

## SYSTEM CALIBRATION VERSUS PHOTOGRAMMETRIC REFRACTION

### ABSTRACT

Nowadays, instruments to measure photo coordinates show a discrimination of half a micrometer. To avoid loss of accuracy, systematic errors must be corrected at the same level. Among the sources of systematic errors is the photogrammetric refraction. The definition of photogrammetric refraction permits to distinguish different components of the phenomena. Recent studies have been demonstrated that the component of refraction due to variation on atmospheric density is not the most important. Other source of refraction - the disturbed air caused by the aircraft displacement - can deviate the optical path ten times more than the atmosphere. If such systematic errors are important for the task of aerial triangulation, it becomes crucial on the procedures of aerial camera calibration as system calibration.

Lens distortion is the non-desirable part of the refraction caused by the lenses. As consequence, both lens distortion and photogrammetric refraction are phenomena of the same nature and consequently physically correlated.

This paper shows how to solve the problem of refraction when calibrating the photogrammetric system. The key results of cameras calibrated by the developed technique show that the level of accuracy reached is compatible to the accuracy of the measured photo coordinates - 1 micron.

### 1. INTRODUCTION

The world's photogrammetric community nowadays has instruments capable of measuring photo coordinates within half a micrometer. However, the accuracy of photogrammetric works doesn't reach the level which those observations can allow. In the last decade D.C. Brown wrote "Much very basic work needs to be done in the formulations and testing of error - Models to be used... For example, ... the precise influence of refractive anomalies caused by atmospheric turbulence needs further study".

Studies in this area were the object of the study for the PhD dissertation of J.B. de Andrade, who modelled the photogrammetric refraction, taking into consideration all its causes, including the atmospheric turbulence caused by the displacement of the aircraft (Andrade, 1977). The proposed model by Andrade was tested by himself on the testing field of Rheidt in Germany showing results for the photogrammetric refraction totally different from those obtained through the standard atmosphere; but, the results of the aerotriangulation show drastically reduced errors, mainly on altitude where it diminished by an average of 60% and in many cases the error was ten times smaller.

By the other side, in the field of treatment of systematic errors in photogrammetry we may count with the tools offered by the methods of camera calibration.

In the area of instrument calibration and the standards Eisenhart, (1963) gives a collaboration which drastically changes the concepts accepted until this time. Searching for a method of calibration which was capable to

attend the concept of system calibration, established by Eisenhart. Merchant, (1972) created the "Method of Mixed Ranges" which revolutionized the subject. This method was improved by Andrade, (1981) in the following aspects:

- a) to use only 1 test range.
- b) use either distance between targets or geometric levelling.

These improvements were possible due to the studies realized on rank deficiency of the design matrix.

The rank deficiency was found to be 8. Therefore, it would be necessary to apply 7 constraints for the realization of the referential frame and 1 to break the correlation between the camera constant and the flying height as a minimum.

This method of calibration assumes that all the methods of calibration, except those concerning the lens system, are appropriately treated either by pre-correction or by modelling and parametrization. Concerning photogrammetric refraction, the parametrization (as developed by Andrade, 1977) was impracticable due to correlations between photogrammetric refraction and the symmetrical radial distortion. The adapted way was the "a priori" correction of the photogrammetric refraction using a standard atmosphere, such procedure made the process viable in spite of the fact that the residuals remained disagreeable. A new way must be followed in order to make the size of the residuals concord with the precision of the observations involved in the process.

Two approaches were tried to solve the problem: the first was to regroup the parameters of the mathematical model for photogrammetric refraction - aiming to break the correlation between parameters of refraction and symmetrical radial distortion; the second was to "a priori" processing the normal equations augmented by the photogrammetric refraction model. In spite of the fact that one of the approaches was unsuccessful, it will also be discussed to make clear important points for the complete understanding of this problem.

