# A CONCEPTION OF NUMERICAL ELABORATION OF TERRESTRIAL PHOTOGRAPHS A BLOCK METHOD "COMPBLOCK"

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Abstract

A method of analytical elaboration of terrestrial photographs has been presented. The derivation and the final form of the formulas based on a solution to the parametric forward spatial intersection have been given. The process of formation of a non-linear set of conditioned equations with unknowns has been presented. Unknown are only adjustments to approximated elements of the outer and inner orientations of the photographs. The adjusted magnitudes are image coordinates as direct observations or computed on the basis of parallaxes. The coordinates of all points in the geodetic system are computed by solving parametric forward spatial intersections after adjusting the elements of orientation of the photographs and the coordinates of the image points. The advantages of the method are a simple algorithm, sets of equations with a relatively small number of unknowns and the ease of programming on a small computer.

## 1. Introduction

The paper presents a method of analytical elaboration of terrestrial photographs arbitrarily oriented in space and taken with any cameras. Derivation and the final form of formulas for the photographs have been given. The formulas were based on equations of intersecting straight lines in a parametric form. For any orientations may be determined by means of analytical adjustment. The adjusted magnitudes are image coordinates as direct observations or computed on the basis of parallaxes adjusted on account of the influence of systematic factors. The process of formation of a non-linear observational set of conditioned equations with unknowns has been presented. In the set the unknowns are only the orientation elements of the photographs. Coordinates of all points in a geodetic system are computed by solving parametric forward spatial intersections after adjusting the orientation elements of the photographs and the coordinates of image points.

A measurable advantage of the method is the relatively small number of unknowns which appears in the set of observational equations. A simple algorithm makes it possible to make the computations on small computers.

## 2. Parametric solution to a forward spatial intersection

The formulas presented below are based on the assumption of simple equations in a parametric form. We prove that this treatment accepts each case of geometrical intersection obviously with the exception of cases when the straight lines are parallel or overlapping each other, since then an intersection does not exist. On the other hand, when the straight lines are skew and the value of the shortest distance between them is sufficiently close to zero and results from measuring errors in geometrical elements then the intersection is determinable. The accuracy of determination of a point`s coordinates depends on the angle of intersection of the straight lines in space.

Let us write the parametric formulas of straight lines originating form points $O_i$, $O_j$ making the basis of intersection (Fig. 1). The points have their definite coordinates in the geodetic system $X_i$, $Y_i$, $Z_i$, $X_j$, $Y_j$, $Z_j$. It is necessary to determine the coordinates of point $X_k$, $Y_k$, $Z_k$ having the following measured data: horizontal angles $(\varphi_{jk}, \varphi_{ik})$, vertical angles $(\omega_{ik}, \omega_{jk})$ and measured by means of levels inclination angles of the instrument`s axis of rotation $(\varkappa_{ik}, \varkappa_{jk})$ at the stations.

The equations of the straight lines have the form

$$\begin{aligned} X_k &= X_i + R_{ik} \cdot a_{ik} \qquad & X_k &= X_j + R_{jk} \cdot a_{jk} \\ Y_k &= Y_i + R_{ik} \cdot b_{ik} \qquad & Y_k &= Y_j + R_{jk} \cdot b_{jk} \\ Z_k &= Z_i + R_{ik} \cdot c_{ik} \qquad & Z_k &= Z_j + R_{jk} \cdot c_{jk} \end{aligned} \tag{1}$$

where: $a_{ik}$, $b_{ik}$, $c_{ik}$ are direction cosines of straight line i-k expressed as functions of measured angles $(\varphi_{ik}, \varkappa_{ik}, \omega_{ik})$,

$a_{jk}$, $b_{jk}$, $c_{jk}$ are direction cosines of straight line j-k expressed as functions of measured angles $(\varphi_{jk}, \varkappa_{jk}, \omega_{jk})$,

$R_{ik}$ is parameter denoting unknown spatial distance between points i-k,

$R_{jk}$ is parameter denoting unknown spatial distance between points j-k.

