THE PRESENT STATUS OF THE THEORY FOR THE EVALUATION OF THE QUALITY OF PHOTOGRAMMETRIC POINT DETERMINATION M. Molenaar Department of Landsurveying and Remote Sensing Wageningen Agricultural University Netherlands

# 1. Introduction.

Some 25 years have passed since the advent of numerical aerotriangulation in the early 1960s. In this period, aerotriangulation has developed into a full-grown geodetic point determination system. Three stages can be distinguished in this development. First, mathematical models were formulated for processing photogrammetric data according to the least squares principle. As soon as more rigourous least squares adjustment methods were available, a first evaluation of the quality of the adjustment output was made by means of variance-covariance matrices computed for coordinates obtained in block simulations. These simulations led to the formulation of directives for the planning of blocks, i.e., for the geometry of blocks, ground control and the use of auxiliary data. The third stage of the development of aerotriangulation began when empirical data became available showing discrepancies with theoretically expected block accuracies. In the search for explanations of these discrepancies, methods were developed for detecting systematic deformations and gross data errors. The testing methods for gross errors led to the analyses of the reliability of blocks, which was a new aspect of accuracy in addition to precision as expressed in the V.C. matrix of the coordinates. To date, reliability studies led mainly to new directives for optimal inner block structures in terms of overlap and tie point geometry. These studies may also lead to new directives for the geometry of the ground control and the use of auxiliary data.

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Now that more advanced techniques for evaluation of quality of photogrammetric coordinates are available, the need is felt to set criteria for precision and reliability in the planning stage. These criteria should represent the requirements for block quality determined by the purpose for which block triangulation is performed. A sketch will be given in this paper of the theoretical problems met in the formulation of these criteria and the problems of translating these criteria (which are often rather abstract) into practical terms.

#### 2. Criteria for precision.

## 2.1. Network optimization.

In modern geodesy, much attention is paid to the problem of network design and optimization. This optimalization is done mainly with respect to precision, using so called criterion matrices. The strategy is as follows: a criterion matrix H is constructed for a point field. Then a geodetic network is designed so that the V.C. matrix G of the newly determined points approximates H as well as possible in the sense that the maximum and minimum eigenvalues  $\lambda \max$ ,  $\lambda \min$  solved from  $|G - \lambda H| = 0$  give a minimum for the functions:  $|\lambda \max - 1|$  and  $(\lambda \max - \lambda \min)$ 

because G and H are positive (semi) definite, we have  $\lambda \min > 0$  ( $\geqslant 0$ ). The research published to date concentrates mainly on planimetric networks. The matrix H represents the precision of an ideal network; it is homogenous and isotropic so that point standard ellipses are all circles of the same size and the size of each relative standard ellipse depends only on the distance between the two points to which it refers. A special case of H is the matrix with a "chaotic" structure; there the relative standard ellipses are

also circles. This structure is most frequently used. The elements of the matrix are:

 $\sigma x_i x_i = \sigma y_i y_i = d^2$ ,  $\sigma x_i x_j = \sigma y_i y_j = d^2 - d_{ij}^2$  for  $i \neq j$ ,  $\sigma x_i y_i = \sigma x_i y_j = 0$ , (2)  $d^2 = \text{constant}$ ,  $d_{ij}^2 = f$  (distance i, j),  $d^2$  and  $d_{ij}^2$  should be chosen so that H is positive definite [4, 16, 17].

The matrix G is a function of the datum choice for the coordinates; it is also a function of the network structure and of the V.C. matrix of the original observations. There are therefore three options for optimizing a network with respect to precision: The first option for choosing an optimal datum is called "zero order design". It is possible to choose a datum so that the G matrix of the coordinates has a minimum trace [2, 7, 9, 14, 20]. This strategy is often advocated for a free network adjustment, i.e., the adjustment of a network without higher order points. The G matrix can become rather homogenous in that case. A serious problem is that it becomes singular with a rank deficiency of four.

The second option for optimizing the structure of the network is called "first order design". The search for optimal network structures did not lead to results  $\lambda \max = \lambda \min = 1$ . The structures which lead to small values for the target functions (1) are not very practical; they can hardly be realized in real fieldwork. It is interesting, though, that aerotriangulation blocks seem to score rather well if they are regularly structured [12, 15].

