THE FORMULAE OF THE RELATIVE ORIENTATION FOR NON-METRIC CAMERA

CHANG BENYI
Xian Research Institute of Surveying and Mapping, China

ABSTRACT

There are two problems to be solved in the relative orientation of non-metric photography for close-range photogrammetry. One is how to calculate the approximations of relative orientation elements, and the other is associated with that the non-metric camera is lacking in fiducial marks and rigorous focal distance.

This paper presents an algorithm for solving the above problems. The formula for determining relative orientation elements is derived by direct linear transformation from coplanar condition equation. The corrections for the image principal point coordinates and for the focal distance are involved as unknowns in conventional relative orientation equation.

A real-time computer program for implementing this algorithm has been developed at Planicomp C-100 analytical plotter and a good result abtained.

INTRODUCTION

The non-metric camera has been widely used in close range photogrammetry. In recent years, the direct linear transformation (DLT) method developed by Dr. Abdle Aziz and Dr. Karara in 1971-1974 (1) is a profound and influential algorithm in this field. This algorithm has overcome the serious difficulty associated with lack of rigorous interior orientation elements in non-metric camera. But for the practical use, a problem still remains in DLT algorithm, that is to say, it needs at least six control points in object space. Can we make use of information from non -metric camera directly to perform the relative orientation? It is the problem that will be treated in this paper. An algorithm for solving this problem consists of two parts:
1. The formula for determining relative orientation elements, which is derived by direct linear transformation from coplanar condition equation;
2. The corrections for the image principal point coordinates and for the focal distance, which are involved as unknowns in conventional relative orientation equation (2). The first formula is used alone in calculating the approximations of relative orientation elements for final rigorous adjustment. Then. the interior orientation elements are involved as unknowns in the relative orientation equation. So far as the elements of interior orientation are correlated with that of relative orientation, it is necessary for weakening such correlation, that the simultaneous equations be constructed by different stereoscopic pairs from a number of photographs. Thus, the problem arised from lacking in rigorous interior orientation elements can be solved. Similar method was put forward by Dubinovski (1972) in (3), but the formula for calculating approximations of the relative orientation elements was not included in his method.

The computer science is making rapid strides in these days and the conditions are already ripe for solving such problem. A real-time computer program for implementing this algorithm has been developed at Planicomp C-100 analytical plotter, and the results of systematic tests proved that our algorithm is correct.

THE DIRECT LINEAR TRANSFORMATIONS OF RELATIVE ORIENTATION ELEMENTS (RDLT)

It is well known that the equation of relative orientation can be derived directly from coplanar condition equation. According to (2), we have

$$\begin{vmatrix} Bx & By & Bz \\ U & V & W \\ U' & V' & W' \end{vmatrix} = 0, \qquad (1)$$

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} x-x_o+xr^2k_1+\ldots\ldots \\ y-y_o+yr^2k_1+\ldots\ldots \\ -f+f_o \end{bmatrix},$$

and

$$\begin{bmatrix} U' \\ V' \\ W' \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} x'-x_o'+x'r'^2k_1'+\ldots\ldots \\ y'-y_o'+y'r'^2k_1'+\ldots\ldots \\ -f'+f_o' \end{bmatrix}$$

By developing the formula (1) and taking no account of the influence of the interior orientation elements and lens distortion, we obtain

$$L_1yx'+L_2yy'+L_3yf'+L_4fx'+L_5fy'+L_6ff'+L_7xx'+L_8xy'+L_9xf'=0 \qquad (2)$$

where

$$\begin{aligned} L_1 &= B_xc_1-B_za_1 \\ L_2 &= B_xc_2-B_za_2 \\ L_3 &= -B_xc_3+B_za_3 \\ L_4 &= B_xb_1-B_ya_1 \\ L_5 &= B_xb_2-B_ya_2 \\ L_6 &= -B_xb_3+B_ya_3 \\ L_7 &= B_zb_1-B_yc_1 \\ L_8 &= B_zb_2-B_yc_2 \\ L_9 &= -B_zb_3+B_yc_3 \end{aligned} \qquad (3)$$

Having been properly transformed the equation (2) can be solved with some term defined in it as a free one. According to formula (3), if $Bx \gg By$ or $Bx \gg Bz$, L_5 can be chosen to divide equation (2); If $By \gg Bx$ or $By \gg Bz$, then L_3 can be chosen; If $Bz \gg Bx$ or $Bz \gg By$, then L_8 can be chosen. When Bx is the greatest, the division of each term in equation (2) by L_5 gives

$$L_1'yx'+L_2'yy'+L_3'yf'+L_4'fx'+L_6'ff'+L_7'xx'+L_8'xy'+L_9'xf'+fy'=0 \qquad (4)$$

where

$$L_i'=\frac{Li}{L_5}, \qquad L_5'=1 \qquad (i=1, 2, \ldots\ldots 9).$$

If more than eight pairs of mutching points are known, the equation (4) can be solved and all L' are obtained.

