjustment. One of the advantages in using a sequential adjustment is that the influence of the constraint is computed and this information can be used to evaluate the integrity of the survey data. Table 1 shows the influence of the constraints on the survey parameters. One can see that in the X-direction, a very high influence occurs with points 1022 and 1024 which involved the shortest distance constraint used in the adjustment. Because of the large influence and since no perceptual change in the residuals for the photo observations can be found, it may be inferred that one or both have an error in their control value. A small error for the length of a short line would have a greater influence in the localized area. Therefore, one can see that this is a convenient mechanism by which the validity of survey information can be evaluated.

Point	X(M)	Y(M)	Z(M)
912	-0.55524D-01	-0.86359D-01	-0.25345D-01
1034	0.21281D-02	0.33918D-02	0.81835D-04
1022	-0.13457D+00	0.75890D-01	0.36800D-03
1024	0.10171D+00	-0.57180D-01	-0.85815D-03
1018	0.27967D-03	-0.66441D-03	0.11454D-05
1021	-0.28548D-03	0.67830D-03	-0.37755D-05
105	0.91258D-03	0.70036D-03	-0.13756D-04
164	-0.19937D-01	-0.15342D-01	0.15054D-03
194	-0.16409D-03	0.20030D-03	0.69309D-06
196	0.16076D-03	-0.19621D-03	0.65347D-06
110	-0.77752D-05	0.20796D-04	0.12227D-07
114	0.78708D-05	-0.21056D-04	-0.55491D-07
100	-0.73337D-04	0.27284D-03	-0.13043D-03
101	0.74963D-04	-0.27153D-03	0.13934D-05
2007	-0.87335D-04	0.12552D-03	0.88801D-06
2008	0.87916D-04	-0.12634D-03	0.30444D-06
2010	-0.90655D-02	0.13570D-01	-0.49895D-04
2012	0.89395D-02	-0.13409D-01	0.36545D-04
2013	0.10339D-02	-0.28302D-03	0.54514D-05
2014	-0.10205D-02	0.27914D-03	-0.18110D-04
209	-0.82115D-02	0.23059D-01	-0.93644D-04
211	0.69617D-02	-0.19681D-01	0.69992D-04
2003	-0.11570D-02	-0.40497D-03	0.92429D-05
2004	0.11574D-02	0.40505D-03	-0.49628D-05
901	0.12803D-03	-0.35583D-03	-0.92439D-06
1035	-0.12797D-03	0.35569D-03	0.19453D-05
2015	-0.58063D-03	0.13438D-03	-0.48305D-05
2017	0.58065D-03	-0.13437D-03	0.78547D-05
210	-0.51677D-04	0.10760D-03	-0.42629D-07
2003	0.52750D-04	-0.10981D-03	-0.60210D-06

Table 1. Influence of the constraints on the survey parameters using the Condition 3 adjustment.

CONCLUSION

The inclusion of a distance constraint within the bundle adjustment with strong peripheral control does not introduce an appreciable loss in accuracy. Therefore, the requirements for control could be held to some minimum value and additional redundancy provided for by the included constraint. This small experiment shows that the potential for accurate control determination is possible. This should be of value for other types of constraints as well. Finally, the sequential approach offers the additional advantage of computing the influence of the constraint upon the adjustment therby providing a convenient tool in the evaluation of the included data.

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