The third option for searching an optimal V.C. matrix for the original observations is called "second order design". Let P be the weight matrix, the inverse of the V.C. matrix of the observations. In most cases P is a diagonal matrix. The propagation laws for V.C. matrices give the relationship between P and G, say: G = f(P). In the second order design, one tries to invert these relationships:  $P = f^-(G)$ . Because G has a lower rank than P, the mapping f is singular, so that  $f^-$  can be found only under certain restrictions. If a solution for  $f^-$  is found, then the optimal weight matrix Po is found by substituting H for G Po =  $f^-(H)$ . The problem has not been solved to date. The weight matrices which seemed to give good scores for (1) have the disadvantage that they led to irregular observation patterns in which different numbers of repetitions were required for the measurement of different sides of a network. No field surveyor will be happy with that.

# 2.2. Network satisfication.

In section 2.1., it became clear that quite a few problems arise when one tries to formulate a strategy for optimizing geodetic networks with respect to the precision of the coordinates. That is why one should try to find another strategy for network design. A first observation to be made in that respect is that a network can be completely described by means of formelements that are angles and length ratios. They define the geometry of a network and therefore its strength. So the analysis of the quality of a network should refer to the quality of the form elements constituting it. It is only a matter of convenience that coordinates are introduced to describe the relative position of points, thus introducing the necessity of a datum definition. That is why the choice of a datum (coordinate definition) should have no effect on the judgement of the quality of the network; it should play no role in the strategy for network design.

This datum problem can be avoided if one always refers to an explicit local datum, which is called an S-base [4]. For levelling networks, the S-base is one point for which the height is fixed as not stochastic. In planimetric networks, the coordinates of two points are fixed as not stochastic. In three dimensional coordinate systems, the S-base consists of two points with non-stochastic coordinates; in addition, the direction must be fixed for the normal vector of the plane containing these two points and a third one [17]. The coordinates computed with respect to an S-base can be considered as a direct function of the form elements of a network.

In section 2.1., we saw that network optimization by first order design, i.e., finding an optimal network structure, does not lead to acceptable results. This strategy does not leave enough freedom to the land surveyor to adapt his network to the topography of the terrain. This fact makes it doubtful whether we should really aim at an optimal network structure in the sense that target functions (1) are minimized.

Let us try to understand the meaning of these target functions somewhat better. If F is an arbitrary function of the coordinates of the network, then the combination of  $\left|\lambda\text{max}-1\right|\simeq 0$  and  $\lambda\text{max}-\lambda\text{min}\simeq 0$  means that GFF  $\simeq$  HFF, in which GFF is the variance of F computed from G and HFF is computed from H. In that case, the actual precision of any function of the coordinates is nearly equal to the criterion given for precision by the criterion matrix H. In practical situations, requirements for precision are hardly ever formulated this way. One would rather give an upperbound for precision; then GFF should not exceed a specified value, hence GFF < HFF. It does not matter if the value of GFF is much smaller than the upperbound.

The problem now is that F is a function of the coordinates and GFF is a function of G the V.C. matrix of the coordinates. The matrix G depends on the network structure and the choice of the S-base in the network. The use of another S-base leads to another G matrix. Earlier, however, we stated that the evaluation of the quality of a network should refer to its form elements its inner geometry. The inequality GFF < HFF should therefore be true for any function F and with respect to any S-base; this is a first requirement when we develop a criterion theory for precision.

