SOLVING THE MAIN PHOTOGRAMMETRIC TASK BY A STEREOPAIR OF PANORAMIC X-RAY PICTURES

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ABSTRACT: The method of panoramic X-raying, when a flexible filmholder is pressed to the object of investigation reproducing its configuration, is used in the medical roentgenology for investigation of thorax organs, skull and jaws. This method helps to decrease projection distortions, to transfer images of investigation details more clear and reveal the smallest defects. At the same time the panoramic X-raying complicated considerably the solving of the task as regards determination of the space coordinates of the points of the inner structure of the investigation object, since the orientation elements of the panoramic picture fail usually to be determined.

This report presents an original method of solving the main photogrammetric task by means of panoramic X-ray pictures; the theory, techniques and methods of survey are considered and the results of measurement error estimation are given.

In medical roentgenology the panoramic X-raying method for investigation of the thorax cavity organs, skull and jaws is used when a flexible filmholder is pressed to the investigation object reproducing its configuration, with a central projection of the object on the cylinder surface.

On the panoramic (cylindrical) picture projection distortions are reduced, the images of investigation parts become clearer and pathological changes of organism become more visible.

It is insufficient to know only the value of the principal distance and the location of the principal point in the system of the picture coordinates to reproduce the connection of projection rays according to the panoramic picture. It is also necessary to know the value of the curvature radius of the picture surface. To determine the orientation elements of the panoramic picture is much more difficult than those of the flat one because the flexible filmholder with the photosensitive material is pressed right to the surface of the investigation object. In some cases when the outer surface survey object has a form of a rotation cylinder and the radius of the picture curvature is constant, this task can be solved by the method of the reverse photogrammetric intersection [1]. However in most cases the object surface form differs from the perfect cylinder which results in the picture curvature radius change on the frame field. As a rule it is impossible to determine the orientation elements of such pictures. As in this case the formulas of the coordinate connection between the panoramic picture points and the object are unknown, to solve the photogrammetric task by these pictures using the traditional methods proves to be impossible.

At the Moscow Tuberculosis Research Institute an original method of solving the photogrammetric task has been developed by
means of panoramic X-ray pictures with the help of an additional stereopair of flat pictures with the known orientation elements [2].

Before the surveying the investigation object surface, for example, a scull surface, is marked by X-ray contrasting base marks. For this purpose it is convenient to use a thin plexiglas surveyor's plane, on the surface of which a few 1 mm diameter lead balls are fixed. It is advisable to locate the base marks at the vertexes of the squares.

The surveyor's plane is pressed to the scull surface and is fixed in this position. Then the patient’s head is placed on a special holder supplied with a pneumatic clamp for a flexible film-holder [3].

The holder is located within the stereoroentgenographic overlap of the X-ray apparatus.

It is more convenient to use a special two-tube apparatus, for example, a stereoroentgenographic device provided with a mechanism to adjust the X-ray tubes [1]. Usually adjustment helps to obtain the coincidence of the main point of the picture with beginning of the coordinates system (x₀ = y₀ = o) and the identity of the focal distance of the left and right pictures (f₁ = f₂).

In this case the conditions of the most simple normal case of stereoroentgenogrammetric survey are provided. The apparatus is completed with a X-ray film-holder to make flat pictures. The film-holder is located under the deck of the X-ray table. The flexible film-holder intended to get panoramic pictures, is pressed to the investigation object from the side of the surveyor's plane by means of a pneumatic clamp.

Fig. 1 shows the geometry of this surveying type.

\[ \text{Fig. 1.} \]
The stereoroentgenography is carried out by means of successive work of the tubes. First the left X-ray tube is switched on, its focus is located at Point \( P_1 \). After the reloading of the flat and flexible film-holders Survey Object 1 is photographed for the second time by means of the right X-ray tube, whose real focus is located at Point \( P_2 \).

As a result of stereoroentgenography a stereopair of panoramic pictures \( P_1' \), \( P_2' \) and a stereopair of flat pictures \( P_1 \) and \( P_2 \) are obtained.

The panoramic pictures are used to interpretate the survey section and to determine the space coordinates of the required point, for example Point \( N \). The stereopair of the flat pictures is intended for determination of the space coordinates of base marks \( M_{11}, M_{12}, \ldots, M_{ij} \) in the photogrammetric coordinate system \( P_1XYL \), the centre of which coincides with the real focus of the left X-ray tube.

