MATHEMATICAL MODEL OF THE ORIENTATION PROGRAM IN THE WILD AVIOLYT PHOTOGRAMMETRIC SYSTEM

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Abstract

The concept of a mathematical rigorous solution for the orientation of a photogrammetric model in an analytical plotter is developed. The realization of this concept in the Aviolyt system is presented. The orientation program is based on a least squares adjustment of the colinearity equation with added conditions in form of object properties or geodetic observations between control points and/or orientation elements.

Zusammenfassung

Eine mathematisch strenge Lösung der Orientierung eines photogrammetrischen Modells ist im Konzept hergeleitet. Weiter ist die Realisierung dieses Konzepts in dem Aviolyt System beschrieben. Die Orientierungsberechnung basiert auf der Kollinearitätsbedingung. In einer Ausgleichung nach der Methode der kleinsten Quadrate werden Objekteigenschaften und geodätische Messungen zwischen Passpunkten und/oder Orientierungsparameter als zusätzliche Bedingungen mit berücksichtigt.

Sommaire

Cet exposé développe une solution mathématique exacte pour l'orientation d'un modèle photogrammétrique dans un plotter analytique. Il présente également la réalisation de ce concept sur le système Aviolyt. Le calcul des éléments d'orientation se base sur les conditions de la collinéarité. A l'aide d'une compensation selon la méthode des moindres carrés, on tient compte comme conditions supplémentaires des propriétés de certains objets et de mesures géodésiques entre des points d'ajustage et/ou des éléments d'orientation.

1. Introduction

The paper reflects the result of a joint project of WILD Heerbrugg and ETH Zürich. The goal of this project was to develop a program for the orientation of a photogrammetric model in an analytical plotter. The program features the following steps:

- Rigorous formulation of the central projection as mathematical model.
- The computed orientation parameters are in an object related system independent from type and attitude of photographs (aerial, terrestrial etc.).

- Inclusion of additional geodetic observation and object properties into the orientation computation.
- Integration of the program into the existing system software and datastructure of the Aviolyt AC1/BC1 instruments. Computation time has to be short enough so that the process of model orientation is not interrupted by the computation (On-line solution).
- Easy to handle
- It has to be possible to introduce the program by stages and to add other features later:
  - other mathematical formulation of the model
  - additional statistical testing
  - single image orientation
  - miniblock adjustment

The program which was developed based on these principles is the ORI program. It is part of the WILD Aviolyt system software. In the following chapters the mathematical model of this program and its realization will be presented.

2. Mathematical model

The function relating the object coordinates to the image coordinates is the well-known colinearity equation here used in the form

$$x = x_0 - c \cdot \frac{(X-X_0)\cdot r_{11} + (Y-Y_0)\cdot r_{12} + (Z-Z_0)\cdot r_{13}}{(X-X_0)\cdot r_{31} + (Y-Y_0)\cdot r_{32} + (Z-Z_0)\cdot r_{33}}$$

$$y = y_0 - c \cdot \frac{(X-X_0)\cdot r_{21} + (Y-Y_0)\cdot r_{22} + (Z-Z_0)\cdot r_{23}}{(X-X_0)\cdot r_{31} + (Y-Y_0)\cdot r_{32} + (Z-Z_0)\cdot r_{33}} \qquad (1)$$

where

x, y	image coordinates
x_0, y_0, c	coordinates of the perspective center with respect to the camera fixed system (x_0, y_0 principal point; c principal distance)
X, Y, Z	object coordinates
X_0, Y_0, Z_0	coordinates of the perspective center with respect to the object coordinate system
$r_{11} \dots r_{33}$	elements of a 3-dimensional orthogonal matrix **R** (rotation matrix)

The rotation matrix is a function of 3 independent rotation angles. The system of rotation angles depends on the geometry of exposure. If the photographs are oriented nearly perpendicular to the XY-plane as it is usual in aerial photogrammetry, the rotations are defined as:

1. rotation ω (omega) about X axis
2. rotation ϕ (phi) about rotated Y axis
3. rotation κ (kappa) about rotated Z axis

This system of rotations breaks down if phi = 100^g. In this case omega and kappa are correlated and can not be determined separately. For this reason another system of rotations is used for all photographs which deviate significantly from the normal case:

1. rotation A (azimuth) about Z axis
2. rotation ν (nadir dist.) about rotated X axis
3. rotation κ (kappa) about rotated Z axis

The second system of orientation angles is also easier to handle for the user working in nontopographic photogrammetry because angles A and ν are easier to visualize than ω and ϕ. In addition the A-ν-κ system corresponds to the system of rotations in a terrestrial camera as e.g. the P31.