A second requirement is that a criterion for precision should not refer to a special network structure. The surveyor should have the freedom to structure a network according to his needs as long as the requirements for precision are fulfilled. In case of a general purpose network, there are no preferences for certain parts of a network; then the criterion should have the same meaning everywhere in the network. This means that the upperbounds for the precision of the form elements, angles and length ratios should depend only on the shape and size of the triangle from which they are computed. The upperbounds should not depend on the location and orientation of the triangle in the network. These requirements will be fulfilled if the criterion V.C. matrix H for the coordinates has a homogenous and isotropic structure, like matrix H in section 2.1. In most cases, one uses in planimetry a matrix as specified by (2) (see also [4]). Such a matrix can not be applied in its original form because it will contain non-zero elements refering to the coordinates of the S-base, which should be notstochastic. This problem can be solved by making these elements zero by matrix reduction; this is a special application of S-transformations as given in [4, 16, 17] . After this transformation, the matrix H is no longer homogeneous (in  $R_3$  it will also lose its isotropy), but because of its meaning for the form elements we say that it represents a "homogeneous and isotropic inner precision". The variance of form elements is invariant after an S-transformation, the transition from one S-base to another. After the S-transformation (reduction), H refers to the same S-base as G. From this transformed H matrix we compute HFF with which GFF is compared.

The inequality GFF < HFF will be true for any function F of the coordinates if the general eigenvalue problem  $|G - \lambda H| = 0$  gives  $\lambda \max < 1$ . (3) Instead of "optimization" which aimed at (1), we call the strategy aiming at (3) "satisfication". We are no longer searching for a network with an optimal ("ideal") precision, but we are searching for a network with a precision which is satisfactory.

If the structure of H before the S-transformation is given by (2) then it is fully specified by the constant  $d^2$  and the function  $d_{ij}^2$  (see [4, 16, 17]). The effect of  $d^2$  is eliminated by the S-transformation so that the actual upperbound is completely defined by  $d_{ij}^2$ . This function can be considered as the variance of the distance between points which have (2) as a V.C. matrix for the coordinates. Because that matrix does not refer to an S-base, it should be considered as the V.C. matrix of a ficticious coordinate system, (a)-system.

The function  $d_{ij}^2$  should be chosen so that the S-transformed matrix H is positive definite. Experience in the Netherlands shows that the function  $d_{ij}^2 = \text{Co} + \text{C}_1 \ l_{ij}$  is a reasonable choice for the criterion matrix.  $l_{ij}$  is the distance between points i and j, Co and  $\text{C}_l$  are parameters. A network classification with respect to precision can be made by giving praticular values to these parameters for each class. Their choice is still a topic of research.

2.3. An application of the criterion theory for precision in photogrammetry. Artificial covariance matrices can be used in two ways: They can be used to formulate criteria for precision and to classify pointfields with respect to precision as discussed in the previous sections. They can also be used as a substitute if the V.C. matrix of the coordinates is not available, as is often the case in photogrammetry. This second posibility will be elaborated in this section.

For substitute matrices, the choice of the function  $d_{i\,j}^2$  is more critical than for the criterion matrices. In addition to the linear function mentioned before, there are other possibilities such as the logarithmic function

 $d_{ij}^2 = Co + C_1C_2 \ln (1 + \frac{l_{ij}}{C_2})$  and the exponential function

 $d_{ij}^2 = C_0^2 + C_1$  (1 - exp (1 -  $C_2^2$   $l_{ij}$ )). The function  $d_{ij}^2$  should always give a positive definite matrix H.

All functions have in common their dependence on the distance between the points i and j to which  $d_{ij}^2$  refers. Furthermore, they depend on the two or three parameters Co,  $C_1$  (and  $C_2$ ). Once a function has been chosen for  $d_{ij}^2$ , values should be given to the parameters to specify a level of precision. The choice of  $d_{ij}^2$  should be based on experiments similar to those performed by Baarda for terrestrial networks ([4] ch.18). Some experiments have already been done by Karadaidis [12] for simulated planimetric independent model blocks. He varied the size of the blocks, the tie point configuration, the control point configuration and the assumptions

for the stochastic model for photogrammetric observations and ground control coordinates. For each block, he computed the V.C. matrix G of the coordinates in the terrain system, which he then compared with three different families of artificial matrices H. Each family was defined by one of the functions  $d_{1\,j}^2$  mentioned before.