It is follows from formula (3):

$$L_5^2 = 2Bx^2/(L_1'^2 + L_2'^2 + L_3'^2 + L_4'^2 + L_5'^2 + L_6'^2 - L_7'^2 - L_8'^2 - L_9'^2)$$

$$By = -(L_1'L_7' + L_2'L_8' + L_3'L_9')L_5^2 \; /Bx \qquad (5)$$

$$Bz = (L_4'L_7' + L_5'L_8' + L_6'L_9')L_5^2 \; /Bx.$$

With L_5 determined, all of the Li can be solved from formula (4).

Although there are nine condition equations, only six of them are independent. For solving nine direction cosines, it is necessary to use following three condition equations obtained from orthogonal matrix property.

$$a_1 = \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} \qquad a_2 = \begin{vmatrix} b_3 & b_1 \\ c_3 & c_1 \end{vmatrix} \qquad a_3 = \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix} \qquad (6)$$

In (3) and (6), all of the direction cosines can be determined. Not having been adjusted strictly, these elements can not be regarded as best possible values, but can be served as preliminary values for further strict adjustment.

When By or Bz is the greatest, another two sets of similar formulae can be derived respectively.

In summary, in the case of using a non-metric camera for close range photogrammetry, when sometimes the values of relative orientation elements are great, it is difficult to assign their initial values. The algorithm can solve the above mentioned problem and it has been proven by test.

A RELATIVE ORIENTATION EQUATION WITH INTERIOR ORIENTATION ELEMENTS AS UNKNOWNS

By using the strict relative orientation equation in (2), and introducing the interior orientation elements as unknown, it follows (only considering Bx to be the greatest):

$$\frac{\partial F}{\partial By}\Delta By + \frac{\partial F}{\partial Bz}\Delta Bz + \frac{\partial F}{\partial \varphi}\Delta\varphi + \frac{\partial F}{\partial \omega}\Delta\omega + \frac{\partial F}{\partial K}\Delta K + \frac{\partial F}{\partial x_0}\Delta x_0 + \frac{\partial F}{\partial y_0}\Delta y_0 + \frac{\partial F}{\partial f}\Delta f + \frac{\partial F}{\partial x_0'}\Delta x_0' + \frac{\partial F}{\partial y_0'}\Delta y_0' +$$

$$+ \frac{\partial F}{\partial f'}\Delta f' + F_0 = -\frac{\partial F}{\partial x}Vx - \frac{\partial F}{\partial y}Vy - \frac{\partial F}{\partial x'}Vx' - \frac{\partial F}{\partial y'}Vy' \qquad (7)$$

For the expressions of partial derivatives one is refferenced to (2), except for the formulae

$$\frac{\partial F}{\partial f} = \begin{vmatrix} Bx & By & Bz \\ 0 & 0 & 1 \\ U' & V' & W' \end{vmatrix} \qquad \frac{\partial F}{\partial f'} = \begin{vmatrix} Bx & By & Bz \\ U & V & W \\ a_3 & b_3 & c_3 \end{vmatrix}$$

The rest is described in (2).

For practical application, the coefficient of lens distortion may be involved into equation (7). According to (4) and taking only k into account, the equation (7) will be enlarged by two additional unknowns:

$$\frac{\partial F}{\partial k_1} = \begin{vmatrix} Bx & By & Bz \\ xr^2 & yr^2 & 0 \\ U' & V' & W' \end{vmatrix}$$

$$\frac{\partial F}{\partial k_1'} = \begin{vmatrix} Bx & By & Bz \\ U & V & W \\ (a_1 x'+a_2 y')r'^2 & (b_1 x'+b_2 y')r'^2 & (c_1 x'+c_2 y')r'^2 \end{vmatrix}$$

