SIMULTANEOUS PHOTOGRAMMETRIC AND GEODETIC MEASUREMENT PROCESSING IN ANALYTICAL AERIAL TRIANGULATION

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Zusammenfassung
Es wird ein mathematisches Modell, das sowohl die Ausgleichung der konventionellen photogrammetrischen, als auch der verschiedenen geodätischen Messungen behandeln kann, beschrieben. Der vorgeschlagene Algorithmus, als auch das Rechenprogramm, sind allgemein verfasst, so dass sie verschiedene geodätische Messungen, in irgendwelcher Anzahl und Anordnung, als auch nur photogrammetrische Messungen umfassen können. Die geodätischen und photogrammetrischen Messungen sind als unabhängige und mit, von den Messfehlern bedingten Gewichten, angenommen.
Für die photogrammetrischen Messungen wird die direkte Ausgleichnng, mit räumlichen photogrammetrischen Bündeln, benützt; für die geodätischen Messungen wird eine Verbesserungsgleichung zur Verarbeitung durch die Methode der vermittelnden Messungen, aufgestellt. Die ausgearbeitete Lösung, under der kodifizierten Benennung PSOFG in AA, ist an fiktiven und reellen Daten, sowohl auf Streifen als auch im Block, getestet worden. Die Endresultate sind in diesem Bericht angeführt.

Problems related to geodetic and photogrammetric measurements are approached independently, considering the common analytical aerial triangulation methods. Geodetic measurements resulting in control point coordinates becoming input data for the photogrammetric solution are made, on the one hand, and, on the other hand, photogrammetric measurements used in analytical aerial triangulation are performed.
The complete approach of the two problems [1,2] brings forth a mathematical model development, able to incorporate, besides the conventional photogrammetric measurements, the geodetic measurements such as: distances, azimuths, horizontal angles, zenithal distances and level differences into the adjustment.

The proposed algorithm and the computational programme are general, being able to comprise either various geodetic measurements considering numbers and locations, or only photogrammetric measurements.
Owing to the simultaneous computation difficulties, only correction equations for photogrammetric measurements are considered in the first part of the algorithm and the contribution of the correction equations for geodetic measurements to the former solutions are added in the second part of the algorithm.

Direct adjustment for space photogrammetric bundles entailing the simultaneous development of a large number of unknowns is used in photogrammetric measurements. The symmetry proprieties, that is to be positively defined or to have a strip - type structure of the normal equation coefficient matrix, aiming at reducing the arithmetic operation number and increasing computation efficiency are used during processings.

A correction equation for least mean square processing, is written for each geodetic measurement using the indirect measurement densities, locations and types, independently considered, are special to each work.
The whole process for simultaneous processing in analytical aerial triangulation is presented in the diagram in Figure 1.

The main stages of the simultaneous processing of photogrammetric and geodetic measurements within analytical aerial triangulation are briefly presented below. Finally, some practical data will be given in support of this presented solution.
Computation algorithm
The geodetic measurements, which correction equations are adjusted at the same time with correction equations for the photogrammetric measurements, using the proposed algorithm, are : distances, azimuths, horizontal angles, zenithal distances and level differences. The whole correction equation set for geodetic measurements can be represented by the following matrix equation:

$$V_g = GX = lg \quad (1)$$

where $\hat{x}$ is the correction vector of the ground coordinates.
The whole correction equation set for photogrammetric measurements can be also represented by the following matrix equation:

$$V_f + \bar{B}\bar{x} + Bx = l_f \quad (2)$$

where $\bar{x}$ is the correction vector of the photograph orientation element parameters, and x is the correction vector of ground coordinates.
The complete mathematical model is obtained by combining (1) and (2) equations:

$$\begin{bmatrix} V_f \\ V_g \end{bmatrix} + \begin{bmatrix} \bar{B} & B \\ 0 & G \end{bmatrix} \begin{bmatrix} \bar{x} \\ x \end{bmatrix} = \begin{bmatrix} l_f \\ l_g \end{bmatrix} \quad \begin{matrix} (3) \\ (4) \end{matrix}$$

that is $V + Ax = l$

In accordance with the least mean square method, the normal equations are:

$$(A^tPA)\hat{x} = A^tPl \quad (5)$$

where P is the weight matrix associated to the measured sizes and has the following shape:

$$P = \begin{bmatrix} P_f & 0 \\ 0 & P_g \end{bmatrix}$$

where P_f and P_g are the matrix diagonals, representing photogrammetric and geodetic measurement weights, respectively.
Equation (5) can be expanded into other two equations, such as:

$$\begin{aligned} L\bar{x} + Rx &= C \\ R^t\bar{x} + Hx &= D \end{aligned} \quad (6)$$

where, we have noted:

$L = \bar{B}^{-t} \; P_f \; \bar{B}$

$R = \bar{B}^{-t} \; P_f \; B$

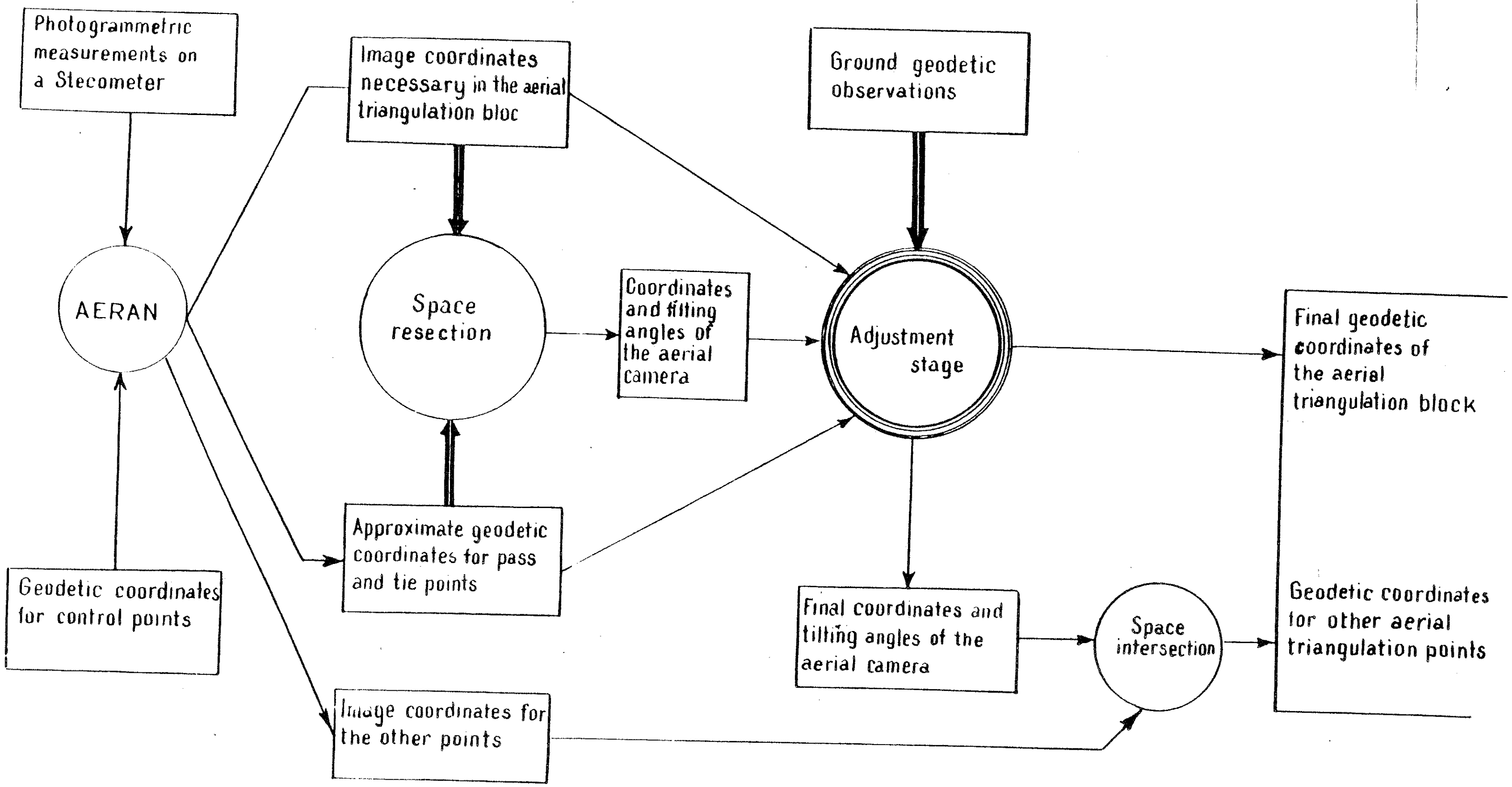

Figure 1

Diagram of the simultaneous processing process in analytical aerial triangulation