Measuring the stereopair of the flat pictures is carried out by means of stereocomparator, where coordinates \( x_m, y_m \) and the difference in horizontal parallaxes \( \Delta p \) of the base mark images are determined. The obtained values are introduced in the formulas of the normal survey case, according to which the space coordinates of the base marks are determined.

\[
\begin{bmatrix}
X_M \\
Y_M \\
Z_M
\end{bmatrix} = \frac{B}{B + \Delta p} \begin{bmatrix}
x_m \\
y_m \\
-f
\end{bmatrix},
\]

where: \( B \) - stereoroentgenography basis
\( f \) - the principal distance of the flat picture.

Let us consider the panoramic picture section, located within the limits of the four base marks, to be a plane (an elementary section). It can be obtained by means of proper density of the base marks.

![Diagram](image-url)

Fig. 2
On each panoramic picture coordinates \( x'_n, y'_n \) of the required point \( N \) of the image are measured. Measuring is carried out by the stereocomparator in the rectangular coordinate system of the respective elementary section \( o'_1, x'_1, y'_1 \) of the left and \( o'_2, x'_2, y'_2 \) of the right panoramic pictures. The beginning of \( o'_1 \) and \( o'_2 \) of these systems coincides with the respective image of Base Mark \( M_{ij} \), and the axes are directed through the base mark images, located in the mutually perpendicular directions (fig. 2).

Since the flexible film-holder is pressed against the surveyor's plane with the base marks \( M_{11}, M_{12}, \ldots M_{ij} \) at the moment of roentgenography, we shall consider conditionally that the images of the respective marks \( m_{11}, m_{12}, \ldots m_{ij} \) have the same coordinates on the panoramic pictures.

According to the transformation formulas of coordinates we shall determine the space coordinates of Points \( n_1 \) and \( n_2 \) which are the images of Point \( N \) of the survey object inner structure on the left and right panoramic pictures.

\[
\begin{bmatrix}
X_n \\
Y_n \\
Z_n
\end{bmatrix} = \begin{bmatrix}
X'_o \\
Y'_o \\
Z'_o
\end{bmatrix} + \begin{bmatrix}
a_1 & a_2 & a_3 \\
b_1 & b_2 & b_3 \\
c_1 & c_2 & c_3
\end{bmatrix} \begin{bmatrix}
x'_n \\
y'_n \\
0
\end{bmatrix},
\]

(2)

where: \( X'_o, Y'_o, Z'_o \) - coordinates of the beginning of the system \( o'x'y' \), which are equal to the coordinates of the \( M \) mark image, with which point \( o' \) is superposed; \( a_i, b_i, c_i \) - the guide cosines, which depend on the orientation angle elements of the elementary section of the panoramic picture in a known way [4].

With help of the proper orientation the roentgenography conditions allow to obtain \( \omega \approx \alpha \approx 0 \) (fig. 1). When according to the survey conditions the stereoroentgenography basis lies in parallel with the longitudinal axis of the survey object \( \alpha \approx \omega \approx 0 \), is obtained by means of proper orientation of the object.

In the first case the formulas (2) will be as follows:

\[
\begin{bmatrix}
X_n \\
Y_n \\
Z_n
\end{bmatrix} = \begin{bmatrix}
X'_o \\
Y'_o \\
Z'_o
\end{bmatrix} + \begin{bmatrix}
\cos \alpha & 0 & -\sin \alpha \\
0 & 1 & 0 \\
\sin \alpha & 0 & \cos \alpha
\end{bmatrix} \begin{bmatrix}
x'_n \\
y'_n \\
0
\end{bmatrix},
\]

(3)

\[\alpha = \arctan \frac{Z_{M_{i+1}j} - Z_{M_{ij}}}{X_{M_{i+1}j} - X_{M_{ij}}}\]

In the second case the formulas (2) can be written as follows:

\[
\begin{bmatrix}
X_n \\
Y_n \\
Z_n
\end{bmatrix} = \begin{bmatrix}
X'_o \\
Y'_o \\
Z'_o
\end{bmatrix} + \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \omega & -\sin \omega \\
0 & \sin \omega & \cos \omega
\end{bmatrix} \begin{bmatrix}
x'_n \\
y'_n \\
0
\end{bmatrix},
\]

