DEFORMATIONS IN ELECTRON MICROGRAPHY
AND THEIR RELATIVE IMPORTANCE

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ABSTRACT

An analytical photogrammetric self-calibration procedure has been successfully used to establish the projection geometry for each of the two systems: Scanning Electron Microscope (SEM) and Transmission Electron Microscope (TEM). This procedure established, inter alia, the various types of distortions inherent in the systems. Specific mathematical model to describe each type of distortion has been established through research. All deviations from parallel projection and deformations in the projected image are considered as distortions. These are categorized as: perspective (being the deviation, if significant, of the projection itself), scale (affinity), radial, spiral and tangential distortions. Their characteristics are presented. The comparative importance of such distortions, in view of their elimination, on-line at an analytical plotter or off-line at a high-speed computer, is discussed. The study is made with regard to x and y photo-coordinates on individual micrographs. All these are for the purpose of precision measurements and mapping with electron micrographs.

INTRODUCTION

The electron microscopes (EM) are almost indispensable investigation tools in fields like metallurgy, microbiology, microelectronics, criminology, etc. Single, two dimensional illustrative micrographs are generally satisfactory for many uses. However, there is a steadily growing demand for precision mensural information on microscopic objects. Many of these seem to be nothing but microscopic versions of normal macroscopic cases usually known to photogrammetrists. However, the projection geometry and inherent deformations are different in the EM imaging systems.

There are basically two types of electron microscopes: Scanning Electron Microscope (SEM) and Transmission Electron Microscope (TEM). While an SEM micrograph depicts the surface details of the specimen (object), a TEM micrograph depicts the inside of a thin specimen, somewhat similar to an X-ray radiograph. Therefore, surface mapping potential with SEM micrography is great. On the other hand, since the TEM has a much better resolving power, it offers larger magnification capability.

An understanding of the various components involved in the EM system used, their geometric forms and their configurations is necessary for precision measurements with electron micrographs. Consideration of various distortions, although in many respects similar to conventional photogrammetry, are uniquely special in both, SEM and TEM.

CALIBRATION OF AN EM SYSTEM

Calibration is a refined form of measurement for the purpose of evaluating the performance of the working system. The necessary parameters and the permissible ranges of their variations need to be established. It is also necessary that a reference "standard" of proven stability is used for such a calibration.
The best available inexpensive standards are carbon replicas made from master
diffraction gratings for the TEM and metal coated calibration specimens for the
SEM. These are cross-ruled grids of fine grooves in a plane surface. The replica
grids often used for high magnification give 2160 lines per mm. These being two-
dimensional, certain convergent configuration (Fig. 1) of two or more micrographs
can be used to "generate" the necessary control in the third dimension. The stereo-
model can thus be fully controlled for assuring precision measurements. Such a
set-up and utilisation of the process of "self-calibration" (Ghosh, 1979) has been
found to yield best results in such calibrations (Adiguzel, 1985; Ghosh, 1975 and
Mauney, 1975).

The collinearity equations so well known in photogrammetry provide the basis for
such an analytical "on-the-job" calibration. These equations are augmented by using
appropriate mathematical models for the involved distortions to best describe the EM
geometry.

RELATED PROJECTIONS

A photograph (micrograph) may be considered as a projected record of information.
It can be considered to be three dimensional (x, y, z), where the third dimension (z)
is a constant (C). One can establish the projective relationship between the photo-
graph and the object by using a 3-D similarity transformation equation or its
variation in some form. In view of scale affinity, one can specify Cx and Cy.

A. Parallel Projection

In view of extremely small objects and extremely large magnifications, an electron
micrograph has an extremely small field angle and the projected bundle of rays can be
viewed as parallel (or near parallel) projection (Fig. 2), the magnification being a
sequence before or after the projection. The parallel projection can be expressed by:

\[
\begin{bmatrix}
    x \\
    y \\
    z
\end{bmatrix} = \begin{bmatrix}
    K_1 & 0 & 0 \\
    0 & K_2 & 0 \\
    0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
    X_R \\
    Y_R \\
    Z_R
\end{bmatrix} \quad \text{Eq. 1}
\]

where x, y
K_1, K_2
and X_R, Y_R, Z_R
are the photo coordinates;
are the scale factors along X_R, Y_R directions,
respectively;
are the object-space coordinates (considering rotations).