$H = H_f + H_g = B^t P_f B + G^t P_g G$

$C = B^{-t} P_f l_f$

$D = D_f + D_g = B^t P_f l_f + G^t P_g l_g$

Considering these operations for a photograph block containing 4 strips and 8 photographs each, and having a cvasi-regular aerial triangulation point locations and some geodetic measurements the normal equation coefficient matrix is presented in Figure 2. The elements equal zero are prevailing. In addition:

L has a diagonal block structure being composed of six order-n_f square submatrices, where n_f is the photograph number on the block;

R has a strip - type structure, being composed of 6 x 3, 6x2 and 6 x 1 - sized submatrices;

H has a diagonal block structure, too, but it is composed of H_g matrix coefficients outside the main diagonal and H_f matrix coefficients, made up of n_p square submatrices having a 3 x 3 maximum size, where n_p is the aerial triangulation point numbers on the block.

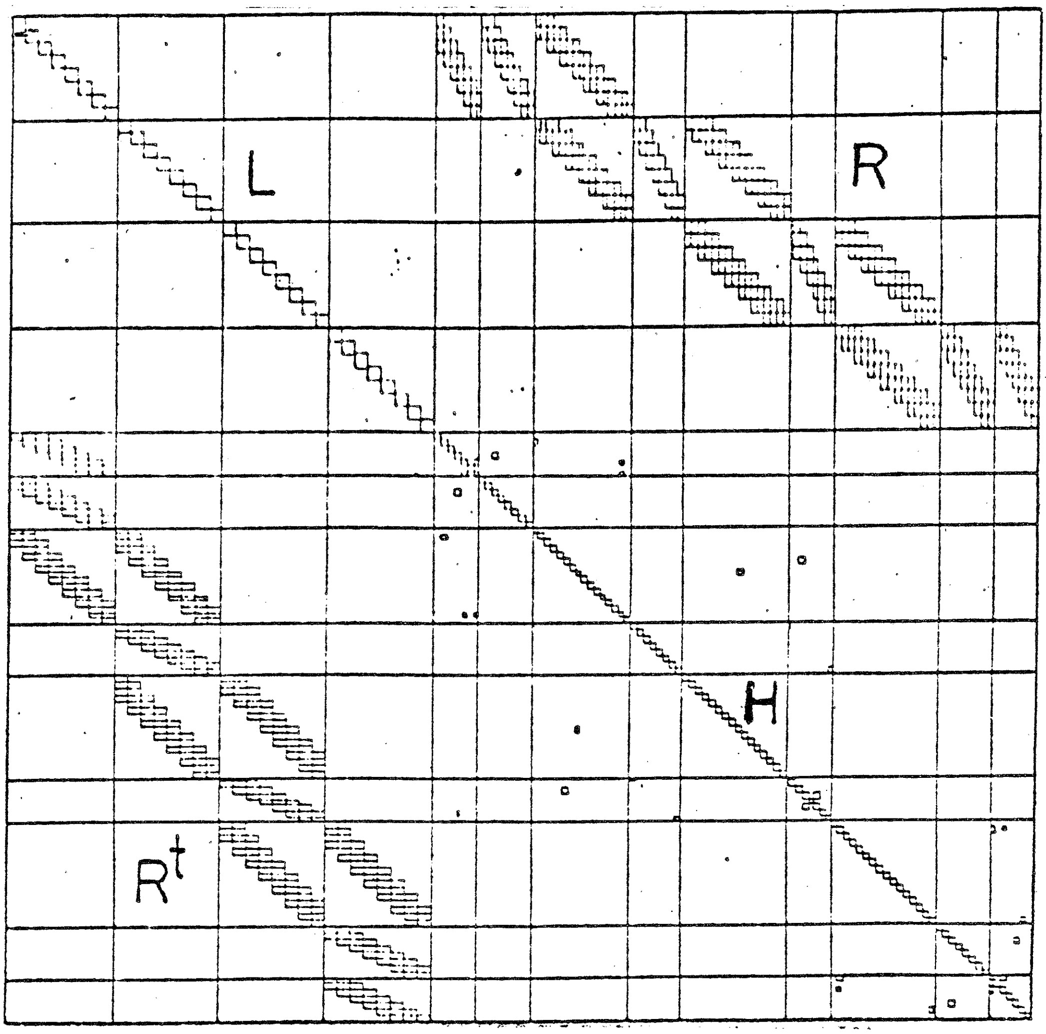

Figure 2 : Normal equation coefficient matrix for a 4 x 8 photograms block

Now, if we note:

$$N = A^t P A$$

then, equation (5) results in :

$$X = N^{-1} A^t P I \qquad (7)$$

but with [3,4] :

$$N^{-1} = \begin{bmatrix} S^{-1} & -S^{-1} T^t \\ -TS^{-1} & H^{-1}+TS^{-1} T^t \end{bmatrix}$$

where:

$S = L - R H^{-1} R^t$

$T = H^{-1} R^t$

Thûs, N inversion is reduced to S and H inversions (which sizes are $6 n_f \times 6 n_g$ and maximum $3 n_p \times 3n_p$, respectively) and to the corresponding matrix multiplications, as well. It is worth mentioning that when there are geodetic measurements S matrix is a complete one, but when there are not geodetic measurements or they are not considered, S matrix is reduced to :

$$S_f = L - R \quad H_f^{-1} \quad R^t \qquad (8)$$

Computation Programme

The simultaneous geodetic and photogrammetric measurement processing for analytical aerial triangulation starts with the following data, considering our case:

- image coordinate corrected and reduced to the main point for all points (control, pass, tie) necessary for an aerial triangulation block, provided by preliminary AERAN programme (Figure 1);
- approximate geodetic coordinates for all aerial triangulation points provided by the same programme;
- geodetic coordinates for control points;
- approximate geodetic coordinates and tilting angles of the aerial camera;
- geodetic measurements (distances, azimuths horizontal angles, zenithal distances and level differences).