In the exterior orientation the six parameters X_0, Y_0, Z_0, ω, ϕ, κ resp. A, ν, κ have to be determined. The parameters of interior orientation x_0, y_0, c which are calculated separately by camera calibration are considered fixed in the current program version. Their determination could be included in the orientation computation with appropriate weights.

There are two groups of information contributing to the determination of the orientation parameters. The first group called the photogrammetric observations are the measured image coordinates of the orientation points. The second group will be called the geodetic observations and includes all types of information about the object e.g. control point coordinates, geodetic observations between object points or perspective centers and other object properties.

The mathematical model for the photogrammetric observations is equation (1) as discussed above. It can be written in vectors as

$$\mathbf{F}(\mathbf{y}) = \mathbf{O} \qquad (2)$$

The vector $\mathbf{y}$ can be split into

$\boldsymbol{\ell}_1$, the vector of photogrammetric observations

$\mathbf{U}$, the vector of unknown parameters which can be further split into

- $\mathbf{O}$ the orientation elements and
- $\mathbf{X}$ the object coordinates of all involved points.

The geodetic information has to be included in the adjustment to compensate for the rank deficiency of a system with photogrammetric observations (image coordinates). This can be done by constraining some of the orientation parameters to their approximate values as we are used from a relative orientation or by weighting the control point coordinates like it is done in a photogrammetric bundle block adjustment. In both cases we add information to the photogrammetric observations in form of direct observations of the unknown parameters. This is the most simple type of a geodetic observation and has the form

$$X - X_B = 0 \qquad (3)$$

X is the unknown parameter (e.g. object point coordinate)

X_B the corresponding observation (e.g. given control coordinates)

Of course more geodetic information than the minimun necessary can be included to improve the orientation. Often this information will not be available in form of coordinates directly, but as functions of these coordinates or the orientation parameters.

Two examples of such functions or conditions between the object points or perspective centers will be discussed in the following paragraphs. Subsequently they will also be called additional conditions.

The first type of additional conditions are the geodetic observations between points of the object or the perspective centers. These observations can be distances, angles, azimuths, height differences etc. In case of a known or measured distance between 2 orientation points the additional condition has the form:

$$s = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2 + (Z_2 - Z_1)^2} \qquad (4)$$

s is the observed distance and

$X_1Y_1Z_1$, $X_2Y_2Z_2$ are the coordinates of the two points involved.

Another type of additional condition can be derived from object properties. For example a group of points can have equal Z coordinates (lake surface) or more generally, they all can be in an arbitrary plane. This condition has the form

$$A \cdot X_i + B \cdot Y_i + C \cdot Z_i + D = 0 \qquad (5)$$

A, B, C, D unknown parameters defining an arbitrary plane

X_i, Y_i, Z_i coordinates of a point in the plane

It is obvious that in case of an arbitrary plane at least 4 points have to be part of the condition to contribute to the determination of the orientation elements and not only be sufficient to solve for the additional unknown parameters introduced into the system by this condition.

The group of geodetic observations for which equations (3) to (5) show some examples, can be written as a second function

$$\mathbf{G}(\mathbf{z}) = \mathbf{0} \qquad (6)$$

The vector $\mathbf{z}$ can be split into the vectors

$\boldsymbol{\ell}_2$, the geodetic observations,

$\mathbf{U}$, the unknown parameters (a subset of $\mathbf{U}$ used in equ. (2) supplemented with zeros)

The orientation parameters are computed from these two groups of observations in a least squares adjustment. The least squares principle minimizes the weighted sum of squares of all residuals:

$$\mathbf{v}_1^T\mathbf{P}_1\mathbf{v}_1 + \mathbf{v}_2^T\mathbf{P}_2\mathbf{v}_2 = \min \qquad (7)$$

$\mathbf{v}_1$ is the vector of residuals of the photogrammetric observations and $\mathbf{v}_2$ of the geodetic observations. The adjustment will yield the best estimate for the orientation elements.