If we compare the right sides of equations (1) we get a set of three equations with two unknown parameters $R_{ik}$, $R_{jk}$:

$$R_{ik} \cdot a_{ik} - R_{jk} \cdot a_{jk} + (X_i - X_j) = 0$$

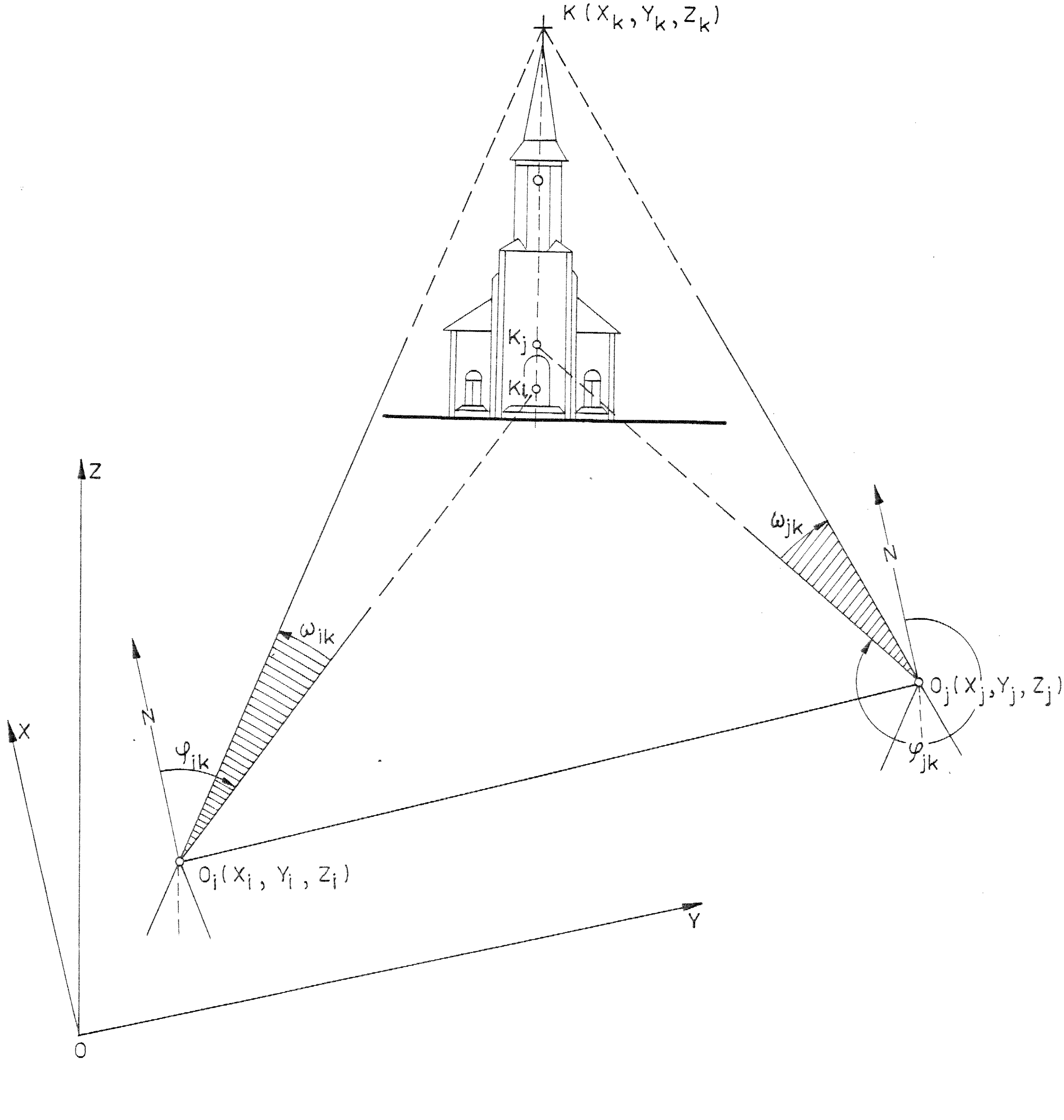


Figure 1.

$$R_{ik} \cdot b_{ik} - R_{jk} \cdot b_{jk} + (Y_i - Y_j) = 0 \tag{2}$$

$$R_{ik} \cdot c_{ik} - R_{jk} \cdot c_{jk} + (Z_i - Z_j) = 0$$

Thus, the resulting set of equations is overdetermined and thus instead of zeros we assing adjustments to the right sides of the equations and apply the condition of least squares.

$$\sum_{i=1}^{3} \delta_i = \min \tag{3}$$

Hence we get normal equations

$$\begin{aligned} +R_{ik} \qquad & -R_{jk} \cdot \cos\gamma = -L_{ik} \\ -R_{ik} \cdot \cos\gamma \quad & +R_{jk} \qquad\quad = +L_{jk} \end{aligned} \tag{4}$$

where: $\gamma$ - is intersection angle of straight lines in space,

$$\cos\gamma = a_{ik} \cdot a_{jk} + b_{ik} \cdot b_{jk} + c_{ik} \cdot c_{jk}$$

$$L_{ik} = a_{ik} \cdot (X_i - X_j) + b_{ik} \cdot (Y_i - Y_j) + c_{ik} \cdot (Z_i - Z_j) \tag{5}$$

$$L_{jk} = a_{jk} \cdot (X_i - X_j) + b_{jk} \cdot (Y_i - Y_j) + c_{jk} \cdot (Z_i - Z_j)$$

