PRECISION PHOTOGRAMMETRY FOR MICROWAVE ANTENNAE MANUFACTURING
Dr. S.F. El-Hakim
Photogrammetric Research Section
National Research Council of Canada
Commission V

Abstract

Large microwave antennae (about 30m diameter) are required to be manufactured with a very high accuracy (0.1-0.15mm) in order to be able to receive clearly certain wave lengths. Photogrammetric techniques are more suitable for this application than any direct measuring method due to the shape of the object and the nature of its material which changes in size with temperature. The program GEBAH has been employed. It allows for spatial distances, height differences and any other external infornation to be used in the adjustment. It also applies a full self calibration approach. The program evaluates the results by computing the variance covariance matrices and the error ellipsoids of the adjusted points. It also checks for blunders by the "data snooping" approach. The accuracy achieved for this application was better than the required accuracy.

Introduction:

Close range photogrammetry, with its flexibility, very large scale, and well defined targets, provides accuracies in all three coordinates that are often difficult to achieve by direct measuring techniques. Many theoretical investigations and practical applications have demonstrated this fact [e.g. Brown, 1980, Paig, 1981, Paig and El-Hakim, 1982, Fraser, 1982, Granshaw, 1980 and Kenefick, 1977]. For a certain project, all the necessary parameters such as camera locations and orientation, control point distribution, and shape and size of targets can be designed and precisely applied to provide the geometrically strongest solution possible. However, in some projects, it is impossible or very difficult to apply these ideal designs particularly when it comes to placing the cameras at the planned locations or establishing control points with the desired accuracy. Therefore, successful close-range photogrammetric systems should be flexible enough to adapt to any situation rather than follow the conventional techniques based on aerial triangulation experience [El-Hakim, 1982a, and Granshaw, 1980]. In particular, the following features should provide a great deal of flexibility, accuracy, and reliability in a close-range system:

1. The camera-station location and orientation elements may be treated as unknown, known, or partially known parameters depending on the possibility of measuring them with certain accuracy. If any of these parameters can be measured with sufficient accuracy, it is of advantage to include this in the solution and reduce the control point requirement.

2. It is often the case that because of lack of intervisibility, tight time schedule, or due to the nature of the project site, it is difficult to establish control points with the required accuracy. However, it is always possible to measure some distances or height elevations which don't necessarily form a geometrically strong network or even form a continuous network. In these cases, rather than utilizing the traditional two step approach of separately adjusting terrestrial measurements to obtain control points then including them in the photogrammetric adjustment, it is of great advantage to be able to utilize the available measurements directly with the photogrammetric data in one adjustment.
3. In many cases, non-metric cameras, and/or a different camera for each exposure station, are desirable. Therefore, the system must be capable of assigning different calibration parameters for each camera exposure and apply the self-calibration approach.

4. Since high accuracy is usually required, and since it is often undesirable or not possible to establish checkpoints to evaluate the accuracy, it is essential that the system is "reliable" and the accuracy assessment is realistic. It should have the capability of detecting gross errors using a statistically sound approach, such as "data snooping" [e.g., Grun, 1978]. In addition, some studies [El-Hakim, 1981] showed that the use of error ellipsoids provides accuracy figures equivalent to those obtained by check point discrepancies. On the other hand, figures computed from other quantities such as scaled residuals follow other patterns and usually provide a higher, misleading accuracy.

The close-range version of the program GEBAT [El-Hakim, 1982] has all the features described above and it has been used for a project that required such flexible features.

The Program GEBAT-V, a Review

This program is designed for close-range applications where a non-metric camera or a number of different cameras, maybe employed. Derived from the general GEBAT system, the geodetic observations are restricted to spatial distances and height differences, while self calibration is arranged in a photo-variant mode (i.e., each photograph has its own calibration parameters: calibrated principal distance, principal point coordinates plus 8 parameters of a harmonic function). The "V" designates the photo-"variant" mode. In addition, gross error detection is applied using the data snooping approach. The program computes the redundancy number for each observation and applies a statistical test as described in [El-Hakim, 1981]. Finally, the variance-covariance matrix and error ellipsoid for each adjusted object point is computed.

The critical value for the data snooping test and the confidence level for the error ellipsoid form part of the input to be provided by the user. Furthermore, any information available about the camera-station parameters as well as ground coordinates of any point, not necessarily a control point, can be utilized in the program. Suitable weight must be introduced for such information. The program allows the weighting of each station parameter or even fixing it, and of each coordinate of each object point.