## 2. PHOTOGAMMETRIC REFRACTION

The angle between the collinear and the target to the actual ray measured at the exterior node, and taken as positive for outward radial displacement of the image, caused by the atmospheric refraction, receives the name of the photogrammetric refraction. We consider only the variation of atmospheric refraction index between the terrain and aircraft (camera), the photogrammetric refraction would always be positive except when there are large inversions in the refraction index. Its maximum value of photogrammetric refraction would be 28 micrometers at sea level and flying heights of 16 km. However, other factors must be taken into consideration: the perturbed air caused by the displacement of the aircraft; the difference of pressure on the filter faces caused by the turbulence of the air in front of the camera; and the difference in air pressure inside and outside when the aircraft is pressurized. Andrade (1977) developed a mathematical model capable of describing the global angular deviation of a ray of light before entering the camera lens system.

$$\epsilon = \frac{\xi}{Z_s - Z_p} (\operatorname{tgn} \alpha - \theta \sec^2 \alpha) \ln \frac{n_s}{n_p} \quad (2.1)$$

where:

$\epsilon$  = photogrammetric refraction

$\xi$  = height of the mass center of the air column between the terrain and the aircraft

$Z_s$  = flying height

$Z_p$  = terrain height

$\alpha$  = nadiral angle

$\theta$  = angle between the boundary layer and the aircraft

$n_s$  = refractive index at the camera level

$n_p$  = refractive index at the ground level

$\theta$  = null, for subsonics velocities

This paper treats only subsonic velocities:

$$\delta x = \epsilon^* \left(1 + \frac{r^2}{c^2}\right) x \quad (2.2)$$

$$\delta y = \epsilon^* \left(1 + \frac{r^2}{c^2}\right) y \quad (2.3)$$

For supersonic velocities the above corrections would be:

$$\delta x = \epsilon^* \left(1 + \frac{r^2}{c^2}\right) x - \epsilon_1^* \left(1 + \frac{r^2}{c^2}\right) \quad (2.4)$$

$$\delta y = \epsilon^* \left(1 + \frac{r^2}{c^2}\right) y - \epsilon_2^* \left(1 + \frac{r^2}{c^2}\right) \quad (2.5)$$

Where:

$\epsilon^*$  = photogrammetric refraction for  $\alpha = 45^\circ$  and subsonic velocities

$\epsilon_1^*$  e  $\epsilon_2^*$  = photogrammetric refraction coefficients (null for subsonic velocities)

Note that for supersonic flights an additive term appears, which is independent of the image coordinates. We are going to analyze only the case of subsonic flights. The atmosphere perturbed by the aircraft adds refractive effects on the light rays entering the camera. These phenomena are strongly related to the type of aircraft in use. The Boundary-Layer Theory (Schlichting, 1968), shows that in a flat plate at zero incidence in parallel flow, the speed of the air varies from zero to a constant value equal to the speed of the flow.

The changes of the speed can be described by Schlichting (1968).

$$\frac{\Delta \rho}{\rho} = \frac{1}{2} M^2, \text{ in the worst situation} \quad (2.6)$$

Where:

$\Delta \rho$  = change in density,

$\rho$  = density,

$M = \frac{v}{c} = \text{"mach number"}$

$v$  = velocity of the flow,

$c$  = velocity of the sound

Based on the above relation, we can find the change in the refraction index due to the flow speed:

$$\frac{n - 1}{n_0 - 1} = \frac{\rho}{\rho_0} = \frac{\rho_0 + \Delta\rho}{\rho_0} = 1 + \frac{\Delta\rho}{\rho} = 1 + \frac{1}{2} M^2 \quad (2.7)$$

or

$$n = 1 + (1 + \frac{1}{2} M^2) (n_0 - 1) \quad (2.8)$$

Where:

$n_0$  = refraction index at  $V = 0$

The conditions in which (2.6) is valid are ideals (laminar flux parallel to a flat plate). Below is an example:

$c = 335,28$  m/s - sound velocity

$v = 400$  Km/h - aircraft velocity

$Z_s = 3.000$ m - flying height

$n_0 = 1,00020543$  - refraction index at the flying height

$n = 1,000239436$

$$\epsilon_{45}^* = \frac{3000 - 1000}{3000 - 1000} \ln \left[ \frac{1,00020543}{1,000239436} \right]$$

$$\epsilon_{45}^* = -3,3998 \times 10^{-5} \text{ radions}$$

$$\delta r = \frac{r^2 + c^2}{c} \epsilon_{45}^* = 2 \times 150 \times (-3,3998 \times 10^{-5})$$

$$\delta r = -0,010 \text{ mm}$$

In practice this value can be ten times greater, because the velocity of flow can be substantially augmented by the action of the propellers; the surface below the aircraft is not flat and the angle of attack is not zero; the flow may not be parallel and certainly isn't below the camera window. Typical values found by Andrade's tests realized in the Rheidt Test Field are in the order of  $\epsilon_{45}^* = -4 \times 10^{-4}$  radians, which corresponds to  $-0.120$ mm of image displacement for a nadiral angle of  $45^\circ$ .

### 3. SYSTEM CALIBRATION

Quoting Eisenhart:

Calibration of instruments and standards is a refined form of measurement. Measurement of some property of a thing is an operation that yields as an end result a number that indicates how much of the property the thing has. Measurement is ordinarily a repeatable operation, so that it is appropriate to regard measurement as a production process, the "product" being the members, i.e., the measurements, that it yields; and to apply to measurements processes in the laboratory the concepts and techniques of statistical process control that have proved so useful in the quality control of industrial production.

"Viewed thus, it becomes evident that a particular measurement operation cannot be regarded as constituting a measurement process unless statistical stability of the type known as a state of statistical control has been attained. In order to determine whether a particular measurement operation is, or is not, in a state of statistical control it is necessary to be

definite on what variations of procedure apparatus, environmental conditions, observers, operators, etc., are allowable in "repeated applications" of what will be considered to be the same measurement process applied to the measurement of the same quantity under the same conditions.

To be realistic, the "allowable variations" must be of sufficient scope to bracket the circumstances likely to be met in practice. Furthermore, any experimental program that aims to determine the standard deviation of a measurement process as an indication of its precision, must be based on appropriate random sampling of this likely range of circumstances.

Ordinarily, the accuracy of a measurement process may be characterized by giving (a) the standard deviation of the process and (b) credible bounds to its likely overall systematic error. Determination of credible bounds to the combined effect of recognized potential sources of systematic error always involves some arbitrariness, not only in the placing of reasonable bounds on the systematic error likely to be contributed by each particular assignable cause, but also in the manner in which these individual contributions are combined. Consequently the "inaccuracy" of end results of measurement cannot be expressed by 'confidence limits' corresponding to a definite numerical 'confidence level', except in those rare instances in which the possible overall systematic error of a final result is negligible in comparison with its imprecision...

Specification of the apparatus and auxiliary equipment to be used, the operations to be performed, the sequence in which they are to be executed, and the conditions under which they are respectively to be carried out - these instructions collectively serve to define a method of measurement.

A measurement process is the realization of a method of measurement in terms of particular apparatus and equipment of the prescribed kinds, particular conditions that as best only approximate the conditions prescribed, and particular persons as operators and observers...

It has long been recognized that, in undertaking to apply a particular method of measurement, a degree of consistency among repeated measurements of a single quantity needs to be attained before the method of measurement concerned can be regarded as meaningfully realized, i.e., before a measurement process can be said to have been established that is a realization of the method of measurement concerned. Indeed, consistency or statistical stability of a very special kind is required: to qualify as a measurement process a measurement operation must have attained what is known in industrial quality control language as a state of statistical control. Until a measurement operation has been 'debugged' to the extent that it has attained a state of statistical control, it cannot be regarded in any logical sense as measuring anything at all. And when it has attained a state of statistical control there may still remain the question of whether it is faithful to the method of measurement of which it is intended to be a realization".

The method of mixed ranges, conceived by Merchant (1972) is the unique applicable to airphotogrammetry which attends to the modern principles established by Eisenhart. The camera is calibrated on the same conditions in which it is used and all the parameters to define the metrical properties of the system of measurement (camera, aircraft, etc.) and the quality of its performance are determined.

