MATHEMATICAL ASPECTS OF RADAR IMAGE RECTIFICATION

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1. Introduction

This paper deals with the geometric approach to restitute sidelooking airborne radar (S.L.A.R.) imagery in a strict manner.
The "projective" equations for SLAR pictures have been derived according to KONECNY (1971), who supplemented the relations for aerial photography, which date back to GAST (1930). The main reason for this so called collinearity approach is the correct calculation of the point hights and the proper formulation of the dynamic exterior orientation parameters. The time varying parameters can be expressed as polynomial functions, Fourier-functions, by the Gauss-Markov process etc. Furthermore systematic errors can be calculated by applying additional parameters within the approach. Single image evaluation is only a special case of a bundle block adjustment approach, which is used for the connection of a ground control point field imaged in different overlapping SLAR pictures. A global task of the image restitution is the image mosaiking, for which the equations are also valid.

2. Geometric radar approach

In a general approach for single SLAR imagery according to KONECNY (1975) the image x' coordinate component follows projective law

$$0 \overset{!}{=} - k_x \frac{a_{11j}\,(x_i - x'_{oj}) + a_{12j}\,(y_i - y'_{oj}) + a_{13j}\,(z_i - z'_{oj})}{a_{31j}\,(x_i - x'_{oj}) + a_{32j}\,(y_i - y'_{oj}) + a_{33j}\,(z_i - z'_{oj})}$$

while the y' component depends upon the distance travelled by the pulse, for slant range follows $y'_{SL} = m_o \cdot r$

$$y'_{SL} = m_o \sqrt{\begin{array}{l}(a_{11}\,(x_i - x'_o) + a_{12}\,(y_i - y'_o) + a_{13}\,(z_i - z'_o))^2 \\ + (a_{21}\,(x_i - x'_o) + a_{22}\,(y_i - y'_o) + a_{23}\,(z_i - z'_o))^2 \\ + (a_{31}\,(x_i - x'_o) + a_{32}\,(y_i - y'_o) + a_{33}\,(z_i - z'_o))^2\end{array}}$$

or, in a short form

$$y'_{SL} = m_o \sqrt{a_x^2 + a_y^2 + a_z^2}$$

This expression is almost equivalent to

$$y'_{SL} = mo \left[\sqrt{(a_{11} (x_i - x'_o))^2 + (a_{22} (y_i - y'_o))^2 + (a_{33} (z_i - z'_o))^2} \right.$$

$$\left. + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}^T \begin{pmatrix} 0 & a_{12} & a_{13} \\ a_{21} & 0 & a_{23} \\ a_{31} & a_{32} & 0 \end{pmatrix} \begin{pmatrix} x_i - x'_o \\ y_i - y'_o \\ z_i - z'_o \end{pmatrix} \right]$$

For ground range follows

$$r_g = \sqrt{r^2 - h^2}$$

thus

$$y'_{GR} = m_o \sqrt{a_x{}^2 + a_y{}^2 + a_z{}^2 - h^2}$$

$$y'_{GR} = \sqrt{y'^2_{SL} - h^2 \cdot m_o^2}$$

As for radar geometrie the roll angle influence is zero, the valid rotation matrix is

$$A = A_\kappa A_\phi = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} \cos\kappa & \sin\kappa & 0 \\ \sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{pmatrix}$$

$$= \begin{pmatrix} \cos\kappa \quad \cos\phi & \sin\kappa - \cos\kappa \sin\phi \\ \sin\kappa \quad \cos\phi & \cos\kappa - \sin\kappa \sin\phi \\ \sin\phi \quad 0 & \cos\phi \end{pmatrix}$$

The index j stands for time depending equal to x' depending parameters, which is valid for all used κ, ϕ, x'_o, y'_o and z'_o values, but neglected in some cases of formula synoptical reasons.
the time functions are in principal

$$\phi_j = \phi = \sum_{i=o}^{i=n} \phi_i \cdot x^i$$

$$\kappa j = \kappa = \sum_{i=o}^{i=n} \kappa_i \cdot x^i$$

$$x'_{oj} = x'_o = x' \cdot \frac{h}{k_x}$$

$$y'_{oj} = y'_o = 0$$

$$z'_{oj} = z'_o = \sum_{i=o}^{i=n} z_{oi} \cdot x^i$$

m_o and k_x are the scale factors for y' and x' respectively.
For practical handling of these formulas according to BAKER and MIKHAIL (1975) instead of applying condition equations for x' values and observation equations for y' values, terrain-hight including polynomials of collinearity equivalent containment can derived from linearized collinearity equations.