The photogrammetric mathematical model is the self-calibration bundle adjustment model:

\[
x_A - x_o + dV_x = -f \frac{(X_A - X_c)m_{11} + (Y_A - Y_c)m_{12} + (Z_A - Z_c)m_{13}}{(X_A - X_c)m_{31} + (Y_A - Y_c)m_{32} + (Z_A - Z_c)m_{33}}
\]

and

\[
y_A - y_o + dV_y = -f \frac{(X_A - X_c)m_{21} + (Y_A - Y_c)m_{22} + (Z_A - Z_c)m_{23}}{(X_A - X_c)m_{31} + (Y_A - Y_c)m_{32} + (Z_A - Z_c)m_{33}}
\]

where: \(m_{ij}\) (\(i, j = 1 \text{ to } 3\)) are the elements of the rotation matrix \(R^T\)

\(x_A, y_A, z_A\) the object coordinates of point A
\(x_c, y_c, z_c\) projection center coordinates
\(x_A, y_A\) image coordinate of point A
\(x_o, y_o\) principal point coordinates
\(f\) principle distance
Correction terms \( dV_x \) and \( dV_y \) are given by:

\[
dV_x = (x_A - x_o) \cdot T
\]

and

\[
dV_y = (y_A - y_o) \cdot T
\]

with \( T \) being the harmonic function,

\[
T = a_{00} + a_{11} \cos \lambda + b_{11} \sin \lambda + a_{20} r + a_{22} \cos 2\lambda + b_{22} \sin 2\lambda + a_{31} r^2 \cos \lambda + b_{31} r^2 \sin \lambda + a_{33} r^2 \cos 3\lambda + b_{33} r^2 \sin 3\lambda + \ldots
\]

and

\[
r = \sqrt{(x_A - x_o)^2 + (y_A - y_o)^2}, \quad \lambda = \arctan \frac{y_A - y_o}{x_A - x_o}
\]

The observation equation for the terrestrial measurements are as follows:

For slope distance, \( S_{ij} \):

\[
V_{s_{ij}} = C_1(dx_j - dx_i) + C_2(dy_j - dy_i) + C_3(dz_j - dz_i) + (S_c - S_o)
\]

where

\[
C_1 = (x_j - x_i)/S = \Delta x/S
\]

\[
C_2 = (y_j - y_i)/S = \Delta y/S
\]

\[
C_3 = (z_j - z_i)/S = \Delta z/S
\]

\( S_c \) is computed as \([ (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 \] \( )^{1/2} \) from approximate coordinates \( X, Y, \)

\( Z \) of points \( i \) and \( j \), and \( S_o \) is the observed value.

For height difference:

\[
V_{h_{ij}} = (Z_j - Z_i) - \Delta h_o
\]

where \( \Delta h_o \) is the observed value.

for details and solution algorithms, see [El-Hakim, 1982a].

**Measurements of Microwave Antennae Subassemblies:**

In order to receive clearly certain wavelengths, large microwave antennae must be manufactured with very high accuracy. An antenna, which has a diameter of more than 30 meters and is a paraboloid in shape, consists of a number of subassemblies, each is approximately 2.7m by 1.5m in size, welded together in a delicate manufacturing process. These subassemblies have to be closely monitored to ensure the high accuracy.

Photogrammetry has been considered as an alternative to the direct measurement technique that was being used by the manufacturer and was regarded as unsatisfactory by the purchaser. Keeping the objective of achieving better than 0.15mm accuracy in mind, the project has been planned and executed as follows:

1. **Camera:** A precision terrestrial camera with minimum distortion and stable parameters was selected. A Wild P-31 with maximum lens distortion of \( \pm 4 \) \( \mu \)m and wide angle lens (\( f = 100\)mm) was available. This camera has 4 fiducial marks and a center cross. Glass plates of dimensions 102 x 127mm, of which 83 x 117mm represents the usable format, were used with emulsion base instead of film. The camera was calibrated shortly before the project.
Figure 1: Two of the Project's Photographs
Figure 2: Targets.

Figure 3: Terrestrial Observations and Control
2. **Targets and Scale:** The assemblies were targeted with 110 points at locations selected by the manufacturer. Figure 1 shows two of these panels and the target shape and locations. To achieve proper contrast to the grey aluminum, a white ring was centered around 1mm clear circle (Figure 2). The exact target point is represented by a cross inside the clear circle. A standard error of 3µm for the measurements, using the zeiss PSK stereocomparator, could easily be achieved for these targets. An average scale of 1:25 (2.5m from the center of the panel) was considered sufficient providing that systematic errors were controlled.

