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Introduction.
The Italian activities during the period 1980-84 for the Commission III of the ISPRS goes along with the recovery of the time lost in Italy as for the mathematical and numerical treatment of geodetic sciences; moreover new developments and experiments on the most important topics of photogrammetry are sought for.

The beginning of this new period can be traced back to the time of a seminar on numerical methods held by CERN (Geneva, 1977), and its results become visible through the international meetings as a seminar on statistical analysis in geodesy and photogrammetry (Stuttgart, 1981), the international symposium of the Commission III of the ISPRS (Helsinki, 1982), the international colloquium of DEM (Stockholm, 1983), and other interesting meetings of geodesy or surveying on the same topics.

Now the researches carry on fast, so much that the Italian Society of Survey and Photogrammetry (SIFET) has established a study and working group on applied computational statistics, of which the writer has become secretary. For the recent cooperations with geologists and architects the control problems take great interest; next year it is likely to be a new study and working group on deformations monitoring.

The present Italian activities concern some different topics: block adjustment, digital terrain models, statistical analysis and numerical methods. These are as well applied to geodesy and surveying, their results being suitable for the photogrammetric applications too. The report shows the principal results obtained in different topics, with a list of references.

1. Block adjustment
The only complete program in Italy of block adjustment by independent models has been prepared by professor Inghilleri before his death.

This program is used for mapping at middle scale (see the block adjustments performed for the Technical Regional Maps) and, experimentally, at great scale and gives excellent results.

The "Istituto di Topografia, Fotogrammetria e Geofisica - Politecnico di Milano" (ITM), in strict cooperation with researchers from the "Istituto di Topografia e Geodesia - Università di Bologna" and the IGMI (Florence), is implementing some new programs:
- block adjustment by independent models and bundles for the aerial or terrestrial photogrammetry;
- network adjustment for planimetric or altimetric or plano- altimetric networks;
- joint adjustment of photogrammetric and geodetic or surveying measurements (see the experiment of the Commission A of OEEPE).

It is to be hoped that these programs will be finished by the end of 1984. Two solution methods are used:
- direct solution, i.e. the exact Cholesky factorization within the profile of the normal matrix and the computation of suitably selected elements of the inverse matrix, after the appli-
cation of a reordering algorithm of modified reverse Kut Hill McKee type to minimize the bandwidth and the profile (see Gibbs Poole and Stockmeyer);
- iterative solution, i.e. conjugate gradient method, after the application of the same reordering algorithm and a preconditioning algorithm, as the incomplete Cholesky factorization to reduce the number of the iterations, and improve the convergence.

Some additional facilities are available:
- fast strategies for the search of the preliminary coordinates;
- free network or block adjustment with minimum norm solution;
- data snooping of Baarda type;
- estimates of the weights of the observation equations with the criterion of the maximum likelihood;
- statistical post-processing for the control problems.

An interesting experiment of joint adjustment for cadastral network densification will be performed next year in strict cooperation with researchers of the "Dipartimento di Georisorse e Territorio - Politecnico di Torino".

2. Digital terrain models and image rectification

The recently formed Italian group, with researchers from ITM and IGMI (for the Commission B of OEEPE), has tried to apply techniques known from other fields, i.e. advanced statistics, to these problems. The technique involved is the least squares collocation. It is obvious that, whichever model and procedure are used to study the digital elevation models, they must be able to analyze a zone of meaningful size. This means to perform a synthesis of the area and recognize its different features and moreover to describe it with the desired accuracy.

Statistical methods look at to perform this task; all this has suggested the use of collocation, which has actually given the following results:
- the empirical estimation of the terrain covariance function using a decreasing number of measured heights gives a preliminary information on the optimum spacing of grid points;
- the stepwise prediction seems to be suitable to describe quantitatively the terrain;
- where the interpolation fails, the residuals are meaningful and show relevant terrain features; the prediction of the slopes and the curvatures using the finite differences method or the Taylor-Karman structure, aids the search for the same features.

By means of collocation, the digital models in control problems, allowing to study in detail the surface deformation, are constructed on the base of the displacements. In such a way one arrives at the knowledge, on a grid, of both plano-altimetric and altimetric displacements; the former, starting from the displacements on well identifiable points; the latter, starting from the singling out of the changes of altitude (in the same planimetric positions). The image rectification involves the mathematical treatment of non-conventional photogrammetry: like radar (SAR) and scanner (SPOT) data. The mathematical treatment applies some statistical techniques, such as interpolation and regression, stationary stochastic processes of the second order (Wiener type), least squares collocation, etc., for giving geometrical corrections to the images. The preliminary examples furnish acceptable results, but the applications are still new, or have again to begin.
3. Statistical analysis
The applications of the statistical analysis concern some different topics: optimisation, gross error detection and systematic error compensation.

The optimization of the configuration and the weights of the measurements, to obtain a satisfying final accuracy of a geodetic or surveying network (examples with photogrammetric data have not yet been performed), involves the techniques of first and second order design. The first order design can be performed by repeating the second order design, rejecting observations with weights tending to zero and stopping the procedure by a test on the reliability. Unfortunately these methods, for the required storage and the execution time, are not useful for realistic examples, particularly with photogrammetric data where the high number of equations and unknowns gives always rise to large systems.

The gross error detection, after clearing of the blunders by topological or geometrical tests, calls for the definition of a strategy just for identifying and testing the gross errors. A robust estimator, as the least sum of the modulus solution seems to be suitable for the identification of the gross errors, but its relevant sample distributions are unknown. Whence statistical tests are at present possible only with the least squares solution, with the draw-back that the least squares mask the gross errors. Therefore the wanted strategy must suitably combine both requirements.

A wide experimentation on the classification of gross errors (work performed in contact with the Commission A of OEEPE) could be useful for the preliminary study of new probability distributions.

The systematic error compensation is performed with the procedures of self-calibration. Instead of the polynomial interpolations, the least squares collocation may be useful to the same aim. This approach seems more flexible, therefore the results could be very interesting.

4. Numerical methods
Several numerical methods are used for preparing and computing the least squares solution in the block or network adjustment (see paragraph 1). However the direct and the iterative methods, the latter with preconditioning, to solve large and sparse systems, as well as the reordering algorithms and the fast strategies to search for the preliminary coordinates, both applying the results of the graph theory, must be mentioned again.

Other very interesting numerical methods are used or studied too. Topological tests using group algebra are applied to detect wrong reference system; as for other blunders, direct generation of the condition equations from the observation equations, using the graph theory, is sometimes applied.

The sequential Cholesky factorisation is also implemented; moreover a well known theorem of matrix algebra provides the relations to compute suitably selected elements of the inverse matrix sequentially; this is useful for gross error detection with recurrent least squares solution.

The simplex method, well known in the linear programming, and some of its variants are performed for gross errors detection by means of robust estimators, as the least sum of the modulus solution.
The last topics are begun to be studied only recently; therefore presumably a long time has to run before the results of these researches will be operative.

Conclusion.
The results so far obtained encourage the Italian researchers to go on in the chosen direction. It is hoped that in the future the supports as well as the menpower in this field will be enough to bring to conclusion all the work undertaken.

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