$$dx_i' = k_x d\phi - y' \, d\kappa + \frac{k_x}{h} dx_o'$$

$$dy_i' = \frac{m_o^2}{2y'} (yd\kappa + hd\phi - 2 \, y \, dy_o' + 2 \, h \, dz_o')$$

After an elimination of high correlated terms for linear orientation elements variation results

$$dx' = \frac{a_o}{h - z_i} + \frac{x'}{h - z_i} a_1 + y' a_2 + x' y' a_3$$

$$dy' = \frac{h - z_i}{y'} b_o + (\frac{h - z_i}{y'}) x' b_1 + \frac{y}{y'} b_2 + \frac{yx'}{y'} b_3$$

where

$$a_o = k_x \cdot dx_o' \qquad b_o = \frac{m_o^2}{2} d\phi_o$$

$$a_1 = k_x \cdot dx_1' \qquad b_1 = \frac{m_o^2}{2} d\phi_1$$

$$a_2 = \kappa_o \qquad b_2 = \frac{m_o^2}{2} d\kappa_o$$

$$a_3 = \kappa_1 \qquad b_3 = \frac{m_o^2}{2} d\kappa_1$$

If second order variations in orientation elements are assumed, the resulting polynomial functions are

$$dx' = \frac{a_o}{h - z_i} + \frac{x'}{h - z_i} a_1 + y'a_2 + x'y'a_3 + y'x'^2 a_4 + x'^2 \frac{a_5}{h - z_i}$$

$$dy' = \frac{h - z_i}{y'} b_o + \frac{h - z_i}{y'} x'b_1 + \frac{y}{y'} b_2 + \frac{yx'}{y'} b_3 + \frac{h - z_i}{y'} x'^2 b_4 + \frac{y \cdot x'^2}{y'} b_5$$

with

$$a_4 = - \kappa_2 \qquad b_4 = \frac{m_o^2}{2} d\phi_2$$

$$a_5 = k_x \cdot dx_2 \qquad b_5 = \frac{m_o^2}{2} d\kappa_2$$

This equations are direct solvable for ground control point coordinate values. For anchor point application an iterative process is necessary.

In figure 1 a digitally rectified part of a SAR 580 image according to LOHMANN is shown.

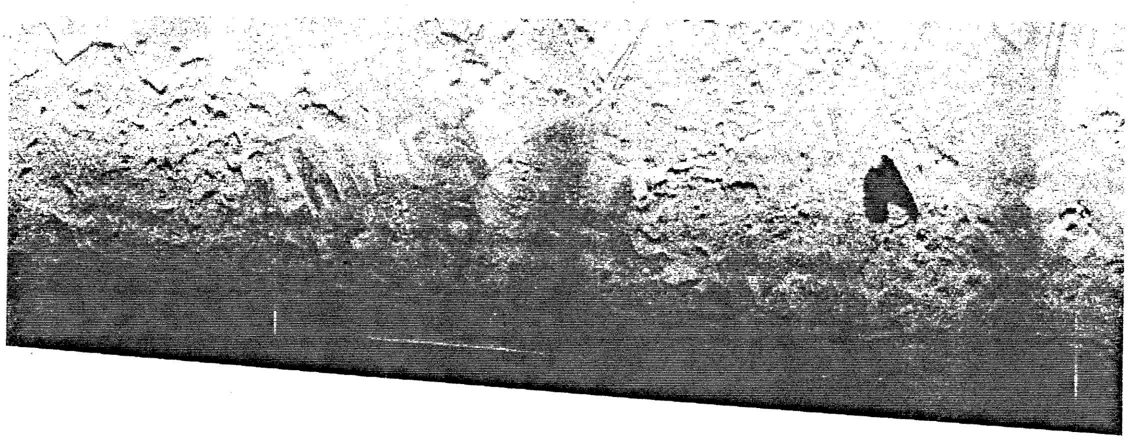

Fig. 1: Digitally rectified SAR image part, scale 1 : 50 000

3. Conclusion

The objective advantage of Radar, the ability of cloud penetration cannot compensate the disadvantage, which is limitation in detail resolution. This is the reason, radar imagery is not even suitable for topographic maps of the scale 1 : 250 000. For this is a principal physical limitation of the radar technology, there will be no increasing progress in future.

Therefore the further development of the geometric radar equations is still a matter of research but also of basic photogrammetric interest.

A suggestion for a futural platform concept are simultaneous flights of a conventional metric camera, a C.C.D.-triplet, a SAR radar sensor and a housekeeping data registration ability, where the conventional images with respect to their high resolution and geometric stability will be used for geometric topographic purposes, including a reference function, while scanner- and radar-data are used more in the sense of additional data.

4. References

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