When two photographs constituting the stereopair are taken with same camera and same focal setting, there is only one set of coefficients of interior orientation elements in (7), which are determined as follows:

$$\frac{\partial F}{\partial x_0} = \begin{vmatrix} Bx & By & Bz \\ -1 & 0 & 0 \\ U' & V' & W' \end{vmatrix} + \begin{vmatrix} Bx & By & Bz \\ U & V & W \\ -a_1 & -b_1 & -c_1 \end{vmatrix}$$

$$\frac{\partial F}{\partial y_0} = \begin{vmatrix} Bx & By & Bz \\ 0 & -1 & 0 \\ U' & V' & W' \end{vmatrix} + \begin{vmatrix} Bx & By & Bz \\ U & V & W \\ -a_2 & -b_2 & -c_2 \end{vmatrix}$$

$$\frac{\partial F}{\partial f} = \begin{vmatrix} Bx & By & Bz \\ 0 & 0 & 1 \\ U' & V' & W' \end{vmatrix} + \begin{vmatrix} Bx & By & Bz \\ U & V & W \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\frac{\partial F}{\partial k_1} = \begin{vmatrix} Bx & By & Bz \\ xr^2 & yr^2 & 0 \\ U' & V' & W' \end{vmatrix} +$$

$$+ \begin{vmatrix} Bx & By & Bz \\ U & V & W \\ (a_1 x'+a_2 y')r'^2 & (b_1 x'+b_2 y')r'^2 & (c_1 x'+c_2 y')r'^2 \end{vmatrix}$$

When a same object is photographyed by one or more cameras, the simultaneous equations can be constructed and written in matrix notation as follows:

$$AX - Q = V \qquad (8)$$

where A=matrix of coefficients,
X=unknowns vector,
Q=free vector,
V=vector of closure.

Let n be the number of cameras, and m be the number of stereopairs, then the number of unknowns is

$$N = 4 \times n + 5 \times m,$$

and the unknown vector is

$$X = (\Delta x_{01} \ \Delta y_{01} \ \Delta f_1 \ \Delta k_1 \ \ldots \ \Delta x_{0n} \ \Delta y_{0n} \ \Delta f n \ \Delta k n \ \Delta By_1 \ \Delta Bz_1 \ \Delta \varphi_1 \ \Delta \omega_1 \ \Delta \kappa_1 \ldots$$
$$\Delta Bym \ \Delta Bzm \ \Delta \varphi m \ \Delta \omega m \ \Delta \kappa m)^T$$

The following normal equation can be derived directly from (8) with P as weight coefficient matrix:

$$A^T PAX = A^T PQ \qquad (9)$$

It is known (2), that the weight of observation for each error equation is determined by

$$p = 1/ \left(\left(\frac{\partial F}{\partial x}\right)^2 + \left(\frac{\partial F}{\partial y}\right)^2 + \left(\frac{\partial F}{\partial x'}\right)^2 + \left(\frac{\partial F}{\partial y'}\right)^2 \right)$$

Since (7) has been linearized, the least squres solution must follow an iterative procedure.

TEST

A real-time computer program has been developed at Planicomp C-100 analytical plotter. The program was tested by use of simulated digital photographys, as well as by true photographys. The results indicate that our algorithm is correct.

The distribution of four simulated photographys is shown in Fig. 1. They are designed to take some object space by same camera. The object space is a transparent cuboid with 18 control points well-distributed on it (Fig. 2). The absolute tilt angle of each photo is more than 5°, and their mutual tilt angles are more than 10°.

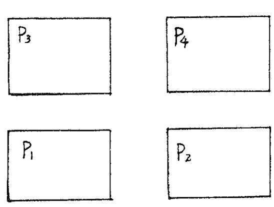

Fig. 1

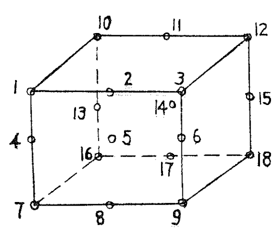

Fig. 2

The stereopairs p1 - p3 and p2 - p3 (see Fig. 1) are calculated by the formula with the By considered as the greatest, and the other stereopairs are treated by the formula with the Bx as the greatest.

The results obtained by comparing our RDLT algorithm (see Eq. 2) with conventional relative orientation equation are listed in Tab. 1, in which "1" denotes the results of RDLT, "2" denotes the results of conventional procedure.

It is obviously from Tab. 1, that our algorithm satisfies the demands for final strict adjustment of relative orientation elements.

