RADAR STERE0 MODEL SET-UP OX THE
ANALYTICAL PLOTTER KERN DSR-1 *)
J. Ragam
Technical University and Graz Research Center
AUSTRIA
Comission III

ABSTRACT

Stereo mapping with radar images became possible with the
advance of analytical plotters. A software package (SMART --
Stereo Mapping with Radar Techniques) for mapping from single
radar images and radar stereo models on the analytical plotter
Kern DSR-1 was developed. Images used for this procedure may
be presented either with ground or slant ranges on film in
analog form. The paper will give a broad description of SMART
and will present the mathematical solution of the radar stereo
model set-up process. Besides examples of results and accuracy
evaluations will be shown.

INTRODUCTION

Analytical plotters use a computer to relate cartesian
X,Y,Z model coordinates to x,y image coordinates and servo
motors for the positioning of the plate carriers in a real time
loop. Photographs with different kinds of imaging geometries
may be addressed to analytical plotters when introducing the
appropriate imaging equations in the loop. This was done for
radar images in the software package SMART (Stereo Mapping with
Radar Techniques), which is installed at the analytical plotter
Kern DSR-1.

The DSR-1 consists of the three computers P1, P2 and P3
(Chapuis, 1980). Processor P1 is the host computer and serves
for development and execution of application programs. Within
these programs the computation of model set-up parameters and
the data transfer between the other processors is performed.
In processor P2 a separate program is executed, which converts
X,Y,Z model coordinates to DSR-1 plate coordinates in a real
time endless loop with orientation parameters received from P1.
This program is denoted as 'plate processor program'. P2 also
transfers data like image, model or object coordinates to
processor P1 if requested from the application program.
Besides the processor P3 is available as a third computer which
may be used for the communication between the operator and the
stereo plotter.

The procedure implemented in SMART for radar stereo model
set-up is based on a radar bundle adjustment for the
determination of the orientation parameters. No relative or
absolute orientation procedures are needed separately. As will

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be shown in the paper this process of model set-up is very convenient and enables also the use of convergent flight lines. This is an important fact since actual radar data acquisition of the Space Shuttle Imaging Radar Experiment (SIR-A) is with crossing flight paths. Viewing limitations for this type of radar stereo were discussed by Domik et al. (1983) and first radargrammetric evaluations with SIR-A data are presented by Kobrick et al. (submitted).

THE SOFTWARE PACKAGE SMART IN A SHORT OVERVIEW

The software package SMART is an interactive program system for either mapping from radar stereo models or single radar images. A first description of the system was presented by Raggam and Leberl (1984). It is organized in several separate modules as follows:

Project Information Module:
... input of specific project parameters
  (origin of local cartesian coordinate system,
   radius of the planet, mean terrain height)
... general information about the project
... image and orbit data information

Control Point Management Module:
... input of geographic or cartesian control
  point coordinates
... manipulation of these data
  (listing, correction, deleting)
... transformation from geographic into cartesian
  coordinates or reverse if necessary

Orbit Data Management Module:
... input and manipulation of geographic or
  cartesian orbit station coordinates
... transformation from geographic into cartesian
  coordinates or reverse if necessary
... computation of approximations for the orbit
  coefficient vectors

Single Radar Image Module:
... input of initial image parameters
... inner orientation
... digitizing of image coordinates
... orientation of a single radar image
... computation of radar image distortion polynomials
... mono-plotting

Stereo Radar Image Module:
... use of single radar image module for inner
  orientation, preliminary single image orientation
  or to determine radar image distortions
... digitizing of homologue or single radar image
  coordinates
... exterior orientation of the stereo model using
  radar intersection conditions for homologue points
... stereo-plotting
Plotting with a single radar image may be done either on a spherical surface of chosen radius or in a three-dimensional X,Y,Z-coordinate system with known constant Z. For a radar stereo model contour-lines or planimetric features are either directly plotted on an xy pen plotter or are entered into a digital data base for further processing, e.g. to create digital elevation models.

**MODEL SET-UP WITH RADAR BUNDLE ADJUSTMENT**

From traditional stereo photogrammetry we know the process of model set-up as a composition of inner, relative and absolute orientation. However, analytical plotters does not need procedures for relative and absolute orientation separately in contrast to analog stereo plotting instruments, but the parameters of these tasks may be determined simultaneously with a bundle adjustment. This approach is also implemented in SMART for radar images and enables a convenient handling even with radar stereo models with non-parallel (intersecting) flight lines.

Therefore the radar stereo model set-up is realized in two steps consisting of the computation of the elements of inner orientation of the radar image pair, followed by an exterior orientation with a radar bundle adjustment.