Considering the programme, the main stages given in the diagram in Figure 1 are performed, such as:

- Space resection stage as a subroutine (RETRO), which is carried out for each photograph on the block;
- considering the adjustment stage, referring to strip-type structure matrices (S_f), FORMSF subroutine and INVSF one for its inversion are used;
- Space intersection stage (INSP) performed independently for each point of interest and which was not used in the adjustment stage.

Information carriers for input data necessary in simultaneous adjustment are the following, considering the present-day computation program variant:

- magnetic tape for image coordinates corrected and reduced to the main point and for approximate geodetic coordinates of all aerial triangulation points, as well; control point geodetic coordinates are also stored on them;
- punched cards for geodetic measurements and for various identifying and control elements.

The process aiming at solution developments requires the largest computation effort as regards the computation algorithm programming into a computer; that is why, some features related to associate matrix solution and inversion [R] have been considered. Three subroutines have been developed: the first for the strip-type structure matrix inversion, the second for equation system solutions and the associate matrix inversion, and the third for equation system solutions, which associate matrix has a strip-type structure.

Practical results

The algorithm and the computation programme have been tested using either fictitious or real data, considering both strip and block. In both cases, the strip has 10 photographs, and the block has 4 strips and 10 photographs each. The mean square errors of the computed coordinates for strip and block, considering fictitious and real data, respectively, are shown in Table 1.

Table 1

Case		m_x [m]	m_y [m]	m_z [m]
Fictitious data	Strip	± 0,15	± 0,15	± 0,20
	Block	± 0,21	± 0,21	± 0,41
Real data	Strip	± 0,29	± 0,31	± 0,46
	Block	± 0,37	± 0,35	± 0,56

The final results certify the usefulness and efficiency of the developed algorithm and computation programme. The possibility to directly use geodetic measurements into adjustment solves one of the most difficult problems regarding control point implementation in zones covered by forest vegetation or where significant details are missing.

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