The method of mixed ranges requires two test fields for calibration: one montaneous, with few signalized 3D-control points and another, flat to facilitate the installation of a great number of targets also with three coordinates carefully determined.

The accurate determination of 3D coordinates on a montaneous terrain (and even on flat terrain) makes the method difficult to be applied in practice. Those difficulties were overcome by the studies that Andrade started in 1977 at The Ohio State University, where was verified that the rank deficiency of the normal equation is 8 for this method - seven to materialize the reference system and one to provide vertical scale. Those studies were followed at the Federal University of Parana (Brazil). A test field (Andrade, 1981) was established in São Luiz do Purunã with 36 points - 30 on the higher level and 6 on the lower level of a scarped terrain, showing a difference in level of 200m in average. The following goals were reached:

- 1) The number of test fields were reduced from 2 to 1.
- 2) The number of photo-images of targeted points were multiplied by taking the photographs in four strips: (N-S); (S-N); (E-W) and (W-E), resulting 12 photographs times 36 ground points, or 432 image-points.
- 3) The 3-D coordinates for each targeted point were substituted by a first order levelling.
- 4) The reference system was arbitrary - only elevation were used to materialize it.

In this method the models for radial symmetric and decentering distortion were added to the projective equation (Bundle Method).

#### 4. PHOTOGRAMMETRIC REFRACTION VERSUS CALIBRATION

To calibrate the Photogrammetric System it becomes necessary to "a priori" correct the other systematic errors or to add adequated parameters to the projective equations.

Concerning Photogrammetric Refraction, "a priori" correction was done using standard atmosphere due to the impossibility of separation of parameters to define refraction from those of symmetrical radial distortion. Refraction and lens distortion are physically the same phenomena and so correlated quantities.

Conrady's model of symmetrical radial distortion reads:

$$\delta r = K_1 r^3 + K_2 r^5 + K_3 r^7 + \dots \quad (4.1)$$

The model for refraction reads:

$$\delta r = \epsilon^* r + \frac{\epsilon^*}{c^2} r^3 \quad (4.2)$$

Comparing (4.2) and (4.1) one can see, that they differ by a constant factor:

Trying to break this correlation, a different agrupment of parameters were tried:

From Andrade (1972):

$$\epsilon^* = \frac{\xi}{Z_s - Z_p} \ln \frac{ns}{np} \quad (4.3)$$

$$Q = \xi \ln \frac{ns}{np}$$

$$\delta r = \frac{Q}{Z_s - Z_p} r + \frac{Q}{c^2 (Z_s - Z_p)} r^3 \quad (4.4)$$

This model was expected to work because  $Z_s - Z_p$  varies up to 200 meters on São Luiz do Purunã Test Field - about 10% of the flying height. However, because  $\xi$  approaches  $Z_s - Z_p$  due to the high air pressure below the aircraft, the correlation cannot be broken. The correlation coefficient becomes only 5% smaller with this approach. So it was abandoned.

The second approach tried was that of computing an "a priori" value of  $\epsilon^*$  by adding it to the projective equation and solving only for it and the elements of exterior orientation. The results were amazing. The values founded by Andrade (1977) for  $\epsilon^*$  were repeated and the residuals drop to the most optimistic levels.

