DETERMINING TENSION FORCES OF GUY-ROPES OF AN AERIAL MAST BY MEANS OF TERROPHOTOGRAMMETRIC MEASUREMENTS

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ABSTRACT

The possibilities to extend the terrestrial photogrammetry applications for calculation of some physical parameters are presented in the paper. Off-line computational method for determining tension forces of the guy-ropes of the aerial mast is proposed.
In this method, traditionally calculated sag of the guy-rope is not necessary to determine the tension forces.

Finally some results of the measurements and conclusuion are presented.

INTRODUCTION

The construction of a steel aerial mast with guy-ropes must be controlled periodically. First of all the quantity of tension forces is checked. These forces may be determined by means of direct or indirect methods. Direct methods are expensive and inconvenient.
Having the results of verticality check of the mast construction and the quantity of sag the guy-ropes it is possible to determine tension forces of the guy-ropes using empirical formulae only.

Deeper analysis of the problem and in particular simplicity of the formula for the quantity of horizontal tension forces of a line :

$$F_{Ox} = F_{Nx} = k\ q \qquad (1)$$

where :

F_{Ox}, F_{Nx} - horizontal tension force at the lower and upper catch points,
k - catenary curve parameter,
q - specific weight of a line 1 running meter

helped the author to come to the idea of determining the quantity of tension forces of guy-ropes using terrestrial photogrammetry.
A homogenous set of points on the line (more than 3) helps to fix "k" parameter of a catenary curve together with accuracy estimation. The "k" parameter is the clue to determine many other physical quantities and geometric characteristics including a sag.

DESCRIPTION OF THE OBJECT

The aerial mast 330 m heigh, which guy-ropes are controlled is placed in the area mining exploitation. The guy-ropes five in each direction lie in three vertical planes. The angle between the planes is 120° (Fig.1). The vertical angles between the ground level and the guy-ropes are in the range of 45° - 55° -(Fig.2).

Terrain conditions make choosing such photogrammetric stations which allow to photograph the whole plane of the guy-ropes with all lower and upper catches impossible, even at the turned and incline axis of the camera.

Only 4 of 15 guy-ropes were photographed in a full length together with the catches. The image of the others were limited to the upper catches and large parts of the lines without the lower catches. However this problem, as it will be shown later, does not influence the determination of the tension forces of the guy-ropes.
Fig.2 shows schematically the construction, photogrammetric stations and the directions in which photos were taken.

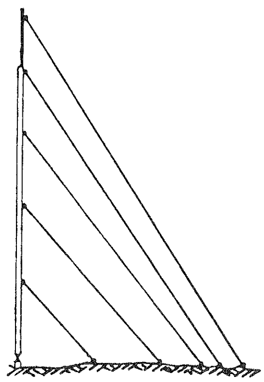

Fig.1. Guy-rope plane vertical section

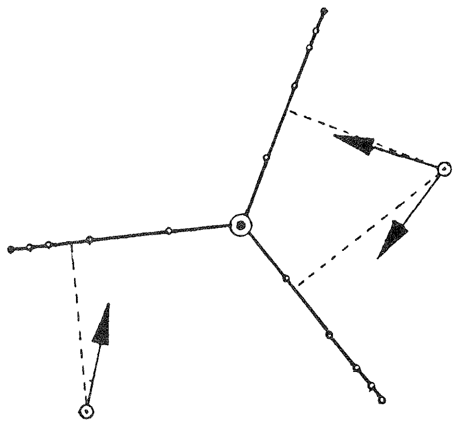

Fig.2. Situation of the guy-rope planes and camera stations

FIELD WORK

Durring the first measuring cycle the angles (Zeiss Jena theodolite of Theo 010A type) and the distances (Wild telemeter DI-3 type) which were necessary to fix X, Y coordinates of:

- photogrammetric stations,
- mast axis,
- lower catches of the guy-ropes,
- ground control points

were measured. The Z coordinates of the above points were determined by means of geometric levelling method.
The photos were taken with Zeiss Jena camera of UMK 10/1318 type on Orwo TO-1 plates, generally with orange filter. The photos were taken at horizontal or vertical position of the camera, always of the inclined axis of the camera $\omega = 15^o$ or $\omega = 30^o$.