Tab.1 The comparison between results of RDLT and conventional procedure (Simulated data)

Stereopair number		P1-P2	P1-P3	P1-P4	P2-P3	P2-P4	P3-P4
Bx	1	1.0	0.009	1.0	1.0	-0.011	1.0
	2	1.0	0.008	1.0	1.0	-0.011	1.0
By	1	-o.001	1.0	0.657	-0.665	1.0	-0.001
	2	-0.002	1.0	0.663	-0.664	1.0	-0.001
Bz	1	-0.103	0.103	-0.046	0.044	0.103	-0.103
	2	-0.102	0.104	-0.040	0.045	0.104	-0.102
a1	1	0.9789	1.0011	0.9792	0.9756	1.0001	0.9789
	2	0.9790	0.9998	0.9773	0.9774	0.9997	0.9790
a2	1	0.0024	0.0176	0.0274	-0.0199	-0.0244	0.0024
	2	0.0026	0.0174	0.0209	-0.0198	-0.0224	0.0024
a3	1	0.2041	-0.0020	0.2100	-0.2083	0.0027	0.2041
	2	0.2036	-0.0028	0.2110	-0.2103	0.0033	0.2039
b1	1	-0.0024	-0.0176	0.0117	-0.0192	0.0225	-0.0024
	2	-0.0024	-0.0176	0.0183	-0.0193	0.0226	-0.0026
b2	1	1.0012	0.9787	0.9874	0.9806	0.9787	1.0012
	2	1.0000	0.9788	0.9831	0.9831	0.9788	1.0000
b3	1	-0.0002	-0.2041	-0.1830	-0.1833	-0.2041	-0.0003
	2	-0.0012	-0.2038	-0.1820	-0.1822	-0.2036	0.0007
c1	1	-0.2039	-0.0018	-0.2104	0.2085	0.0025	-0.2039
	2	-0.2036	-0.0008	-0.2112	0.2104	0.0013	-0.2039
c2	1	-0.0005	0.2039	0.1979	0.1889	0.2038	-0.0003
	2	0.0007	0.2038	0.1817	0.1822	0.2036	-0.0012
c3	1	0.9777	0.9777	0.9550	0.9578	0.9778	0.9777
	2	0.9791	0.9790	0.9604	0.9665	0.9791	0.9790

Tab.2 The calculated corrections for interior orientation elements (Simulated data)

Number of photos	Number of pairs	Introduced errors of interior orientation elements			Corrections for interior orientation elements		
		Vx	Vy	Vf	x	y	f
4	6	0.5mm	0.5mm	0.5mm	0.500mm	0.500mm	0.999mm
4	6	-0.5	-0.5	-1.0	-0.499	-0.500	-1.000
3	3	-0.5	0.5	-1.0	-0.500	0.500	-1.000
3	3	0.5	-0.5	1.0	0.499	-0.500	0.999
2*	1	0.5	0.5	1.0	0.499	0.491	0.235
2*	1	-0.5	0.5	-1.0	-0.499	0.474	0.735

The errors of interior orientation elements are introduced into simulated data, and the simultaneous equations constituted from Eq. 7 are solved. The results are listed in Tab. 2. The equations have only one set of interior orientation elements, and have no coefficient of the lens distortion.

It is seen from Tab.2, that the simultaneous equations composed of four or three photos can all give good results. In the case of single pair, this algorithm can not provide correct results, especially the value of principal distance of camera.

Based on the successful test with simulated digital photographys, the processing of true photos taken by non-metric camera was performed. The focal distance of camera is 41 mm, and the format size is 36 x 24 mm. The distribution of true photos was the same as simulated photos (see Fig. 1). The object space selected is a corner of wall with 15 well-distributed control points (Fig.3). The coordinates of control points in object space have not been determined. In Tab.3, the comparison between results obtained by RDLT algorithm and conventional relative orientation procedure is given. As Tab.3 showns, in the case of relative orientation of six photopairs, the φ angle has a value up to 56.2°,and ω angle up to 15.9°. Therefore, it is difficult to implement relative orientation in close range photogrammetry without correct preliminary values of orientation elements prorided by RDLT algorithm.

Among the four photos taken by same camera, p1, p2 have one set of interior orientation elements and p3, p4 have the another. This is done by resetting of focal distance and introducing of lens distortion coefficient, when p3 and p4 were taking. Therefore, the two sets of interior orientation elements are involved as unknown in Eq. 7, herein incorporated also the coefficient of lens distortion. The image coordinates are measured at Planicomp C-100. The six sets of preliminary values of the interior orientation elements are corrupted intentionally by different errors. The results listed in Tab. 4 show, that they are in good coincidence with test conditions given, that is, the two sets of photos have approximately equal coordinates of principal points, but slightly different principal distances.