The comparison was made by means of the general eigenvalue problem  $|\text{G}-\lambda\text{H}|=0.$  For each family, the parameters Co, C<sub>1</sub> (and C<sub>2</sub>) were chosen so that  $\lambda\text{max}=1$  and  $\lambda\text{max}/\lambda\text{min}$  is as close to 1 as possible. The parameter Co is the constant part in each function; it should be chosen in relation to the precision of the measurement of the model coordinates. The other parameters belong to the distance-dependent part of  $d_{ij}^2$ , whose characteristics depend on the actual block structure.

Karadaidis' experiments show that Co should be chosen first; then  $C_1$  can be used to scale the matrix H with respect to G. The parameter  $C_2$  in the logarithmic and exponential functions determines the curvature of these functions. After fixing Co, the parameter  $C_1$  is chosen to make  $\lambda \text{max}=1$  and then  $C_2$  is chosen so that  $\lambda \text{max}/\lambda \text{min}$  gets a value as close to 1 as possible. The experiments gave the best results for the logarithmic and exponential functions. The difference between the results for these two were small compared with the difference between these results and those for the linear function. More experiments should be done, however, because the blocks were of only limited sizes: maximum three strips with six models each. The experiments were done only for planimetry and a simple V.C. matrix has been assumed for ground control.

More experiments should be performed to study the effect of larger block sizes, different assumptions for the V.C. matrix of ground control and auxilary data. Experiments should also be done for heights and for three-dimensional point determination and bundle blocks. All these exercises should lead to a proper choice for the  $d_{ij}^2$  function and its parameters for different types of blocks. The results of these experiments can then be used to describe the precision of photogrammetric block coordinates by means of artificial matrices. These are easy to generate, so that the computational effort to obtain the real matrix G for each block can be avoided in practice. The function  $d_{ij}^2$  with its parameters can at the same time serve as a classification of the precision of photogrammetric point fields. The matrix H generated with a specified  $d_{ij}^2$  can then be used as a criterion for the precision of the densification masurements; this criterion should be used as in section 2.2.

One should be aware of the fact that the target function  $\lambda max - 1$  and  $\lambda max/\lambda min - 1$  are used in a way different from what was done in section 2.1. Here they are used to find the matrix H which fits best the real matrix G for a given block. Our aim is not to optimize the observation pattern, but we want to find the best suitable substitute matrix. The second function can be minimized only as far as the structure of the H matrix allows. In this experiments, Karadaidis hardly ever found results better than  $\lambda max/\lambda min = 3$ ; values larger than 10 have been found frequently.

In section 2.2., we stated that  $\lambda \max$  < 1 means that for any function F of the coordinates we find GFF < HFF. If  $\lambda \max$  and  $\lambda \min$  are both known, we can state that  $\lambda \min \cdot \text{HFF}$  < GFF <  $\lambda \max \cdot \text{HFF}$ , so that we have an indication in which range the value of GFF is situated.

The theoretical tools exist to find a best substitute H for G. The practical problem is indicating how accurate information about the precision of block coordinates should be; that is, how small should we make the range defined by  $\lambda$ max HFF and  $\lambda$ min.HFF.

If the strategy for the classification of point fields by means of artificial matrices is accepted, then people working in applied photogrammetry should be trained to use the function  $d_{ij}^2$  with its parameters to specify the precision of their products. This will be easier for them if the relationship between the parameters and the actual block structure is better understood than it is now. This understanding can be obtained only by means of experiments, as mentioned before. It would be good to use some of our research capacity to solve this problem in the near future. At the same time, photogrammetrists should try to combine their efforts with those of land surveyors in this respect so that a consistent set of criteria and classification parameters can be formulated for the precision of photogrammetric blocks and terrestrial networks.

### 3. Criteria for reliability.

#### 3.1. Testing procedures.

The previous sections concentrated on the criterion theory for precision because in the last decade it did not get much attention in photogrammetric literature. This is quite different from literature on geodetic network design. Most likely this difference is due to the fact that geodesists have more freedom in designing their networks than we have for photogrammetric blocks. Because of this fact, it is very likely that the possibilities which the theory gives for classification and for generating substitute matrices will be more important for photogrammetrists than the directives which can be derived for block design.