Having solved them we get:

1) formulas for determination of parameters $R_{ik}$, $R_{jk}$

$$R_{ik} = \frac{L_{jk} \cdot \cos\gamma - L_{ik}}{1 - \cos^2\gamma}$$

$$R_{jk} = \frac{L_{jk} - L_{ik} \cdot \cos\gamma}{1 - \cos^2\gamma} \tag{6}$$

2) variance-covariance weight matrix in the form

$$Q = \frac{1}{1 - \cos^2\gamma} \begin{bmatrix} 1 & \cos\gamma \\ \cos\gamma & 1 \end{bmatrix}. \tag{7}$$

From an analysis of the weight matrix it follows that errors in determination of $R_{ik}$, $R_{jk}$ parameters are identical. This allows us to average the coordinates of the intersection points. Having set the right sides of formula (1) we get the dependencies

$$
\begin{aligned}
X_k &= (X_i + R_{ik} \cdot a_{ik} + R_{jk} \cdot a_{jk}) / 2 \\
Y_k &= (Y_i + R_{ik} \cdot b_{ik} + R_{jk} \cdot b_{jk}) / 2 \\
Z_k &= (Z_i + R_{ik} \cdot c_{ik} + R_{jk} \cdot c_{jk}) / 2
\end{aligned}
\qquad (8)
$$

## 3. Geometrical dependencies of a pair of photographs

Let us take into consideration a pair of photographs taken form two different stations $O_i$, $O_j$ with known elements of interior and exterior orientation defined in the folloving way (Fig. 2)

X, Y, Z - system of geodetic coordinates. This is a dextrorotatory system,

x, y, z - coordinate system of a photograph (axis y is identical with the camera's axis which is defined as a line crossing the main point and the centre of projections). This is a laevorotatory system.

Axis of nodes - this is a trace of intersection of XOY plane of the geodetic system by zOy plane crossing the photograph's axis and the camera's axis;

$\varphi$ - turn angle of camera axis contained between node axis and positive sense of axis OX of geodetic system. Turn around OZ axis $(0 \leq \varphi < 2\pi)$

$\varkappa$ - inclination angle of Ox axis of the photograph against OXY plane of the geodetic system. Turn around node axis. $(0 \leq \varkappa < \pi)$ (The angle usually assumes values close to zero withim the range of error of levelling of the camera);

$\omega$ - inclination angle of camera axis contained between node axis and camera axis (Oy axis). Turn around Ox axis. $(0 \leq \omega < 2\pi)$

Point K (Fig.2) has been contained in the photographs. Making use of dependencies occurring in a single photograph between the image coordinates and the horizontal and vertical angles we get:

$$
\begin{aligned}
\alpha_{ik} &= \operatorname{arc\,tg}\left(\frac{x'_{ik}}{-f_i}\right) & \beta_{ik} &= \operatorname{arc\,tg}\left(\frac{z'_{ik}}{\sqrt{f_i^2 + x'^2_{ik}}}\right) \\
\alpha_{jk} &= \operatorname{arc\,tg}\left(\frac{x'_{jk}}{-f_j}\right) & \beta_{jk} &= \operatorname{arc\,tg}\left(\frac{z'_{jk}}{\sqrt{f_j^2 + x'^2_{jk}}}\right)
\end{aligned}
\qquad (9)
$$

where: $x'_{ik}$, $z'_{ik}$, $x'_{jk}$, $z'_{jk}$ - reduced image coordinates computed from the formulas