As it is shown in Fig.2 the camera was not perpendicularly orientated to the vertical planes of the guy-ropes. Air temperature, wind direction and velocity were noted.

MEASUREMENT OF IMAGE COORDINATES AND THEIR TRANSFORMATION INTO SPACE COORDINATES

Coordinates X,Y,Z of all points were calculated in a local system of coordinates and local reference level. Additionally, for each plane of the guy-ropes, three coordinate system X,Y,Z with the common starting point (axis of mast) were established.

The measurement of image coordinates were performed with Zeiss Jena Steocometer with Coordimeter F type. In next measuring cycles an improvement of image coordinates observations was introduced.
The improvement concerned the plate-carrier which was turned to an

angle $\varkappa = 50^g$ when the line image in a photo was paralel to one of the instrument movements. It improved the precision of placing the measuring mark on the line image at about 30 %.
10 points (including two catches) regularly distributed were observed on each rope. Apart from 5 guy-ropes also steady checking points were registred in each pictures.

When discrepancies between two measurements, exceeded 5 μm, more observation were taken.

Clear positional numbering system of all 150 points, 15 guy-ropes were established.

0 1	5
number of the guy-ropes	point number on the line

Fig.3 presents schematicly the system of coordinates used in calculations as well as physical and geometric values.

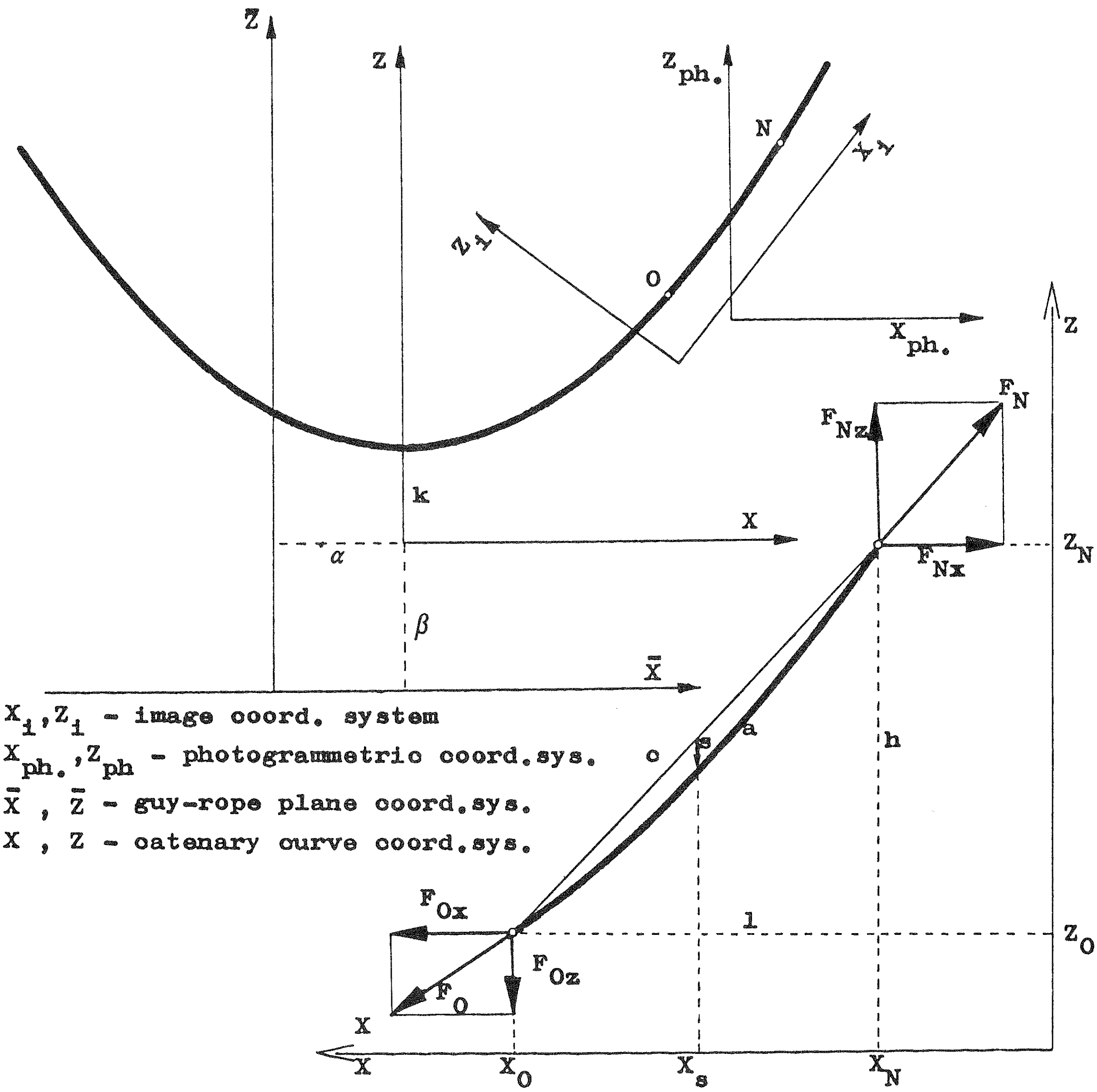

Fig.3. Used coordinate system and signs.

After initial check of the method, the whole calculations were continued according to two programmes (Ziobro,1985) "PRIM" and "LINA". "PRIM" programme gives the mean observation, rejects fals ones, makes isometric transformations into the image coordinates system, calculates iteratively an element of external orientation and geodetic coordinates in a system of the guy-ropes plane. Non linear system of equations is solved with the ordinary Newton method when the linear one by means of Banachiewicz's transformation - Cracovian calculus.