If discrepancies are existing between the two functions $\mathbf{F}(\mathbf{y}) = \mathbf{O}$ and $\mathbf{G}(\mathbf{z}) = \mathbf{O}$ - this can happen if systematically wrong control points have been introduced - the geodetic observations can disturb the photogrammetric model. Such distortions will show up in form of parallaxes and height differences during compilation. For this reason a second adjustment model is provided in the ORI program. It is based on the positioning of a free net and minimizes the sum of squares of the residuals in two independent steps:

$$\dot{\mathbf{v}}_1^T\mathbf{P}_1\dot{\mathbf{v}}_1 = \min$$

and <u>independently</u>

$$\mathbf{\Delta}_X^T\mathbf{P}_X\mathbf{\Delta}_X = \min. \qquad (8)$$

$\dot{\mathbf{v}}_1$ is the vector of photogrammetric residuals from a minimum constraint solution

$\mathbf{\Delta}_X$ the vector of discrepancies between the given coordinates and the adjusted model coordinates as described below.

This model will be called "Two Step Orientation" in contrast to the "One Step Orientation" based on equ. (7).

In the 2-step orientation the geodetic observations are restricted to control points only. The image coordinates are adjusted in a minimum constraint solution in which only 7 parameters are constrained to their given control values in order to eliminate the rank deficiency of the purely photogrammetric system. The result of this adjustment is the vector $\mathbf{X}'$ which is only influenced by the noise of the photogrammetric observations. In a second step the model is transformed onto the control points minimizing $\mathbf{\Delta}_X^T\mathbf{P}_X\mathbf{\Delta}_X$, the sum of squares of the weighted discrepancies between the adjusted model coordinates and the control point coordinates:

$$\mathbf{\Delta}_X = \mathbf{X}_C - \mathbf{X}$$

$$\mathbf{X} = s\mathbf{D}\mathbf{X}' + \Delta\mathbf{X} \qquad (9)$$

s scaling factor

$\mathbf{D}$ orthogonal matrix based on 3 independent rotations

$\Delta\mathbf{X}$ translation vector

$\mathbf{X}_C$ vector of given control point coordinates

Equation (9) corresponds to a 7-parameter coordinate transformation which does not distort the photogrammetric model. The orientation elements are correspondingly determined from the transformation parameters.

Since the least squares adjustment is a linear estimator all nonlinear colinearity conditions and most of the geodetic observations have to be linearized. To keep the number of iterations small, good approximations of all unknown parameters should be available. Two methods for computation of approximate values are part of the orientation program:

- 2-dimensional similarity transformation

  This model is used for photographs nearly perpendicular to the XY-plane. ω and ϕ are set to zero, Z_0 is taken from the flight plan and X_0, Y_0 and κ are determined by a 2-dimensional similarity transformation of the photo coordinates of the control points to the corresponding object coordinates. The approximations allow deviations from the normal case up to 20^g - 30^g.

- direct solution of the space resection

  The solution of the space resection problem which is used in the program is based on an approach published by [Müller, 1925]. In a first step the coordinates of the perspective center are determined from the pyramid defined by 3 control points (base) and the perspective center (vertex). This problem leads to a 4th order equation. To avoid ambiguous solutions a second pyramid with control points 1 and 2 and a 4th point as base is used. The simultaneous solution of these two equations yields the perspective center directly. Once the perspective center is known, the elements r_{11} .. r_{33} of the rotation matrix are computed from equation (1). The orientation angles can be determined from these 9 elements after orthogonalization.

  The method of space resection allows the computation of good approximations of the orientation parameters for an arbitrarily oriented photograph without needing any information about the geometry of the exposing situation.

If known, the approximate orientation parameters can also be entered by the user.