3. **Control:** Due to an unsufficient number of terrestrial observations, no point could be coordinated with sufficient reliability to be used as a control point for such a high accuracy project. Since no particular coordinate system was of interest, one of the points at one end of the network was chosen as origin of the coordinate system, while the X-coordinate of another point at the opposite end was given an arbitrary value of zero, to define the azimuth. These three coordinates plus three height control points represented the only fixed control values. Distances and height differences between six precisely established points were measured, as well as two perpendicular measuring tapes placed flat on the panel surface, to be adjusted simultaneously with the photogrammetric observations. This provided excellent control. Figure 3 shows the distribution of the various control and observations (see also the photos in figure 1).

**Figure 4:** Camera Station Arrangement
4. Photography: Highly convergent photography, due to its improved geometrical strength, has been employed. Four convergent photographs were taken from the corners of the panel, as shown in figure 4. A platform was used to raise the exposure station to about 3 meters above ground. The photo stations were placed in such a way that all points appear in each photograph. However, some points, due to wide range of scale variation within the photograph, were precisely measurable from only three photographs.

The place negatives were measured directly without converting them into diapositives to reduce the processing operations and thus the chance of additional systematic errors.

Results and Concluding Remarks

Image coordinates of the four photographs, distances, and height differences have been adjusted simultaneously with GEBAT-V. Applying the data snooping approach resulted in the rejection of an average 8 observations out of an average 1050 image coordinates (about 0.75%) for each panel. This rejection was based on the test:

\[
\omega_i = \frac{v_i}{\sigma_v} = \frac{v_i}{(\sigma_i / \sqrt{q_i})} \geq C
\]  

(6)

where \( \omega_i \) is the standardized residual, \( v_i \) is the residual, \( q_i \) is the diagonal element \( i \) of matrix \( Q_{vv} \) (the weight copactor matrix of the residuals) and \( C \) is a critical value for a specific confidence level. For this data, \( C \) has been chosen to be 3.29.

The adjustment was carried out with self-calibration. The focal length has been given a large weight since the recently-calibrated focal length was used. Table 1 displays the results of one of the panels. The accuracy is evaluated by the axes of the error ellipsoid at 95% confidence level, however the table displays other figures for comparison purpose. These figures are for the standard error of unit weight \( \sigma_q \) and the average standard deviations of the adjusted coordinates. The case of basic adjustment without self calibration is also shown. For this panel the accuracy is:

0.12mm (X), 0.13 mm (Y) and 0.15mm (Z),

which is within the requirements. Self calibration improved the results by about 25%, although the absolute value of the improvement is only 1.5\( \mu \)m in image coordinates.

To obtain the panel deformations, or its deviation from the ideal surface, a best fitting paraboloid was determined from the adjusted coordinates of the targets by the method of least squares. The paraboloid surface equation is:

\[
x^2 + y^2 = 4CZ
\]  

(7)

where \( C \) is a constant. The coordinates of the center of the surface, the orientation angles with XYZ coordinate systems and \( C \) are the unknowns.

The RMS of the residuals at the targeted points did not exceed 0.10 mm for the tested panels.

In conclusion, it has been demonstrated that a flexible close-range photogrammetric system can achieve a high accuracy with minimum control and
### Table 1: Accuracy Figures

<table>
<thead>
<tr>
<th>standard errors</th>
<th>no self calibration</th>
<th>with self calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td>- of unit wt. ( \hat{\sigma}_o )</td>
<td>6 ( \mu )m</td>
<td>4.5 ( \mu )m</td>
</tr>
<tr>
<td>- of adjusted coordinates:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \hat{\sigma}(x) )</td>
<td>0.07 ( \mu )m</td>
<td>0.05 ( \mu )m</td>
</tr>
<tr>
<td>( \hat{\sigma}(y) )</td>
<td>0.07 ( \mu )m</td>
<td>0.05 ( \mu )m</td>
</tr>
<tr>
<td>( \hat{\sigma}(z) )</td>
<td>0.07 ( \mu )m</td>
<td>0.05 ( \mu )m</td>
</tr>
<tr>
<td>- Error Ellipsoid Axes:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( e(x) )</td>
<td>0.16 ( \mu )m</td>
<td>0.12 ( \mu )m</td>
</tr>
<tr>
<td>( e(y) )</td>
<td>0.17 ( \mu )m</td>
<td>0.13 ( \mu )m</td>
</tr>
<tr>
<td>( e(z) )</td>
<td>0.20 ( \mu )m</td>
<td>0.15 ( \mu )m</td>
</tr>
</tbody>
</table>

utilizing any available terrestrial observations. This reduces effort, time and cost compared to a direct measurement technique or a traditional photogrammetric approach.

The author wishes to thank Al Way-Nee, of his affiliation, for his help with the field work.

### References