(4)

\[\omega = \arctan \frac{Z_{M_{i+1}j} - Z_{M_{ij}}}{Y_{M_{i+1}j} - Y_{M_{ij}}}\]
The coordinates of the required point \( N \) can be determined according to the following formulas[2]:

\[
\begin{bmatrix}
X_N \\
Y_N \\
Z_N
\end{bmatrix} = \frac{Z_{n_2} B}{X_{n_1} Z_{n_2} + (B - X_{n_2}) Z_{n_1}} \begin{bmatrix}
X_{n_1} \\
Y_{n_1} \\
Z_{n_1}
\end{bmatrix},
\]

(5)

where \( X_{n_1}, Y_{n_1}, Z_{n_1} \) stand for the space coordinates of Point \( n_1 \) of the left panoramic picture in the system \( F_1 \) \( XYZ \); \( X_{n_2}, Z_{n_2} \) stand for space coordinates of point \( n_2 \) of the right panoramic picture in the system \( F_2 \) \( XYZ \).

The mean square errors of determination of the space coordinates of the required point \( N \) can be found according to the formulas (5). As a result we obtain

\[
\begin{align*}
m_X & \approx \frac{X_{n_1} m_{x_n} \sqrt{2}}{X_{n_1} + X_{n_2}'}; \\
m_Y & \approx m_Y; \\
m_z & \approx \frac{Z_{n_1} m_{x_n} \sqrt{2}}{X_{n_1} + X_{n_2}'};
\end{align*}
\]

(6)

where \( X_{n_2}' = B - X_{n_2} \).

Let \( B/f = 1/4 \), \( X_{n_1} = 0.5 \text{ B} \). Then, if \( m_X = m_Y = 0.20 \text{ mm} \), we obtain \( m_X = 0.14 \text{ mm}, m_Y = 0.20 \text{ mm}, m_Z = 0.84 \text{ mm} \) according to formulas (6).

The location of the required point \( N \) of the investigation object inner structure is usually determined with regard to the orientation point on its surface. For this purpose the differences in the coordinates \( \Delta X, \Delta Y, \Delta Z \) between the required point and the orientation one are calculated.

It is known that the mean square error of determination of coordinate difference increases proportionally to \( \sqrt{2} \), therefore \( m_{\Delta X} = 0.20 \text{ mm}, m_{\Delta Y} = 0.28 \text{ mm}, m_{\Delta Z} = 1.18 \text{ mm} \).

For the practical estimation of accuracy a test object was used, made of plexiglas in the form of a cylinder segment with 15 steel balls of 1.5 mm diameter fixed inside as control points. The ball centres were coordinated with regard to point \( N \) 1 by means of direct instrumental measurements with the accuracy of 0.05 mm. The test object was placed on the special locator within the area of the stereoroentgenographic overlap of the X-ray apparatus TUR-1000 and
was oriented in such a way that the test-object cylinder surface
generants should be perpendicular to the survey basis. On the
test-object cylinder surface, turned to the flat film-holder, a
surveyor’s plane with a field of X-ray contrast marks was fixed,
after which a flexible film-holder was fixed. The stereorooent-
genographic survey was carried out with $B = 200$ mm and $f = 800$
mm. The flat X-ray pictures meet the requirements of the normal
case of the stereorooentgenogrammetric survey. The obtained flat
and panoramic pictures were measured by means of stereocomparator
STR-3.

The measurements according to the 15 control points allowed
to calculate some mean square errors of determination of the
difference in space coordinates.

The Gauss formula was used. As a result of the accuracy esti-
mation the following mean square errors were obtained.

$$m_{\Delta x} = 0.27, \quad m_{\Delta y} = 0.34, \quad m_{\Delta z} = 1.32$$

Thus, the new method of the stereorooentgenogrammetric survey
allows to solve with a sufficient accuracy a complicated roentgeno-
topographic task which arises in medicine in panoramic roentgeno-
graphy and in industry, in radio-graphic inspection of tubes and
casting blanks of cylinder form.

The precise photogrammetric data will improve the X-ray method
information possibilities and allow specialists to pass from purely
qualitative estimation of the investigation object to quantative
one which is exceptionally important both in medicine and industry.

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