Each of the coordinates X,Y,Z of the checking points may be freely eliminated from the equation or treated as a constant.
A photo may not contain any checking points - in this case constant element of external orientation should be given. According to printed deviations it is possible to eliminate incorrect points from the equation.

"LINA" programme is used to calculate tension forces of the rope. Using the results of "PRIM" programme - a list of coordinates of line points lying on one plane this programme determines, by means of least square adjustment methods, 3 unknown quantities : "k" parameter of the catenary curve and two integration constants α and β . It transforms coordinates of points of each guy-rope which were expressed in the geodetic plane system of the guy-ropes into coordinates of the catenary curve system, separately for each line.

AN ALGORITHM OF "LINA" PROGRAMME

When a set of coordinates of line points consist of more than 3 it is possible to calculate "k" parameter of the catenary curve, two integration constants α and β , correction V_z with the precission estimation of unknown quatities.
Catenary curve equation

$$\bar{Z} - \beta = \frac{k}{2}\left(e^{\frac{\bar{X}-\alpha}{k}} + e^{-\frac{\bar{X}-\alpha}{k}}\right) \qquad (2)$$

when integration constants are calculated and the $\bar{X},\bar{Z}$ system is transformed for this value, the equation (2) takes the form :

$$Z = \frac{k}{2}\left(e^{\frac{X}{k}} + e^{-\frac{X}{k}}\right) = k \cosh \frac{X}{k} \qquad (3)$$

where :
X,Z - coordinates of the catenary curve system

For each point of the line, according to the algorithm (Gogoliński, 1976) we make error equations in the following form :

$$v_i = a_i dk + b_i d\alpha + c_i d\beta + l_i \qquad (4)$$

where :

$$a_i = \cosh \frac{\bar{X}_i - \alpha_o}{k_o} - \frac{\bar{X}_i - \alpha_o}{k_o} \sinh \frac{\bar{X}_i - \alpha_o}{k_o}$$

$$b_i = - \sinh \frac{\bar{X}_i - \alpha_o}{k_o}$$

$$c_i = 1$$

$$l_i = k_o \cosh \frac{\bar{X}_i - \alpha_o}{k_o} + \beta_o - \bar{Z}_i$$

where : k_o , α_o, β_o - approximate values.