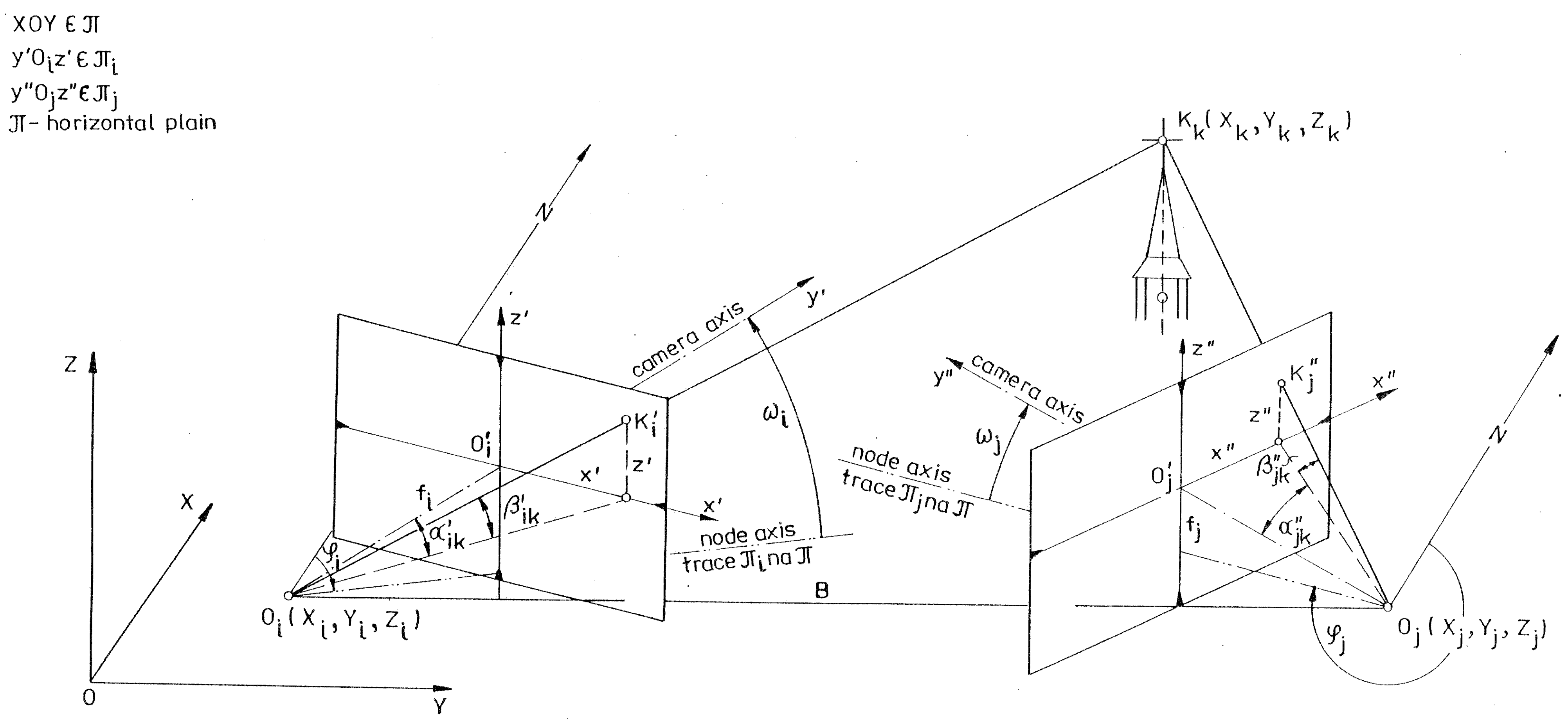
XOY ∈ π
y'O_iz' ∈ π_i
y"O_jz" ∈ π_j
π- horizontal plain
K_k(X_k, Y_k, Z_k)
N
z'
camera axis
y'
Z
O'_i
K'_i
z'
x'
ω_i
X
f_i
β'_ik
α'_ik
φ_i
node axis
trace π_i na π
O_i(X_i, Y_i, Z_i)
B
O
Y
y"
camera axis
ω_j
node axis
trace π_j na π
z"
K"_j
x"
z"
x"
O'_j
β"_jk
f_j
α"_jk
N
φ_j
O_j(X_j, Y_j, Z_j)

Figure 2.

a) for monocularly observed photographs

$$x'_{ik} = x_{ik} - xo_i \qquad x'_{jk} = x_{jk} - xo_j$$
$$z'_{ik} = z_{ik} - zo_i \qquad z'_{jk} = z_{jk} - zo_j \qquad (10)$$

b) for stereograms

$$x'_{ik} = x_{ik} - xo_i \qquad x'_{jk} = x_{ik} - xo_i + xo_j - p_{ijk}$$
$$z'_{ik} = z_{ik} - zo_i \qquad z'_{jk} = z_{ik} - zo_i + zo_j - q_{ijk} \qquad (11)$$

$x_{ik}$, $z_{ik}$, $x_{jk}$, $z_{jk}$, $p_{ijk}$, $q_{ijk}$ - image coordinates and measured parallaxes,
$xo_i$, $zo_i$, $xo_j$, $zo_j$ - points coordinates of main photographs,
$f_i$, $f_j$ - distances of pictures.