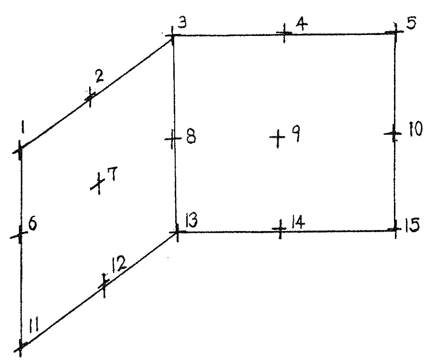

Fig. 3

Tab.3 The comparison between results of RDLT and conventional procedure (True photos)

Stereopair number		P1-P2	P1-P3	P1-P4	P2-P3	P2-P4	P3-P4
Bx	1	1.0	0.171	1.0	1.0	0.103	1.0
	2	1.0	0.179	1.0	1.0	0.125	1.0
By	1	-0.102	1.0	0.168	-0.278	1.0	0.107
	2	-0.102	1.0	0.165	-0.281	1.0	0.096
Bz	1	-0.483	0.227	-0.534	0.498	0.072	-0.552
	2	-0.462	0.272	-0.487	0.504	0.038	-0.533
a1	1	0.5451	1.0030	0.5210	0.5834	0.9944	0.5600
	2	0.5536	0.9983	0.5469	0.5791	0.9970	0.5690
a2	1	0.0476	0.0238	-0.1391	0.1647	-0.0779	-0.1741
	2	0.0478	0.0233	-0.1421	0.1663	-0.0773	-0.1735
a3	1	0.8324	0.0510	0.8339	-0.7971	-0.0112	0.8041
	2	0.8314	0.0528	0.8256	-0.7981	-0.0070	0.8034
b1	1	-0.0756	-0.0367	-0.0048	0.0138	0.0780	0.2426
	2	-0.0761	-0.0369	0.0009	0.0131	0.0773	0.2395
b2	1	1.0011	0.9605	0.9944	0.9771	0.9826	0.9716
	2	0.9971	0.9608	0.9856	0.9769	0.9822	0.9701
b3	1	-0.0078	0.2734	0.1689	0.2120	0.1692	0.0415
	2	-0.0066	0.2747	0.1690	0.2131	0.1714	0.0397
c1	1	-0.8266	-0.0449	-0.8381	0.8138	0.0103	-0.7816
	2	-0.8293	-0.0443	-0.8378	0.8151	-0.0064	-0.7863
c2	1	-0.0639	-0.2737	-0.0906	-0.1381	-0.1710	0.1841
	2	-0.0596	-0.2762	-0.0915	-0.1339	-0.1714	0.1698
c3	1	0.5449	0.9554	0.5085	0.5671	0.9880	0.5843
	2	0.5556	0.9601	0.5383	0.5636	0.9852	0.5941

Tab.4 Calculating results of interior orientation elements (True photos)

Test option No	Approximations of orient. elements				Corrections of interior orient. elements.							
	x	y	f	k	x	y	f	k	x'	y'	f'	k'
	mm	mm	mm		mm	mm	mm		mm	mm	mm	
1	1	-1	41	0	0.593	-0.258	-2.557	1.071	0.574	-0.324	-2.813	0.958
2	1	1	42	0	0.594	1.734	-1.553	1.076	0.574	1.668	-1.806	0.965
3	0	0	43	0	-0.406	0.735	-0.553	1.075	-0.426	0.668	-0.806	0.965
4	-1	-1	44	0	-1.406	-0.270	0.447	1.077	-1.425	-0.337	0.195	0.968
5	-1	1	45	0	-1.406	1.738	1.447	1.075	-1.425	1.672	1.192	0.964
6	2	-2	46	0	1.594	-1.266	2.447	1.076	1.575	-1.332	2.194	0.965

SUMMARY

There is certainly a great advantage to use the stereophotogrammetric technique directly into close range photogrammetry with non-metric camera. The method provided by this paper has made a beneficial attempt in this field, and much work remains to be done, such as the accuracy estimation, practical applications, and so on.

Finally, I wish to express my hearty thank, to Mr. Yuan Gen, my dear colleague, for his remakable doing of full test work.

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