In view of rotations and translations to a selected origin the last term in Eq. 1 becomes:

\[
\begin{bmatrix}
    X_R \\
    Y_R \\
    Z_R
\end{bmatrix} = M \begin{bmatrix}
    X - X_o \\
    Y - Y_o \\
    Z - Z_o
\end{bmatrix} \quad \text{Eq. 2}
\]

where X, Y, Z are the coordinates of the selected origin in the object systems;
and M is the orientation matrix.

If the height datum (level reference) is defined by setting Z_o = 0, after sub-
titutions and simplifications (Ghosh, 1979), one obtains the following projective
equations in the case of parallel projection:
\[ x = K_1 (X - X_0) (c^\phi c^\kappa + s^\omega s^\phi s^\kappa) + K_1 (Y - Y_0) c^\omega s^\kappa + K_1 Z (s^\omega c^\phi s^\kappa - s^\phi c^\kappa) \]  
\[ y = K_2 (X - X_0) (s^\omega s^\phi s^\kappa - c^\phi c^\kappa) + K_2 (Y - Y_0) c^\omega c^\kappa + K_2 Z (s^\phi c^\kappa + s^\omega c^\phi c^\kappa) \]  
\[ \text{Eq. 3} \]

where \( c \) and \( s \) prefixes mean cosine and sine functions, respectively, of the rotation angles as indicated.

\[ x = x_R \frac{C_x}{\pi - Z_R} \quad \text{and} \quad y = y_R \frac{C_y}{\pi - Z_R} \]  
\[ \text{Eq. 4} \]

where \( Z \) is the projection distance to the reference datum of the object, and the rest of the terms are as defined previously.

By substituting the expansion for \( Z_R \) from Eq. 2 into Eq. 4, after rearranging,

\[ x = \frac{(C_x/Z) x_R}{1 - \frac{1}{Z} \{(X - X_0) c^\omega s^\phi - (Y - Y_0) s^\omega + (Z - Z_0) c^\omega c^\phi\}} \]  
\[ \text{Eq. 5} \]

\[ y = \frac{(C_y/Z) y_R}{1 - \frac{1}{Z} \{(X - X_0) c^\omega s^\phi - (Y - Y_0) s^\omega + (Z - Z_0) c^\omega c^\phi\}} \]

One can substitute \( K_1 \) for \( C_x/Z \) and \( K_2 \) for \( C_y/Z \). Furthermore, the second term in the denominator being smaller than unity, the expression can be expanded in a series (cf. \( 1/(1-q) = 1 + q + q^2 + q^3 + \ldots \)). This gives the final expressions for a perspective projection:

\[ x = K_1 x_R \{1 + \eta + \eta^2 + \ldots\} \]  
\[ \text{Eq. 6} \]

\[ y = K_2 y_R \{1 + \eta + \eta^2 + \ldots\} \]

where \( \eta \) is the second term in the denominators of Eqs. 5. Details of this derivation and the validity of the various approximations are credited to Nagaraja (1974).

**DISTORTIONS**

With regard to electron micrography, it is practical and convenient to consider the parallel projection as appropriate. The deviations from parallel projection may be considered as distortions. The various distortions and their mathematical models are presented below in accordance with their relative magnitudes (see Adiguzel, 1985; El Ghazali, 1978; Ghosh and Nagaraja, 1976). Some distortion patterns are given in Fig. 4.
1. Perspective Distortion

This defines the difference between the perspective and parallel projections. One may note that the first terms of Eq. 6 constitute Eq. 1. This gives:

\[
\begin{align*}
    x_{\text{parallel}} &= x_{\text{pers}} - x_{\text{pers}} \{ \nabla + \nabla^2 + \ldots \} \\
    y_{\text{parallel}} &= y_{\text{pers}} - y_{\text{pers}} \{ \nabla + \nabla^2 + \ldots \}
\end{align*}
\]

Eq. 7

The representative values of \( \nabla, \nabla^2 \) etc. can be obtained from calibration studies of the specific EM imaging system.