From Figure 2 it follows that

$$\varphi_{ik} = \varphi_i + \alpha_{ik} \qquad \omega_{ik} = \omega_i + \beta_{ik}$$
$$\varphi_{jk} = \varphi_j + \alpha_{jk} \qquad \omega_{jk} = \omega_j + \beta_{jk} \qquad (12)$$

where: $\varphi_i$, $\varphi_j$ - turn angles of camera axes;
$\omega_i$, $\omega_j$ - inclination angles of camera axes;
$\alpha_{ik}$, $\alpha_{jk}$ - horizontal angles computed from image coordinates,
$\beta_{ik}$, $\beta_{jk}$ - vertical angles computed from image coordinates.

Regarding the fact that the geodetic system and the system of a photograph have different turns the direction cosines of the straight lines will have the form

$$a_{ik} = +\cos(\varphi_i + \alpha_{ik})\cdot\cos(\omega_i + \beta_{ik}) + \sin(\varphi_i + \alpha_{ik})\cdot\sin\varkappa_i\cdot\sin(\omega_i + \beta_{ik})$$
$$b_{ik} = +\sin(\varphi_i + \alpha_{ik})\cdot\cos(\omega_i + \beta_{ik}) - \cos(\varphi_i + \alpha_{ik})\cdot\sin\varkappa_i\cdot\sin(\omega_i + \beta_{ik})$$
$$c_{ik} = +\sin(\omega_i + \beta_{ik})\cdot\cos\varkappa_i \qquad (13)$$
$$a_{jk} = +\cos(\varphi_j + \alpha_{jk})\cdot\cos(\omega_j + \beta_{jk}) + \sin(\varphi_j + \alpha_{jk})\cdot\sin\varkappa_j\cdot\sin(\omega_j + \beta_{jk})$$
$$b_{jk} = +\sin(\varphi_j + \alpha_{jk})\cdot\cos(\omega_j + \beta_{jk}) - \cos(\varphi_j + \alpha_{jk})\cdot\sin\varkappa_j\cdot\sin(\omega_j + \beta_{jk})$$
$$c_{jk} = +\sin(\omega_j + \beta_{jk})\cdot\cos\varkappa_j$$

Having set dependencies (13) into formulas (8), the latter will become basic formulas. Formulas (8) are general and may be used for determination of geodetic coordinates of points of any case of terrestrial photographs arbitrarily oriented in space.

## 4. Algorithmic-numerical solution to the problem

We shall now formulate conditions for points which are found at least in two photographs and points situated in a single photograph but with three known geodetic coordinates. For k-th point photographed in $r_k$ photos we can formulate $3/2 \cdot r_k \cdot (r_k - 1)$ conditioned equations with unknowns in the form

$$\begin{aligned} R_{ik} \cdot a_{ik} - R_{jk} \cdot a_{jk} + (X_i - X_j) &= 0 \\ R_{ik} \cdot b_{ik} - R_{jk} \cdot b_{jk} + (Y_i - Y_j) &= 0 \\ R_{ik} \cdot c_{ik} - R_{jk} \cdot c_{jk} + (Z_i - Z_j) &= 0 \end{aligned} \qquad (14)$$

where: $R_{ik}$, $R_{jk}$ - calculated form formula (6).

Besides, for a point with known geodetic coordinates ($s_k$ = 1,2, 3) we shall get $s_k \cdot r_k$ additional conditioned equations with unknowns of the type

$$\begin{aligned} R_{ik} \cdot a_{ik} + (X_i - X_k) &= 0 \\ R_{ik} \cdot b_{ik} + (Y_i - Y_k) &= 0 \\ R_{ik} \cdot c_{ik} + (Z_i - Z_k) &= 0 \end{aligned} \qquad (15)$$

where $R_{ik}$ is determined from equation

$$R_{ik} = \sqrt{(X_k - X_i)^2 + (Y_k - Y_i)^2 + (Z_k - Z_i)^2} \qquad (16)$$

k - point indicator,
i,j - indicators of photographs.

Thus, for a single point photographed in $r_k$ photos with known geodetic coordinates ($s_k$ = 0, 1, 2, 3) we can jointly write

$$w_k = 3/2 \cdot r_k \cdot (r_k - 1) + s_k \cdot r_k \qquad (17)$$

of conditioned equations with unknowns expressing dependencies between observations with unknown adjustments to approximated orientation elements of photographs.

For the whole block of photographs containing all m points we can formulate

$$w = \sum_{k=1}^{m} w_k \tag{18}$$

conditioned equations with unknowns of the type (14) and (15). Assuming that in each i-th photograph we have t unknown orientation elements ($t = 1, 2, \ldots, 9$), the number of unknowns will be determined from the dependence

$$U = \sum_{i=1}^{N} t_i \tag{19}$$

where: N - number of photographs.