Now a short summary of the least squares adjustment can be given. Equations (2) and (6) can be written in linearized form

$$\mathbf{A}_1\mathbf{v}_1 + \mathbf{B}\boldsymbol{\Delta} = \mathbf{w}_1$$
$$\mathbf{A}_2\mathbf{v}_2 + \mathbf{C}\boldsymbol{\Delta} = \mathbf{w}_2$$

$$\mathbf{A}_1 = \frac{\partial \mathbf{F}}{\partial \boldsymbol{\ell}_1} \qquad \mathbf{B} = \frac{\partial \mathbf{F}}{\partial \mathbf{U}} \qquad \mathbf{w}_1 = -\mathbf{F}(\boldsymbol{\ell}_1^0\,\mathbf{U}^0)$$
$$\mathbf{A}_2 = \frac{\partial \mathbf{G}}{\partial \boldsymbol{\ell}_2} \qquad \mathbf{C} = \frac{\partial \mathbf{G}}{\partial \mathbf{U}} \qquad \mathbf{w}_2 = -\mathbf{G}(\boldsymbol{\ell}_2^0\,\mathbf{U}^0) \qquad (10)$$

A detailed description of the algorithm of forming and reducing the normal equation system can be found in [Schmid, 1977] and [Schmid, 1980].

The weight matrices $\mathbf{P}_1$ and $\mathbf{P}_2$ are considered to be diagonal matrices with the weights computed from the standard deviations using equ. (11)

$$p_i = \frac{\sigma_o^2}{\sigma_i^2} \qquad (11)$$

σ_o^2 variance of unit weight

σ_i standard deviation of an observation (image coordinates or geodetic observations)

The standard deviations of control points, known orientation parameters and geodetic observations can be defined by the user. All image coordinates are assigned with equal weights.

After the adjustment of the orientation parameters the following statistical information is displayed to give the user the possibility to judge the quality of the orientation:

- estimated standard reference deviation and redundancy
- residuals of image coordinates, of control point coordinates and of additional conditions.
- mean errors of the orientation elements

More statistical features like data snooping or aposteriori variance component estimation could readily be included in the package. Most submatrices which are necessary for this purpose are already computed and stored on temporary files.

3. Realization

In the Aviolyt system software the process of outer orientation is devided into three sections:

1. Input and editing of control point coordinates and geodetic observations in the program INP.
2. Measurement of image coordinates and definition of object properties in the program PMO.
3. Computation of the orientation parameters in the program ORI.

This paper only discusses section 3 of the total orientation package. After the mathematical model has been developed in the previous chapter, a few points for the application of this model will be presented below.

- Flexibility: The ORI program can process observations of up to 30 points. Every point can be control point and part of one or several geodetic observations. Groups of up to 5 points can be correlated by such additional conditions. There are no restrictions on the size and range of the orientation elements. If enough control points are available no approximate values of the orientation parameters have to be known.
- Modularity: The current version of the program consists of about 90 subroutines, each performing just one well defined

function. The modules can easily be changed and new modules can be added to expand the potential of the program.

- Human interface: The program is integrated into the menu driven Aviolyt software. In the main menue the user selects the type of computation he wants ORI to perform or he switches to the INP program to edit control points. In a second menu he can set program control parameters.
- Off line mode: Although integrated into the PMO program, ORI can also be used off-line to compute a model orientation from image coordinates which have been measured earlier e.g. in a photogrammetric block triangulation.

4. Concluding remarks

In the previous chapters the mathematics and concept of the ORI program have been presented. Emphasis was put to show the flexibility of the chosen model. This flexibility has been proven by many different topographical and nontopographical applications. From the mathematical point of view, almost any kind of additional condition can be included into the adjustment. Chapter 2 shows what can be done in an on-line solution today. The question which of these additional conditions are useful and economical remains open. In this aspect the model orientation has to be treated differently from a photogrammetric block adjustment. Practical work and experiments have to show what types of additional geodetic observations contribute best to an economical model orientation.

A problem which arises with the growing flexibility and potential of such a software package is its complexity; specially the difficulty for the user to handle the program. Of course the manufacturers of software are paying attention to a userfriendly design of the human interface but more has to be done in education of the photogrammetrists. Specially training centers have to put more emphasis towards modern analytical photogrammetry in the sense of the model presented in this paper.

References

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