THE DIGITAL ELEVATION MODEL (DEM) AND ITS UTILIZATION
FOR CONTOUR LINES PLOTTING AND THE BUILDING-UP OF THE
3D REPRESENTATIONS OF THE RELIEF

Nicolae OPRESCU
Ion IONESCU
College of Civil Engineering, Bucharest
Romania

Commission III

ABSTRACT

We present the design of the programs system I-GSR₃S. The system
allows input data randomly or uniformly sampled. For data with
random structure, the collocation method is applied where the
trend surface is statistically determined and the covariance
function may be automatically changed to the purpose of adapting
it to the various terrain classes. Uniform structures are process-
shed by program based on the approximation theory with spline
functions.

After the computation of the DEM, optional, conventional represen-
tation of the relief, or 3D ones may be generated. The method of
plotting contour lines make use the digitizing principles in
the square schema. 3D representations are carried out by an opti-
mized variant of the Wright method.

Maps with digitally plotted contour lines, as well as examples
of 3D representations are given for exemplification.

RESUME

On presente la conception du systeme de programmes I-GSR₃S. Le
systeme admet des donnees initiales echantillonnees de facon
uniforme ou aleatoire. Pour les donnees a structure aleatoire on
applique la methode de la collocalation, ou la superficie de ten-
dence est probabilisement determinee, tandis que la fonction de
covariance peut etre automatiquement changee en vue de son adap-
tation aux differentes classes de terrain. Les structures uni-
iformes sont traitees a partir d’un programme basé sur la theorie
de l'approximation a fonctions spline.

Apres le calcul du DEM, optionnel peuvent etre engendrees des re-
presentations conventionnelles du relief ou 3D. La procedure de
traçage des courbes de niveau utilise les principes de nume-
riser en schéma quadratique. Les representations 3D sont reali-
sées par une variante optimisée de la méthode Wright.

En guise d'exemple on presente des sections de plan à courbes des
niveau restituées de facon numérique ainsi que des exemples de
représentations 3D.

ZUSAMMENFASSUNG

Es wird das Konzept des Programmierungssystem I-GSR₃S dargestellt.
Das System lässt sich als Muster aleatorisch oder regelmäßig
genommenen Anfangsdaten zu. Für die Daten mit aleatorischer Struk-
tur wendet man das Kollokasionsverfahren an, wobei die Tendenz-
fläche statistisch bestimmt wird und die Kovarianzfunktion auto-
matisch verändert sein kann, um ihre Anpassung an verschiedene
Geländeklassen zu erreichen. Die regelmässigen Strukturen werden
mit Hilfe eines Programms verarbeitet dass sich auf die Annäh-
erungsfunktionen mit Spline-Funktionen gründet.

Nach der Berechnung des DEM, optional können konventionelle Dar-
stellungen des Reliefs oder 3D Darstellungen erzeugt werden.
1. INTRODUCTION

Analyzed from the informative point of view, the digital terrain model (DTM) represents a system that comprises different categories of informations (heights, slopes, land values, soil type, hydrological, geological and geotechnical data, etc.) corresponding to the classes of objects and phenomena that form the surface of the earth. In use, they will be reference data, necessary to the building of different types of special digital models. Out of the multitude of models with special character, the digital elevation model (DEM) presents a particular importance to the activities of topographic mapping as well as those belonging to civil engineering. This includes an ordered multitude of informations concerning the position and the height of certain points on the terrain surface, sampled according to the structural characteristics of the relief and algorithms for approximation and reconstruction of the terrain surface in the new points. Besides the altimetric data, that have an importance of a primary element, DEM requires and comprises some specific features of planimetry e.g. shore lines of lakes and rivers, geomorphologic lines for shape contouring, valley bottoms, break lines, etc.