Taking into consideration all the measured magnitudes we get a set of equations which must fulfil the condition $w > U$.

Let us describe the subvector of approximated orientation elements of photographs by

$$\varrho_{ij} = (X_i, Y_i, Z_i, \varphi_i, \varkappa_i, \omega_i, xo_i, f_i, zo_i, X_j, Y_j, Z_j, \varphi_j, \varkappa_j, \omega_j, xo_j, f_j, zo_j) \tag{20}$$

and by $\tilde{x}_{ik}$, $\tilde{z}_{ik}$, $\tilde{x}_{jk}$, $\tilde{z}_{jk}$ the measured image coordinates whose adjusted values will be equal to $x_{ik} = \tilde{x}_{ik} + vx_{ik}$; $z_{ik} = \tilde{z}_{ik} + vz_{ik}$; $x_{jk} = \tilde{x}_{jk} + vx_{jk}$; $z_{jk} = \tilde{z}_{jk} + vz_{jk}$; then the observation equations will assume the form

$$F_{ijk}(\varrho_{ij} + \delta\varrho_{ij}, \tilde{x}_{ik} + vx_{ik}, \tilde{z}_{ik} + vz_{ik}, \tilde{x}_{jk} + vx_{jk}, \tilde{z}_{jk} + vz_{jk}) = 0 \tag{21}$$

Assuming that in a close neighbourhood of observed magnitudes $\tilde{x}_{ik}$, $\tilde{z}_{ik}$, $\tilde{x}_{jk}$, $\tilde{z}_{jk}$ function F changes almost linearly, and thus we get

$$F_{ijk}(\varrho_{ij}, \tilde{x}_{ik}, \tilde{z}_{ik}, \tilde{x}_{jk}, \tilde{z}_{jk}) = \Delta_{ijk} \tag{22}$$

where:

$$\Delta_{ijk} = -\frac{\partial F}{\partial x_{ik}} vx_{ik} - \frac{\partial F}{\partial z_{ik}} vz_{ik} - \frac{\partial F}{\partial x_{jk}} vx_{jk} - \frac{\partial F}{\partial z_{jk}} vz_{jk} \tag{23}$$

the partial derivatives being functions of parameters $\varrho_{ij}$, $\tilde{x}_{ik}$,

$\tilde{z}_{ik}$, $\tilde{x}_{jk}$, $\tilde{z}_{jk}$.

Upon set (22) we impose the condition

$$\Delta^T \cdot P \cdot \Delta = \min \tag{24}$$

Because of the form of formulas (14) and (15) generalized in (21) both observation equations and the set of normal equations obtained form (24) have a non-linear form. In order to solve the set we will use one of the algorithms of the least square method Gauss-Newton's algorithm.

$$\eta^{(s+1)} = \eta^{(s)} - \left(A^{(s)T} \cdot P^{(s)} A^{(s)}\right)^{-1} \cdot A^{(s)T} \cdot P^{(s)} \Delta^{(s)} \tag{25}$$

where: s - cycle indicators s = 0, 1, 2, . . . ,

$\eta$ - vector of unknown orientation elements of the photographs,

$A_{(w,U)}$ - matrix of vector partial derivatives, $A(\eta) \stackrel{df}{=} \frac{\partial F(\eta)}{\partial \eta}$

$P_{(w,w)}$ - weight matrix determined with (23) taken into consideration, with given observation vector covariant matrix of image coordinates its form will be presented below,

$\Delta_{(w,1)}$ - rest vector,

$T$ - symbol of transposition.

We shall now define the form of the weight matrix (Kadaj R., Rychlewski G. 1982). For the whole block, between the vector of corrections V and the vector of rests $\Delta$ the following dependence obtains

$$-\Delta = C \cdot V \tag{26}$$

where: C - matrix of partial derivatives of function $F_{ijk}$ versus image coordinates.