It is striking, though, that photogrammetrists paid much more attention to the second aspect of quality control than geodesists do. This second aspect is the detection of errors and, related to that, reliability studies. This area is well-known to land surveyors, but its importance is apparently largely under-estimated by geodesists working on research on network design. This is striking because the most successfull techniques for error detection have been formulated by geodesists [3, 13]. One wonders whether this is because people working in research on aerotriangulation have a more direct contact with practice than many of the researchers on geodetic network design. Whatever the reason may be, many publications are available so that the interested photogrammetrist can easily become informed about the status of this research. We will therefore be brief about this status, but we will give more attention to the problem of the formulation of criteria for reliability.

The most successful error detection techniques published to date are the "data snooping" technique [3] and the "Danish method" [13]. The data snooping technique is a straight-forward application of the statistical theory of hypothesis testing. It is based on a well formulated null hypothesis Ho and an alternative hypothesis Ha which is tested against Ho. In geodesy and photogrammetry, Ho is, in general, the assumption that the observations are normally distributed with a know V.C. matrix, while the expectations should fulfill certain conditions which represent the mathematical model of the measured network or block. The data snooping technique consists then of a series of best tests (most powerful tests), one for each

member of a family of alternative hypothesis Ha, e.g. the assumption that observational errors occur. Methods based on robust statistics, such as the Danish method, are less restrictive in the formulation of Ho and Ha [8, 11, 13]. They search for inconsistencies among the observations which may indicate different error sources, e.g., observational errors or systematical deformation. Whereas the data snooping technique is strongly related to least squares estimation, robust methods allow other estimation procedures also. Both methods however, are based on the formulation of a mathematical model from which condition equations can be derived for the observations. These condition equations give the essential information required for errordetection, or more generally for hypothesis testing. It is possible to prove that for a properly formulated Ho and Ha, the data snooping technique always gives a most powerful test, although it is often difficult to decide which Ha occurs in case of a rejection of Ho. The robust methods lead in many cases to a quick data screening, but their sensitivity for errors cannot be better than that of data snooping. The separability of error sources, alternative hypothesis Ha, depends mainly on the structure of the condition equations rather than on the testing technique. Hypothesis testing therefore has two aspects: the first consists of the optimal structuring of the condition equations so that they give an optimal separability of those hypotheses in which one is interested. This is a matter of block or network design. The second aspect is the choice of an optimal testing procedure, which gives most powerful (most sensitive) tests. In practice, one often adds the restriction that the testing procedure should not lead to too large a computational effort.

# 3.2. Reliability studies.

An advantage of the data snooping technique over the robust methods is that it gives a tool for evaluating the quality of a block or network in terms of reliability [3, 5]. In the design stage, one can evaluate what magnitude of errors can be found, the probability of their detection and what distortion they will cause in the final coordinates if they are not found. It would be quite a challenge to develop similar tools for the robust methods, for only then the different testing techniques will be comparable.

The reliability studies related to data snooping can be used in two ways. Primarily, one can design a network or a block so that the tests become so sensitive that there is a high probability of finding even small observational errors or deformations. In that case the system has a high "internal reliability". Secondly, one can design a network or block so that undetected errors or deformations do not lead to serious distortions of the final coordinates. Then the system has high "external reliability". This means in fact that one makes the estimation of the coordinates robust, i.e., insensitive with respect to changes in the means of the density functions of the observations.

Reliability studies are usually done as follows. For each  $\mathrm{Ha}_p$  a best test is formulated with a test value Wp which possesses under Ho a standard normal distribution: Wp = N (0, 1). Then a significance level is specified:  $\alpha_0$  = probability of rejecting Ho when it is true (type I error) and the power is specified:  $\beta o$  = probability that Ho is rejected if  $\mathrm{Ha}_p$  is true (1 -  $\beta o$  = probability of a type II error). The critical value C of the test is a function of  $\alpha_0$  and the boundary value  $\nabla \widetilde{\mathrm{Wp}} = \sqrt{\lambda}$  is a function of C and  $\beta o$  (and thus of  $\alpha o$  and  $\beta o$ ). The boundary value is the shift in the test value  $\nabla \widetilde{\mathrm{Wp}}$  which can just be detected with probability  $\beta o$ . For each  $\mathrm{Ha}_p$ , one can compute to which distortion of the observations the value  $\lambda o$  corresponds and one can also compute what the effect on the final coordinates  $(\nabla \widetilde{\mathrm{x}}, \ \nabla \widetilde{\mathrm{y}})$ 

will be if this distortion is not detected.