2. BUILDING DIGITAL ELEVATION MODEL (DEM) WITHIN THE COMPLEX PROGRAMS I^3GSR^3S

The automatization and the methods of analytical photogrammetry, give new valencies to topographic informations that once acquired according to the principle of checking maximum amount of information, recorded in a unique code and digitally stored, subsequently allow to be endlessly used without any restriction in different applications. It gives the possibility to generate by means an automatic processing and output systems of specific products for different aims, using same data basis resulting in a obvious increase yield and decrease of prices. The implementation of such technologies require the application of digital restitution methods within the building of digital models is the main component. Following this line, a complex programs for the calculation of DEM using the acquired photogrammetric data was achieved having as first application procedure the generation of contour lines. In the present variant I^3GSR^3S (Intercol, Intercub, Interlin, Genmatre, Sol, Rot and Relief-S) the issued modular complex programs has an advanced universality degree since admits input data with random and uniform structures. Data sampling are semiautomatically achieved using a system maked of a stereometerograph connected with Independent-102F minicomputer where a composite sampling technique is applied. Arbitrary located points are sampled by operator subsequent to the photointerpretation of the relief contained in the stereo model or uniformly placed in a square network whose step is determined using transfer function criterion. In both cases the points of the morphologic terrain features which will further complete the structure of the points of current surface description previously recorded are separately measured,
The first stage of data processing consist in their checking and
partitioning at calculation units, then, follows the building of
the digital model, that for random distributed data uses the Inter-
col program conceived on interpolation basis using the colloca-
tion method. The range of interpolation (fig. 1) within blocks
points are determined, is of circular shape and obtained getting
the maximum correlation distance between points equal with (1/2.5
... 1/4)λ for even or relatively even terrain, or (1/4.5... 1/6.5)λ
for uneven terrain, λ being medium wave length of terrain oscilla-
tion in comparison with the general trend surface.

The choice of trend surface corresponding to every domain is
made analysing the standard devia-
tion of differences between the
measured heights and their esti-

dated values. These are inferred by
means all variants resulted after
the combination of a basis surface
composed of the first six terms
of the incomplete bicubic poly nom
(A_{22}=A_{23}=A_{32}=A_{33}=0), with the rest
of the terms.

At the beginning, the 12 polynomial
coefficients are determined using
the method of the least squares:

\[ Z = [1 \times x^2 x^3][A][1 y y^2 y^3] \]

where \( A_{22}=A_{23}=A_{32}=A_{33}=0 \)

\[ Ax=L+v; (A^A)x=A^L=0; x=(A^A)^{-1}A^L \]

Then we use the following applied notations:

\[
\begin{align*}
Z &= A_{00} + A_{10}x + A_{01}y + A_{20}x^2 + A_{11}xy + A_{02}y^2 + A_{21}x^2y + A_{21}xy^2 + A_{03}y^3 + A_{12}x^2y^2 + A_{32}xy^3 + A_{33}x^3y + A_{33}x^3y
\end{align*}
\]

\[ \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{21} & A_{22} & A_{23} & A_{31} & A_{32} & A_{33} \end{bmatrix} \]

According to these, the trend surface variants is obtained by
combining \( A_1 \ldots A_7 \) elements.

\[ T_1(x,y), T_2(x,y) \ldots T_m(x,y) \] surfaces will successively result, and
owing to them new \( Z_i \) height values for \( N \) reference points from the
interpolation range will be calculated, and subsequently the
standard deviation of heights differences, \( \sigma \).

\[ \sigma = \frac{1}{N} \sum_{i=I}^{N} (Z_i - \bar{Z}_i)^2 \]

The surface corresponding to \( \sigma \) value will be selected as
trend surface \( \min T_C(x,y) \). Its coefficients will be
redetermined assigned for every equation in the system an weight
equal with the reverse value of height difference previously
obtained in reference point whose corresponds the equation.

\[ (A^T PA)x - A^T P L = 0 \]

where \( p_i = 1/(\bar{Z}_i - Z_i) \), in the \( P \) matrix weight.

Then the \( z_i \) height values are calculated as well as the \( dz_i \)
differences which participate in the interpolation process using
the collocation method
\[ Z(i) = T_c(X,Y) + c^T C^{-1} L \]  
(6)

Since the inverse \( C^{-1} \) of covariance matrix and \( L \) vector remain
constant for all interpolated points in a range, the interpolation
vector \( N = C^{-1} L \) is computed so that the interpolation is prac-
tically reduced to the product \( C^T N \).

The use of this method has the advantage of interpolating a group
of points (fig.1) in every range with a diminished time necessary
for the computation, since the covariance matrix need to be inver-
ted once.