Solving the set of 3 normal equations we obtain finally the values of k, α and β .

The line points coordinate we express in the catenary curve system X, Z from which we calculate tension forces and geomtric characteristics of the line according to the formulae :

- horizontal component of tension forces at the lower and upper catches :

$$F_{0x} = F_{Nx} = k\ q \qquad (5) = (1)$$

- vertical component of tension forces at the lower and upper catches :

$$F_{0z} = k\ q\ m_0 \quad , \quad F_{Nz} = k\ q\ m_N \qquad (6) \quad (7)$$

where:

m_0, m_N - directional coefficients tangent to the curve at lower and upper catch according to the formulae :

$$m_0 = \left| \sinh \ -\left(\frac{X_0}{k}\right) \right| , \quad m_N = \left| \sinh \ \frac{X_N}{k} \right| \qquad (8) \quad (9)$$

where:

X_0 , X_N - coordinates of lower and upper catches

- resultant tension forces of both catch points :

$$F_0 = \sqrt{F_{0x}^2 + F_{0z}^2} = k\ q \ \sqrt{1 + m_0^2} \qquad (10)$$

$$F_N = \sqrt{F_{Nx}^2 + F_{Nz}^2} = k\ q \ \sqrt{1 + m_N^2} \qquad (11)$$

- the length of the horizontal projection of the line :

$$l = X_N - X_0 \qquad (12)$$

- the length of the vertical projection :

$$h = Z_N - Z_0 = k\left(\cosh \frac{X_N}{k} - \cosh \frac{X_0}{k}\right) \qquad (13)$$

- the length of the chord :

$$c = \sqrt{l^2 + h^2} \qquad (14)$$

- the length of the line arc :

$$a = k\left(\sinh \frac{X_N}{k} - \sinh \frac{X_0}{k}\right) \qquad (15)$$

- vertical sag of the line in any point :

$$s = m\left(X_i - X_N\right) + Z_N - k \cosh \frac{X_i}{k} \qquad (16)$$

where:

m - directional coefficient according to the formula :

$$m = \frac{Z_N - Z_O}{X_N - X_O} \qquad (17)$$

- the ordinate X_s , where the sag is maximum :

$$X_s = k \text{ arc sinh } m \qquad (18)$$

- maximum sag :

$$s = m\,(X_s - X_N) + Z_N - k \cosh \frac{X_s}{k} \qquad (19)$$

When the photo of the lower catch is not taken, "LINA" programme, basing on the given distance from this point to the mast axis, computes the lacking coordinate Z and other values depending functionally on these coordinates.

SAMPLE OF CALCULATION - RESULT OF "PRIM" PROGRAMME

PHOTO NO 53

FIDUCIAL MARKS NO	CORRECTIONS VX	VY
1	-0.001	-0.000
2	0.001	0.000
3	0.001	0.000
4	0.000	-0.000

INCREASES OF UNKNOWN QUANTITIES

ITER.	FI	OMEGA	KAPPA	XO	YO	ZO
1	161.88	6.70	-0.65	0.000	0.000	0.000
2	0.14	0.01	0.44	0.000	0.000	0.000
3	-0.00	-0.00	-0.00	0.000	0.000	0.000

IMAGE COORDINATES NO	CORRECTIONS VX	VY
2571	0.2	-0.1
2572	0.2	-0.1
2573	0.0	0.0
2574	-0.1	0.0
3612	-0.3	0.1
1622	0.0	0.0

NUMBER OF UNKNOWN QUANTITIES N = 8
NUMBER OF OBSERVATIONS OB = 12
MEAN ERROR M = 0.2
MAX. CORRECTION V MAX = 0.3 POINT 3621