In section 2.2., it was stated that the coordinates should be defined in an S-system. The effect  $(\nabla \tilde{\mathbf{x}}, \ \nabla \tilde{\mathbf{y}})$  for individual points will change in the transition from one S-system to another. The evaluation of these quantities is therefore useful only in special purpose networks where a particular S-base must be chosen. For general purpose networks, one should make use of a parameter  $\tilde{\lambda}$  which is computed from the quadratic form with the inverse of the V.C. matrix of the coordinates as the central matrix, which is then post-multiplied by the vector containing the distortions  $(\nabla \tilde{\mathbf{x}}, \ \nabla \tilde{\mathbf{y}})$  of all the points and it is pre-multiplied by the transposition of this vector.  $\tilde{\lambda}$  is invariant under S-transformations [2, 5].

3.3. Risk minimization.

The techniques for evaluating reliability are useful for the design of networks. Once the measurements have been made, one could question, however, whether hypothesis testing should really be performed with a fixed  $\alpha$ o. There are other testing strategies which minimize the risks of making wrong decisions [1, 6, 18, 19] based on a different line of thought. In section 3.2., one should first choose  $\alpha$ o and  $\beta$ o and then compute what distortions can be expected in the final coordinates caused by undetected errors. For risk minimization, however, one should first set a tolerance for those distortions. In a special purpose network, one could set tolerances for individual coordinates, say  $\forall$ tol  $\tilde{\mathbf{x}}$  and  $\forall$ tol  $\tilde{\mathbf{y}}$ . In a general purpose network, one could set a tolerance for  $\bar{\lambda}$ , say  $\bar{\lambda}$ tol.

Error detection should aim at finding observational errors which give distortions larger than these tolerances. The tolerances for the test values can be found from  $\forall \text{tol } \widetilde{\text{Wp}} = \sqrt{\lambda} \text{tol with } \lambda \text{tol} = \lambda \; (\overline{\lambda} \text{tol}) \text{ or } \lambda \text{tol} = \lambda \; (\forall \text{tol } \widetilde{\mathbf{x}}, \lambda \text{tol } \widetilde{\mathbf{y}})$ , see [18]. If one knows by experience that, in general, a fraction p of the observations is erronous, then a fraction 1-p must be correct. When testing is done with significance level  $\alpha$ , the probability of making a type I error (rejection of correct observations) is p (type I) = (1-p). $\alpha$ . The financial risk of a type I error is then  $R_1 = (1-p).\alpha.L_1$ , in which  $L_1$  is the cost factor related to such an error. For a type II error (not finding errors), we find similarly  $R_2 = p(1-\beta) \; L_2$ ; here  $\beta = power of the test, <math display="inline">L_2 = cost$  factor of type II error. If testing is done with a critical value C, we find

$$\alpha_{\rm C} = \alpha$$
 (C) =  $\int_{\rm C}^{\infty} f (Wp | Ho) dWp and  $\beta_{\rm C} = \beta$  (c) =  $\int_{\rm C}^{\infty} f (Wp | Ho_p) dWp$ .$ 

The total risk function of the test is  $r_{c}=R_{1}+R_{2}=(1-p)\alpha_{c}L_{1}+P(1-\beta_{c})L_{2}$ . If Hap supposes a distortion with the magnitude of the tolerance, then  $r_{c}$  will be minimal if we choose (see [18]):

will be minimal if we choose (see [18]): 
$$C = \frac{1}{\sqrt{\lambda} \text{tol}} \left[ \ln \left( \frac{1-p}{p} \right) + \ln \left( \frac{L_1}{L_2} \right) \right] + \frac{\sqrt{\lambda} \text{tol}}{2}$$

assuming Ho :  $\underline{\text{Wp}} = N$  (0, 1) and Ha :  $\underline{\text{Wp}} = N$  ( $\sqrt{\lambda}$ tol, 1). For distortions larger than  $\sqrt{\lambda}$ tol, we find  $\beta > \beta_{\text{C}}$  and thus r < r<sub>C</sub>; hence r<sub>C</sub> is an upper bound for the risk function. This upper bound is minimized.