Let us group the components of vector V for the particular network points and arrange the groups according to the succession of the points. By analogy to (26) there is a dependence between true errors $\underline{\varepsilon}_\Delta$, $\underline{\varepsilon}$ replacing vectors $\underline{\Delta}$, $\underline{V}$. Hence we can present the dependence between the covariant matrixes.

$$E\{\underline{\varepsilon}_\Delta \cdot \underline{\varepsilon}_\Delta^T\} = C \cdot E\{\varepsilon \cdot \varepsilon^T\} \cdot C^T$$

Setting this to $E\{\varepsilon\varepsilon^T\} = \mu^2 \cdot I$, where I - unit matrix, $\mu$ - mean error of a typical observation we get:

$$P = \left(E\{\varepsilon_\Delta \ \varepsilon_\Delta^T\}\right)^{-1} \cdot \text{const} = \left(C \cdot C^T\right)^{-1} \cdot \frac{1}{\mu^2} \cdot \text{const}$$

or, assuming const $= \mu^2$:

$$P = \left(C \cdot C^T\right)^{-1} \tag{27}$$

As we can notice, in the investigated system, matrix $(C \cdot C^T)$ has an almost diagonal form. To each network point corresponds a non-zero block measuring $(w_k \times w_k)$, on the main diagonal, $w_k$ - number of conditions with k-th point. Thus, it is an easily invertible matrix and it does not require a large area of the computer's memory. The particular diagonal blocks may be riewed separately, ie, with a respective group of observation equations referring to a given point. Because of a non-linear character of observation equations we use an iterative procedure to solve them. In an iterative process $(s = 0,1,2,\dots)$ we form linearized observation equations for each ordered group of points $(k = 1, 2, \dots)$, m and then we impose condition (24) on the whole system. We solve the set of normal equations by determining the increase in and successive approximations of the vector of unknowns (25). If the norm of the vector fulfils the condition $\|\delta\eta\| > \varepsilon$ (where $\varepsilon$ - numerical accuracy) then we realize the next iterative cycle $s = s + 1$, otherwise the iterative process comes to an end. The subsequent final computations are performed in the following sequence:

- determination of rest vector (22),
- computation of corrections to measured image coordinates from the system

$$\begin{cases} \Delta^{(s)} = C^{(s)} \cdot V \\ V^T \cdot V = \min \end{cases}$$

  or from the dependence:

$$V = C^{(s)T} \cdot \left( C^{(s)} \cdot C^{(s)T} \right)^{-1} \cdot \Delta^{(s)} \qquad (28)$$

  It is easy to notice that matrix $C^T \cdot (C \cdot C^T)^{-1}$ contains independent non-zero blocks measuring $(2r_k \times w_k)$, $(k = 1,2,\dots,m)$ which multiplied by a respective subvector of vector $\Delta^{(s)}$ give a subvector of the correction vector corresponding to k-th point of the network. Thus, computation of corrections is not a problem of solving a large system but in fact it is realized independently for each subset of image coordinates corresponding to k-th point of the network.
- geodetic coordinates of network points are computed from (8) after adjusting image coordinates,
- mean errors of coordinates are determined as functions of unknowns of orientation elements of photographs.

In the last place, in order to complete the algorithmical-numerical procedure a simplified block scheme of the presented method has been given. (Fig.3)