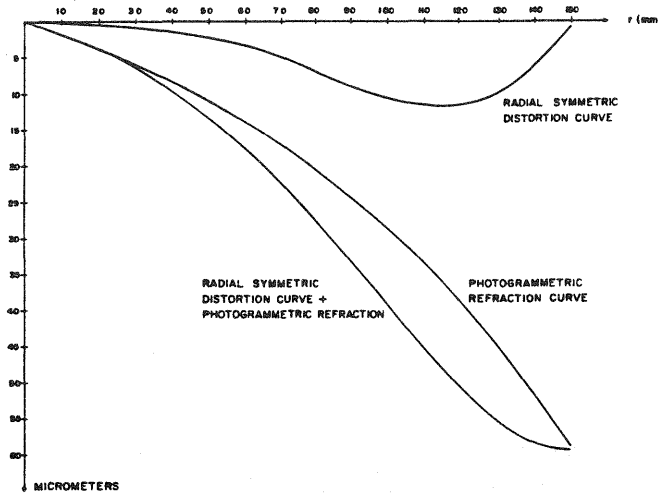


Figure 1: Photogrammetric Refraction and radial symmetric distortion curves

RESIDUAL IN PHOTOCOORDINATES					RESIDUAL IN GROUND COORDINATES				
POINT	V <sub>x</sub>	V <sub>y</sub>	POINT	V <sub>x</sub>	V <sub>y</sub>	POINT	V <sub>z</sub>	POINT	V <sub>z</sub>
3	-0,0026	0,0025	18	0,0002	-0,0004	1	-0,0031	20	0,0001
4	-0,0034	0,0002	19	0,0034	-0,0006	2	0,0006	21	-0,0002
1	-0,0007	0,0009	36	0,0020	-0,0014	3	0,0005	22	-0,0010
6	0,0015	0,0004	20	0,0002	-0,0014	4	-0,0014	23	-0,0011
8	-0,0011	0,0013	21	0,0027	0,0009	5	-0,0058	24	-0,0033
10	-0,0007	0,0035	22	-0,0041	0,0009	6	0,0029	25	0,0006
5	-0,0013	-0,0001	23	0,0010	-0,0045	7	0,0017	26	0,0012
11	0,0004	0,0008	24	-0,0012	-0,0011	8	0,0016	27	0,0007
12	0,0001	0,0022	33	-0,0004	-0,0012	9	-0,0023	28	0,0020
13	-0,0023	0,0007	25	-0,0046	0,0002	10	0,0036	29	0,0012
15	0,0021	-0,0003	38	-0,0012	-0,0028	11	0,0000	30	-0,0013
14	0,0016	0,0016	37	0,0015	0,0010	12	0,0014	31	0,0000
16	0,0033	-0,0022	26	0,0016	-0,0001	13	-0,0013	32	0,0008
35	0,0014	-0,0016	27	-0,0036	0,0008	14	0,0000	33	0,0018
17	0,0026	-0,0007	28	0,0009	-0,0019	15	-0,0024	34	0,0017
9	0,0009	0,0038	29	0,0021	-0,0023	16	0,0019	35	0,0017
7	0,0009	0,0003	30	0,0009	-0,0038	17	0,0011	36	-0,0001
2	0,0032	0,0007	31	-0,0032	-0,0021	18	-0,0009	37	-0,0041
32	0,0020	-0,0008	34	-0,0019	0,0009	19	-0,0011	38	-0,0003

Table 1: Residuals obtained through this method.

The relative accuracy reached was 1:1.000.000 of the flying height - the theoretical limit obtainable.

The values of photogrammetric refraction in four different experiments were:

$\epsilon_{45}$	$\sigma^2 \epsilon_{45}$	Camera	Scale	Aircraft	Velocity
$-4,160.10^{-4}$ rad	$4,239.10^{-9}$ rad <sup>2</sup>	Zeiss RMK-AR	1:8.000	-	-
$-4,265.10^{-4}$ rad	$6,7814.10^{-10}$ rad <sup>2</sup>	Wild RC-10	1:12.000	Navajo	300Km/h
$-5,199.10^{-4}$ rad	$8,376.10^{-10}$ rad <sup>2</sup>	Wild RC-5/RC-8	1:12.000	Seneca	300Km/h
$-5,068.10^{-4}$ rad	$5,542.10^{-9}$ rad <sup>2</sup>	Wild RC-5	1:12.000	Seneca	300Km/h

## 5. CONCLUSIONS

The results of this research show the possibility of performing aerial triangulation with a very high level of accuracy. Naturally, one cannot expect that in aerial triangulation the same level of accuracy as a test field may offer, can be reached. However, there is no doubt that the proper treatment of the systematic errors allowed compensatory good results. It is important to remember that the total effect of photogrammetric refraction (including air disturbed by aircraft motion) is on the order of 10 times greater than the one computed through a standard atmosphere and with opposite sign (negative).

## 6. REFERENCES

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