For each Ho $_p$ , the parameters c and r $_c$  can be evaluated. If r $_c \simeq R_1$  then  $R_2 \simeq 0$ . In that case, one is quite sure that Ha $_p$  will be detected. If r $_c \simeq R_2$ , then  $R_c \simeq 0$ . Then it is almost impossible to detect Ha.

Different  $\alpha_{\text{C}}$  and  $\beta_{\text{C}}$  are used for different  $\text{Ha}_{\text{D}}$  which means that if we fix the tolerance for the coordinates, the reliability of the tests varies because  $\beta_{\text{C}}$  varies. This is quite different from section 3.2. where  $\beta_{\text{C}}$  was

fixed and  $(\nabla \tilde{\mathbf{x}}, \nabla \tilde{\mathbf{y}})$  or  $\bar{\lambda}$  varied. One has to take care of this when evaluating the overall reliability of the whole system.

The evaluation of several parameters is required for risk minimization, such as the tolerance and the cost factors  $L_1$  and  $L_2$  and the error rate p. These values can be obtained only by experience, and even then it will be difficult to define them sharply. In fact one should first study how sensitive the risk function r is with respect to variations of the values of these parameters. Only then one can decide how sharply they should be defined.

There are more problems to be solved: At first, there is the fact that test variates for different  $\mathrm{Ha}_\mathrm{p}$  are often strongly correlated, so that rejection of one  $\mathrm{Ha}_\mathrm{p}$  involves the rejection of others. The evaluation of the risk of total testing procedure involving more than one test should take this into account. Secondly, error detection is often performed in different stages of a project; the risk and tolerances for the tests in each stage should be chosen in regard to those in other stages [19].

If a strategy is to be formulated for the design of a block or network to minimize the risk functions, one faces the following problem: this reduction often requires that the structure of a block or network is improved by the addition of extra observations, which means an increase in cost.

How far should one go? How should one weigh the deterministic costs of the measurements with respect to the probabilistic risk functions? The answer to this question is far from clear yet. This is in fact a central problem in the design of blocks or networks to meet criteria for quality: how to weigh these criteria with respect to the actual costs.

#### 4. Conclusions.

In our discussion of the status of the theory for geodetic network (photogrammetric block) design, a separation was made between precision and reliability. We should try to integrate these two in our further research. In section 3.3., the theory for reliability was worked out to the extent that economic factors could be linked to abstract parameters. This is not yet possible for the criterion theory for precision. This linking is important because cost factors are the final terms in which a photogrammetrist is interested when he has to discuss his products with his costumers. One should keep in mind, however, that the validity of the applied functional and stochastic models is limited; they give only an approximate description of the physical reality. When economic cost factors are linked with the parameters in these models another inaccuracy is introduced in our evaluation methods. One should therefore use these methods with care; they can give only coarse directives for network or block design. Because of this fact, the question arises how far we should go with further refining the theory for quality control and our strategies for network design. We soon come to the point where the scientific challenge and effort are no longer in balance with their economic importance. That is why research in this field should be done in close contact with practice, so that instead of developing our theories to a too high level of sophistication, we intry to find a better translation of these theories and their essential parameters into practical terms.

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practical decision models within the frame work given by the theory for the The paper discusses the problems which arise when one tries to formulate evaluation of the quality of point determination systems as it has been developed to date.

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review of modern methods for the evaluation of the precision and reliabi-Primarily a review is given of the present status of this theory, i.e. a lity of the photogrammetric point-determination systems. A comparison is is discussed how target functions for the optimization of point-determimade with methods applied for the evaluation of landsurveying networks. defined evaluation criteria into terms which are useful in practice. It photogrammetrist. The role of the costfactors in this formulation will Secondly attention is paid to the problem of translating theoretically be sketched. An outline will be given of topics for further research. nation systems can be formulated which are useful for the practicing

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