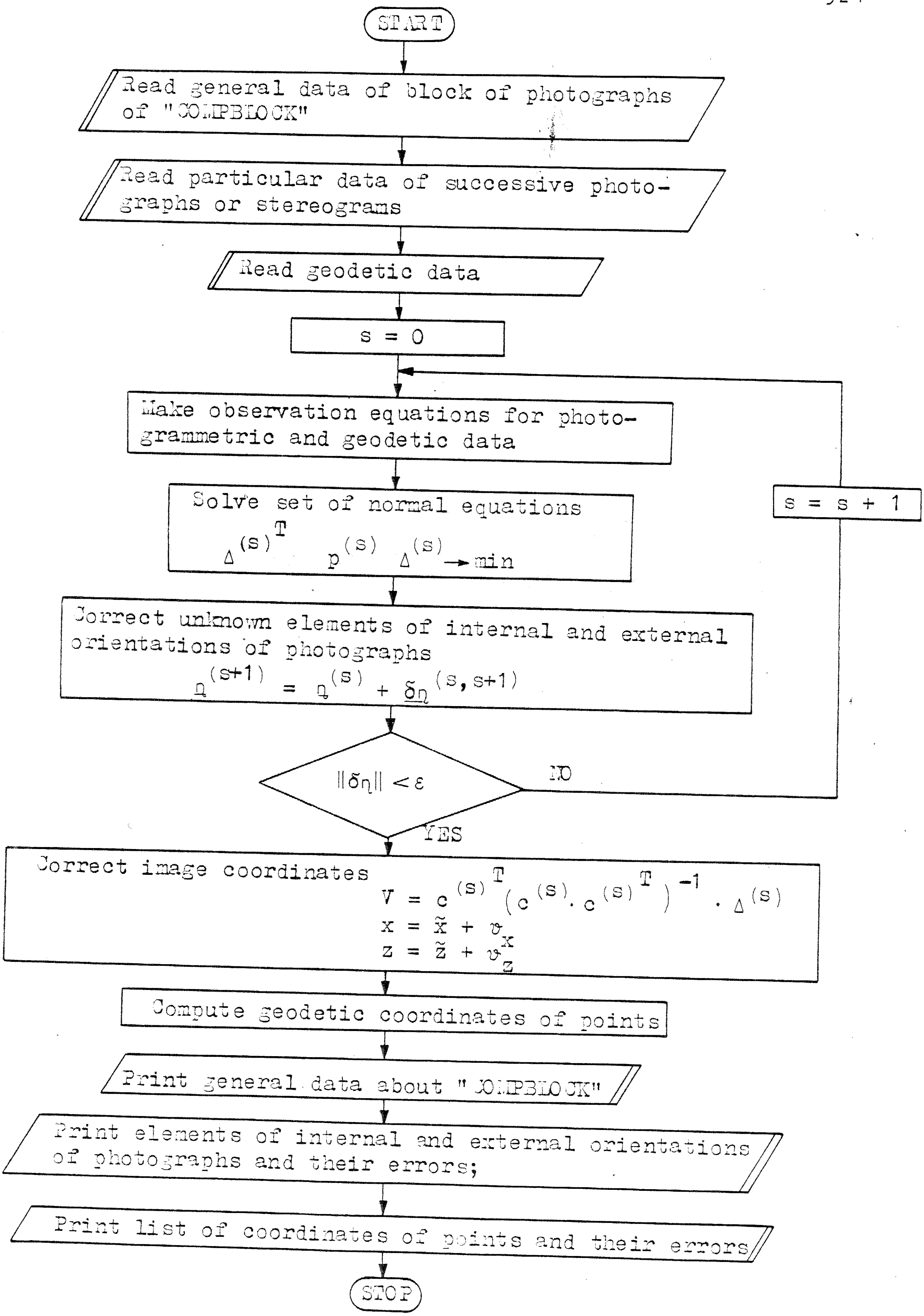
START
Read general data of block of photographs of "COMPBLOCK"
Read particular data of successive photographs or stereograms
Read geodetic data
s = 0
Make observation equations for photogrammetric and geodetic data
Solve set of normal equations
$\Delta^{(s)T} p^{(s)} \Delta^{(s)} \rightarrow min$
Correct unknown elements of internal and external orientations of photographs
$\eta^{(s+1)} = \eta^{(s)} + \delta\eta^{(s,s+1)}$
$\|\delta\eta\| < \varepsilon$
NO
YES
s = s + 1
Correct image coordinates
$V = c^{(s)T} (c^{(s)} \cdot c^{(s)T})^{-1} \cdot \Delta^{(s)}$
$x = \tilde{x} + \vartheta_x$
$z = \tilde{z} + \vartheta_z$
Compute geodetic coordinates of points
Print general data about "COMPBLOCK"
Print elements of internal and external orientations of photographs and their errors;
Print list of coordinates of points and their errors
STOP

Fig. 3

Literature

Kadaj, R., Rychlewski, G. (1982) : "An Algorithm of An Aerotriangular Network "PHOTONET"", Int. Symp. Mathematical Models, Accuracy Aspects and Quality Control, June 7 - 11, 1982, Helsinki, s.276 - 283.

Moniwa, H. (1976) : "A flexible method of self-calibration for aerial and close-range photogrammetric systems". XIII Congress ISPRS Commission V, Helsinki 1976.

Rychlewski, G. (1979) : "Określenie zmian kształtu lotni w locie metodami fotogrametrii analitycznej". "Defining changes of shape of a paraglider in flight by methods of analytical photogrammetry" . Przegląd Geodezyjny Pol. , nr.4, 1979. s.21-24.

Rychlewski, G (1981) : "Parametryczne rozwiązanie wcięcia przestrzennego w przód i jego zastosowanie w fotogrametrii naziemnej". "A parametric solution of spatial intersection and its application in terrestrial photogrammetry". XI Narada Szkoleniowa nt. "Informatyka w Geodezji i Kartografii", SGP. NOT Katowice 1981, s.75 - 82.

Rychlewski, G. (1982) : "Technology of Numerical Elaboration of Terrestrial Photographs by Means of a Block Method "TERROBLOCK"" Int. Symp. Mathematical Models, Accuracy and Quality Control , June 7 - 11, 1982, Helsinki, s.406 - 424.

Wong, K., W. (1976) :"Mathematical formulation and digital analysis in close-range photogrammetry". XIII Congress ISPRS Helsinki 1976.