**Inner Orientation:**

The system distinguishes between a new and an old inner orientation process. While a new inner orientation always is used for the first model set-up of two radar images, the old one is for resetting the radar stereo model. The relationship between DSR-1 plate coordinates \( (x_p, y_p) \) and physical radar measurements (range \( r \) and time \( t \)) is assumed in SMART as follows:

\[
\begin{align*}
    x_i &= a_{11} * x_p + a_{12} * y_p + a_{13} \\
    y_i &= a_{21} * x_p + a_{22} * y_p + a_{23} \\
    t &= x_i * m_x + t_0 \\
    r &= y_i * m_y + r_0 \quad \text{(slant ranges)} \\
    r &= \sqrt{(y_i * m_y + r_0)^2 + H^2} \quad \text{(ground ranges)}
\end{align*}
\]

\( x_i, y_i \) ... radar image coordinates

In the program system a so-called 'range reference line' is used at the near range edge of the image as the x-axis of the radar image coordinate system, which is pointing in the flight direction of the satellite. It corresponds to the start of the sweep on the image recorder and should be defined by some distinct tick marks (fiducials). The y-axis is perpendicular to it and depending on the look direction of the
radar system it points either to the right or to the left of
the range reference line.

A new inner orientation procedure consists of the
measurement of tick marks and determination of the range
reference line by minimizing the perpendicular distances of the
measured ticks from a straight line. In the case of an old
inner orientation for model reset at least two of the tick
marks have to be remeasured, so that the conformal
transformation of plate coordinates to radar image coordinates
can be recalculated. In the event that no tick marks exist on
the radar image the operator has to create artificial marks and
to determine the inner orientation in a process of
self-calibration together with the exterior orientation.

**Exterior Orientation:**

The parameters of exterior orientation are first of all
the coefficients of the orbit time polynomials. But also the
parameters to relate rectangular radar image coordinates \(x, y\) to
physical radar measurements range and time (equations 1) as
well as a value for the squint angle are determined within the
radar bundle adjustment. Rigorous radargrammetric formulations
are used for the model set-up process, using intersection
conditions for homologue image points to create a parallax-free
stereo model. The procedure requires either sufficient ground
control or well known flight parameters to get good
approximations for the radar image orientation parameters.

This kind of solution of exterior radar image orientation
is the same for single radar images as well as for radar stereo
models. So the individual images may be processed first to
obtain better approximate values for a bundle adjustment for
the radar stereo model. The following observations may be used
for a bundle adjustment with a radar stereo image pair (compare
Figure 1):

... sensor position measurements for one or both
orbits
... image coordinate measurements of control points
in one or both images
... homologue image coordinate measurements of
stereo model orientation points

**Unknowns within the adjustment are:**
... the coefficients of the polynomials representing
the flight lines for left and right radar image
... the radar imaging parameters to convert radar
image coordinates to physical measurements
and a value for the squint angle for both images
... \(X, Y, Z\) - coordinates of homologue orientation
points
Figure 1: Illustration of observations to be used in the radar bundle adjustment

Since the flight lines are represented with time polynomials the sensor position measurements may be expressed as functions of time $t$:

$$s_x = a_x + b_x * t + c_x * t^2 + ...$$
$$s_y = a_y + b_y * t + c_y * t^2 + ...$$
$$s_z = a_z + b_z * t + c_z * t^2 + ...$$

(EQ. 2)

Approximations for the orbit polynomial coefficients are known and so a Taylor series development of equations (2) results in:

$$\mathbf{v_1} = \mathbf{A_1} * \Delta \mathbf{u} - \mathbf{w_1}$$

(EQ. 3)

$\mathbf{v_1}$ is the vector of corrections for sensor position measurements, vector $\Delta \mathbf{u}$ contains increments for the unknowns (orientation parameters and $x, y, z$ object coordinates of the orientation points) and $\mathbf{w_1}$ is a vector of the observation equation discrepancies. $\mathbf{A_1}$ is a coefficient matrix, but it contains only non-zero elements in the columns corresponding to the unknown polynomial coefficients.

For radar image coordinate measurements $x, y$ the basic equations for the radar bundle adjustment consist of two types:

1) the squint angle condition:

$$\hat{\mathbf{s}} * (\mathbf{p} - \mathbf{s}) - \sin \tau * |\hat{\mathbf{s}}| * |\mathbf{p} - \mathbf{s}| = 0$$

the range condition:

\[(y_i * m_y + r_o) - |p - s| = 0\]  \[\text{(for slant range presentations)}\]

\[\sqrt{(y_i * m_y + r_o)^2 + h^2} - |p - s| = 0\]  \[\text{(for ground range presentations)}\]

(Eq. 4)

General linearization of these equations results in the following system of correction equations:

\[C * \Delta u + D * \Delta u + w = 0\]  \[\text{(Eq. 5)}\]

\(C\) and \(D\) are coefficient matrices, \(\Delta u\) contains the corrections of image coordinate measurements, \(\Delta u\) is a vector of increments for the unknowns and the elements of vector \(w\) are the contradictions of the basic observation equations (4). The elements of matrix \(D\) corresponding to unknown \(x, y, z\) object coordinates obviously are zero in the case of measured image coordinates of a control point.