2. Scale Distortion

Scale distortion or affinity (in view of different scales in different directions) is contained in Eqs 1, 4, 5 or 6 and is, therefore, best dealt with directly in the mathematical model for the projection.

3. Radial Distortion

This is considered, conventionally, with regard to the fiducial center/principal point of the micrograph, either positive (in the outward direction) or negative (in the inward direction). Similar to conventional photogrammetry, it is expressed by a polynomial:

\[
\Delta r = k_o r + k_1 r^3 + k_2 r^5 + \ldots
\]

Eq. 8

where \( r \) is the radial distance of the image point from the photo center; and \( k \)'s are certain constants. The first term \( (k_0) \), however, is equivalent to a scale factor and, in a sequential application of corrections, can be considered as contained in the terms \( k_1 \) or \( k_2 \).

It has been found from actual studies that terms of 5 and higher order in \( r \) can be neglected in normal applications. For the x and y components, therefore, one gets:

\[
\begin{align*}
    \Delta x &= \Delta r \frac{x}{r} = k_1 r^3 \frac{x}{r} = k_1 (x^3 + xy^2) = D_1 x^3 + D_2 xy^2 \\
    \Delta y &= \Delta r \frac{y}{r} = k_1 r^3 \frac{y}{r} = k_1 (x^2 y + y^3) = D_3 y^3 + D_4 x^2 y
\end{align*}
\]

Eq. 9

where the \( D \)'s are certain constants.

4. Spiral Distortion

Klemperer and Barnett (1971), expressed the spiral or rotational twist of the electron beam by

\[
\Delta d = C (y' / y) r^3
\]

Eq. 10

where \( \Delta d \) is the displacement of the ray lateral to the radial direction on the micrograph; \( C \) is the spiral distortion coefficient; \( y' / y \) is the magnification and \( r \) is the radial distance of the point from the principal electron axis.
Magnification being already contained in the \( K \) factors, a substitution, \( S = C y'/y \), may be considered. Furthermore, in view of possible affinity, it may be pertinent to consider two values of \( S \): \( S_1 \) and \( S_2 \) for the \( x \) and \( y \) directions, respectively. This gives:

\[
\Delta x = S_1 \frac{y}{r} r^3 = S_1 (x^2 y + y^3) \quad \text{Eq. 11}
\]

\[
\Delta y = S_2 \frac{x}{r} r^3 = S_2 (x^3 + xy^2)
\]

where \( S_1 \) and \( S_2 \) are the spiral constants.

5. Tangential Distortion

Maune (1973) modelled this pattern of distortion after that of a conventional photogrammetric camera (effect due to decentering of lens) and obtained satisfactory results. However, Nagaraja (1974) found that this type of distortion can be effectively accommodated in the mathematical model for spiral distortion. Adiguzel (1985) found some (although not very significant) improvement in the mensural accuracy by considering this distortion. The effect of tangential distortion being negligible, and the mathematical model being lengthy, it is not presented here. The ASPRS Manual of Photogrammetry (1980) would give ideas on this.

The degree of stability of the EM system has a significant influence on the distortions. The reliability of the mathematical models, therefore, would depend on the calibration and evaluation of the specific imaging system performed under specific working conditions. An idea on the degree of refinement attainable by considering such distortions is presented in Fig. 5.

Figure 5, as an example, illustrates interesting facts on relative accuracies with regard to various types of distortions and their combinations. However, with regard to the absolute limiting accuracy, a strong role is played by the magnification on the micrograph and by the measuring instrument.

To an EM user, comparative results from different specimens are often more important than the absolute accuracy values. This causes a general reluctance for considering accuracy related aspects. However, more and more EM users find that these are important considerations.

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Figure 1.
Convergent configuration of micrographs by using one pre-tilt ($\theta_0$) and several rotations ($\kappa$). Note: I, II and III represent the perspective centres.

Figure 2.
Parallel projection and Magnification

Figure 3.
Perspective projection, Collinearity condition
Figure 4.
Some distortion patterns in electron micrography

Figure 5.
RMS deviations of image coordinates with regard to various combinations of correction parameters. [Example from Adiguzel, 1985]. Note: Here, E = exterior orientation (to include the magnification), R = radial distortion, S = spiral distortion, and T = tangential distortion. This is a case of an SEM and the magnification was 2900x.

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