The program comprises five covariance functions:
\[ F_{C_1} = C(0)-k d^2, \text{ where } C(0)=V, \text{ or with filtering } C(0)=(1-10^{-3})V \]
\[ F_{C_2} = (1-(d^2/m^2))^{1/2}; \quad F_{C_3} = 1/(1 + d^2/m^2); \quad F_{C_4} = e^{-2d/D}; \quad F_{C_5} = 1/(1+d/m) \]  
(7)

Having as decision parameter the comparison between the error of
evaluation and the maximum admissible error:
\[ m = C(0) - c^T C^{-1} c \leq m_{\text{adm}} \]  
(8)

the covariance function can be automatically changed in view of
adapting to the terrain surface configuration.

When the range of interpolation is crossed by break lines, this
part in subranges that are separately analysed. The computation
units process in Intercol program comprises up to 1000 points,
and the result of the processing is a model generated as a square
network.

The relief modelling starting from the uniformly structured input
coming from the sampling process or from Intercol program is
processed using Intercol program. Its algorithm is based upon ap-
proximation theory with spline functions used in different vari-
ants for the achievement of the digital elevation models(Ref.1,4,5)
that combine controlled polynomial approximation with the contin-
uity and smoothness of representation. A completely bicubic poly-
nom is used:
\[ Z = F(x,y) = [ X ][ A ][ Y ]^T \]  
(9)

the 16 parameters being determined by imposing four conditions
in each of the corner points of a network element which expresses
the connection in reference points \( Z_i \), the continuity of slope
\( Z'_x \) and \( Z'_y \) and the smoothness \( Z''_{xy} \). Considering the origin of
the coordinates system in the left corner of every square element
and the length of the side of thus is equal to a unit, the \( [A] \)
matrix elements result as functions only of \( Z, Z'_x, Z'_y, Z''_{xy} \)
numerical values (fig.2 and(10) formulas).

\[ A_{00} = Z_1; \quad A_{01} = Z'_x; \quad A_{10} = Z'_y; \quad A_{11} = Z''_{xy}; \]
\[ A_{20} = \frac{1}{2}[Z'_x Z'_{x2} - Z_x]; \quad A_{30} = \frac{1}{2}[Z'_y Z'_{y2} - Z_y]; \]
\[ A_{21} = 3[Z'_x Z'_{y2} - Z'_x Z_y]; \quad A_{31} = \]
\[ A_{33} = \]

\[ \text{Fig.2.} \]
The consideration of break lines within this program is obtained adding some supplementary conditions concerning the curvature continuity. The processed computation units are dimensioned up to 1000 points.

After the obtaining of digital model with one of the two procedures above described, the data are stored on magnetic tape or disc in a basis that includes only \(Z_{(i,j)}\) heights, the step value \(h\), the \((X_0, Y_0)\) origin and \((i,j)\) dimension. Is not necessary to store the \(X_{(i,j)}\), \(Y_{(i,j)}\) coordinates, since they may be obtained by computation formulas: \(X_{ij} = X_0 + ih; \quad Y_{ij} = Y_0 + jh\).

3. APPLICATIVE WORKING

According to digital model informations, optionally, conventional relief representations through contour lines at different scales or spatial representations (3D) can be obtained.

The levelling generation proceeding first call Interlin program which computes the maximum and minimum height, the number of contour lines and the points belonging to them on each side of the existing squares of the network, as long as on the break lines conforming to the equidistance introduced by the initial parameter. The new points determined by the linear interpolation are provisionally stored in files labeled according to the values of contour lines.

Consequently will be called the Gemmatre program that generate the representation matrix of every contour line. The element of the matrix represents the number of intersections that makes the curve with the sides of the grid squares, their value varying from 0 to 4. Besides is performed the testing of contour line configuration by comparing elements on every row, column and diagonal of the matrix to detect the areas where the curves have a low determination. The existence of such areas point out the fact that the network of the digital model has a not adequate density thus it doesn't contain a sufficient quantity of information and automatically should be thickened with Intercub program, the step of the network becoming \(h/2\). Then the computation sequences from Interlin and Gemmatre programs are repeated.