Since \(C\) is a regular quadratic matrix, the following conversion of equation (5) is possible to separate the corrections of image coordinate measurements:

\[z_2 = A_2 * \Delta u - w_2\]  \[\text{(Eq. 6)}\]

with

\[A_2 = -C^{-1} * D\]

\[w_2 = C^{-1} * w\]

\[z_2 = v\]

So two equations are obtained from measured image coordinates of a control point in the single image. Otherwise each pair of homologue image coordinate measurements of orientation points results in a pair of these equations with the object coordinate increments as additional unknowns and produces one additional condition. Equations (3) for orbit station measurements and equations (6) for image coordinate measurements together are used in a least squares bundle adjustment. The system of correction equations for orbit station measurements and image coordinate measurements for control points or homologue orientation points is solved with a least squares algorithm. The method of conjugated gradients (Schwarz, 1970) is used for the adjustment process instead of a common Gauss solution of normal equations. This method is iterative and may have some advantages against the Gaussian algorithm, especially for larger equation systems when a small computer must be used and computing times and storage requirements are limited.

To perform an exterior orientation for a radar stereo model with radar bundle adjustment the operator first has to measure the image coordinates of a set of control points and probably of homologue orientation points. This measurement
should be done in a stereo comparator mode. Then the single radar image module might be entered to solve the exterior orientation for the stereo partners separately to get better approximations for the orientation parameters of the images. This process is only possible with a sufficient number of control points available. At last the bundle adjustment procedure for the radar stereo model is used to determine final values for the orientation parameters.

Depending on the quality of the approximate values for the unknowns and on the range of overdetermination the whole adjustment process has to be repeated iteratively. At each iteration step the approximations are corrected and the operator may decide, if the results satisfy subjective termination criteria or if the next iteration should be started.

RESULTS OF MODEL SET-UP WITH BUNDLE ADJUSTMENT

The procedure described above for stereo model set-up within the software package SMART was applied to a SEASAT radar stereo model of the area of Los Angeles (see Figure 2) and to a SIR-A radar stereo model of the Greek islands Cephalonia and Ithaka (see Figure 3).

![Figure 2: SEASAT stereo model of Los Angeles](image)

- image scale 1:750,000, digital correlation, control points from
- map 1: 24,000, parallel flight lines
The bundle adjustment was solved for both stereo models with different distributions of control and orientation points. The measured radar image coordinates of all identified control points were transformed to object coordinates $X_t$, $Y_t$, $Z_t$ with the adjusted orientation parameters and compared with the given values $X$, $Y$, $Z$. Table 1 summarizes the results of this procedure for both the SEASAT and the SIR-A stereo model.

<table>
<thead>
<tr>
<th>Radar Imaging System</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>RMS-residuals (meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>SEASAT</td>
<td>2</td>
<td>2</td>
<td>16</td>
<td>10</td>
<td>134</td>
</tr>
<tr>
<td></td>
<td>2</td>
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<td>4</td>
<td>0</td>
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<td>2</td>
<td>2</td>
<td>2</td>
<td>8</td>
<td>169</td>
</tr>
<tr>
<td>SIR-A</td>
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<td>31</td>
<td>6</td>
<td>92</td>
</tr>
<tr>
<td></td>
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<td></td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>56</td>
</tr>
</tbody>
</table>

- a ... number of left orbit station measurements
- b ... number of right orbit station measurements
- c ... number of homologue control points
- d ... number of homologue orientation points

Table 1: Root mean square errors in object coordinates of control points after bundle adjustment
It can be seen, that even with a minimum number of control points the results obtained are quite reasonable for the SEASAT as well as for the SIR-A stereo model. The accuracies generally correspond to what could have been expected from the radar input. The use of higher control point densities improves the results whether for SEASAT nor for SIR-A data with an equivalent factor. However, it has emphasized, that a number of well distributed homologue orientation points should be measured to obtain a sufficiently parallax-free stereo model.

CONCLUSION

A short overview is given of the capabilities of the radar stereo mapping program system SMART, which was developed for the analytical plotter Kern DSR-1. The procedure of radar stereo model set-up used in the system is described. Rigorous radargrammetric formulations are used in a radar bundle adjustment to determine the orientation parameters of the stereo model. Tests have shown, that this is a very powerful method for model orientation, which works also with convergent radar stereo images. This is of relevance in all satellite radar projects since at higher latitudes radar stereo imagery is with non-parallel flight lines. Convergent stereo is also actual for the Space Shuttle Imaging Radar experiments SIR-A and SIR-B or for the forthcoming Venus Radar Mapping mission.

After model set-up it is possible to create a digital elevation model by digitizing of contour-lines and supplementary terrain features. One effort of these 'stereo-derived' height models may be to support the rectification of digital radar images (Domik and Raggam, 1984), but certainly the accuracy of heights obtained from actual radar stereo images is limited due to the radar input.
REFERENCES