E L E M E N T S O F O R I E N T A T I O N

FI	OMEGA	KAPPA	XO	YO	ZO
7637.98	3340.04	-0.021	834.568	783.816	84 393
M FI	M OMEGA	M KAPPA	M XO	M YO	M ZO
17.58	9.38	31.62	0.000	0.000	0.000

A = 0.000 B = 0.000 CK = 98.970

RESULTS OF "LINA" PROGRAMME

GUY - ROPE NO 1

C O O R D I N A T E S

NO	GUY-ROPE PLANE SYSTEM $\bar{X}$	$\bar{Z}$	CATENARY SYSTEM X	Z	CORRECTIONS V
10	924.272	101.646	1046.416	1699.067	0.004
11	930.845	108.006	1057.989	1705.423	0.001
12	937.719	114.709	1059.863	1712.122	-0.004
13	944.945	121.811	1067.089	1719.222	-0.005
14	952.548	129.339	1074.692	1726.758	0.003
15	960.530	137.330	1082.674	1743.741	-0.005
16	968.954	145.827	1091.098	1743.246	0.003
17	977.824	154.874	1099.968	1752.291	0.000
18	987.202	164.525	1109.346	1761.954	0.012
19	997.108	174.866	1119.252	1772.273	-0.009

N = 10

U N K N O W N   Q U A N T I T I E S

K	ALFA	BETA	
1223.740	-122.144	-1597.417	
M K	M ALFA	M BETA	M
9.588	8.483	13.589	0.007

TENSION FORCES FOR Q = 8.16 KG

FOX = FON = 9985.7 KG = 97.927 KN
FOZ = 9618.0 KG = 94.320 KN
FNZ = 10460.8 KG =102.585 KN
FO = 13864.4 KG =135.963 KN
FN = 14461.7 KG =141.821 KN

MO = 0.963180
MN = 1.047574

GEOMETRICAL FEATURES OF THE GUY-ROPE

L = 72.836 M
H = 73.206 M
C = 103.268 M
A = 103.275 M
XS = 1082.962 M
S MAX = 0.768 M

CONCLUSIONS

1. It is not necessary to determine the sag to calculate, the most important for the user, values of the horizontal components of tension forces at both catches.

2. The value of the vertical component of tension force at the upper catch is also possible to determine this point is always visible .

3. The value of the "k" parameter of the catenary curve is also possible to determine by means of terrestrial photogrammetry in very unconvenient conditions, even there, where it is not possible to take a photo of the whole line.

4. Photogrammetric method of registration of dynamic structures should be the only data source for such objects. Comparing the results and accuracy of the above method with the trigonometric method is simply misunderstanding.

5. To obtain the best standarization of the results, the photos should be taken synchronically in all three planes, and the results should be supplied with additional data (temperature, wind direction and velocity).

REFERENCES

Gocał, J. (1982), "Eliminowanie wpływów termicznych w pomiarach strzałki zwisu lin" (Elimination of Thermal Effects in Rope Sag Measurements) - in Polish, English summary, Zeszyty Naukowe AGH, Geodezja, Z.73, Kraków 1982 pp 33 - 41.

Gogolińska, M., Gogoliński, W. (1976) "Krzywa łańcuchowa" (Catenary curve) in Polish, Przegląd Geodezyjny No 7/1976 pp 257 - 259.

Kolondra, L. (1984) "Próba kompleksowego opracowania wyników terrofotogrametrycznego pomiaru sił naciągowych masztu antenowego" (Complex Attempt to Study the Results of Terrophotogrammetric Measurements for Determining Tension Forces of Guy-Ropes of an Aerial Mast), in Polish, English summary, Zeszyty Naukowe AGH, Geodezja, Z.84, Kraków, 1984 pp 31 - 44.

Ziobro, J. (1985), Programy "PRIM" i "LINA" w opracowaniu Zespołu Rzeczoznawców SGP ("PRIM" and "LINA" Programmes), in Polish, typescript, Warszawa 1985.