After that the curves fulfill the required conditions imposed of the Gemmatre program, the files of temporary stored points are processed with Sol sorting program which are achieved on the principle contour lines digitizing in the square schema. The program is composed by subroutines for most plausible forms of the curves analyzing with respect of the characteristic morphological terrain surface features. The existing contour lines within a computation unity can be closed, the first point being identical with the end point or open, when the end points are set on computation limits unity. Corresponding to that classification, the accurate contour line tract is determined through logical order sorting the points investigating each square using St, Dr, Su and Inf subroutines, not generalizing the shape of the curve. The network elements in which the terrain has a saddle form, characterised by the existence of four points belonging to a curve on the square sides are analysed with specific Parabol subroutine, that assimilates the terrain with a hyperbolic paraboloid. The position of the four points in a local coordinated system having its origin in the center of the hyperbole got by intersection of the paraboloid with a horizontal plan equivalent in height with the curve.
value and the configuration of the points within neighbouring squares establishing the order for correct points connection. The subroutine makes also the distinction between saddle shapes and the cases in which the four points of a square are due to the break line. The files of the points obtained by Sol program makes the curve under the polygon form. The usual contour line aspect is obtained consequently to Rot program that generates additional points on every side by a sliding interpolations method with a 5 degree polynomials. The number of additional points is established according to the characteristics of the used plotter for mapping. The second application procedure makes the generation of spatial representation (3D) by Relief-S program according to a variant of Wright method. The input data are composed of the corresponding heights network to the area that is to be represented and extracted from the general DEM in a [R] matrix, $(X_o, Y_o)$ origin, $(h)$ step and $(i,j)$ the network dimensions. The points are transformed in a special coordinate system for the obtaining 3D view effect:

\[
\begin{align*}
X_{i,j}^i & = X_o + (i-1)T_x + (j-1)h \\
Y_{i,j}^i & = (i-1)T_y + Z_{i,j} - Z_{min}
\end{align*}
\]  

This is equivalent with the displacement on the horizontal $(T_x)$ - optionally toward the left or the right and on $(T_y)$ vertical, $Y_{i,j}$ coordinate being replaced with the value of $Z_{i,j}$ height. The points along $i=1$ line that belong to $[R_{1,j}]$ matrix with transformed data, define the first minimum visibility limit. In comparison with this, computing interval by interval a function of the form:

\[
F(\Theta) = \left[ \Theta_{P_i,j,P_{i+1},j} - \Theta_{P_i,j',P_{i+1},j'} \right]
\]

the visible points of $i=2$ line are determined. The negative values of the function points out the hidden points and in their place new points are linearly interpolated and they will correct the longitudinal section initially drawn by the point of $i=2$ line obtaining the following minimal visibility limit. This represents the correct representation line on the horizontal. During the processing its points are consequently located in the line files of representation in crossing direction. The procedure is repeated with the next line of the matrix $[R_{1,j}]$. Different perspective variants can by obtained by applying some rotation of the data. Fig. 3 presented spatial plots - 3D corresponding to a sheet of map at 1:10,000 scale acquired with I2GSR2S complex of program and plotted to Aristo automatic drawing installation.

4. CONCLUSIONS

The applications achieved with I2GSR2S complex program have proved the possibility getting digitally contour lines which fulfill all the accuracy and mapping quality imposed for levelling representations on great scale maps. As compared to conventional restitution, digital restitution is more accurate in even, relatively even and moderately irregular terrain areas, but less adequate to thoroughly irregular terrain, crossed by a large number of break lines.

Data sampling is a fundamental element in the technique of DEM's and it always conditions the accuracy, fidelity, yield and price.
Fig. 3 Spatial plots - 3D (top) corresponding to sheet of topographic map at 1:10,000 scale (bottom), acquired with 1_2\text{GSR}_2S complex of programs.

Thus the use of interfaced devices with computers prove to be the most efficient means of getting the sampling, offering more rapid on line checking, testing and correcting by an interactive process. It also allows tremendous opportunities for the implementation of complex sampling procedures that allow the getting of height information according to the structural characteristics of each terrain class.

Nowadays the digital models impose themselves ever more since they represent a modern data basis, having the capacity of offering informations to a large range of technical and engineering activities.
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