A PROPOSED POLYNOMIAL FOR GEOMETRIC RECTIFICATION OF ORBITAL SAR IMAGERY

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ABSTRACT

Geometrical characteristics of orbiting SAR imagery was described and geometric distortions were analysed. A proposed polynomial for geometric rectification of SAR images was developed, tested and compared with other mathematical models. A film of Seasat-SAR swath was used. About 100 ground control points were measured in both the SAR film and the 1:50,000 topographic maps. Transformation of image co-ordinates into ground co-ordinates was performed using the proposed polynomial and other mathematical models. The results show that due to the stability of the Seasat's platform the minimum number of control points can be used for geometric rectification of SAR imagery. Rectified SAR imagery may be used for map revision of very small scale topographic maps and for land-use mapping. Also, it was appeared that the geometric accuracy of SAR imagery is comparable with Landsat-MSS image accuracy.

INTRODUCTION

Seasat-1 was launched in July 26, 1978 by NASA for sea and ocean monitoring applications with main objective to measure global ocean dynamics and physical characteristics and to determine key functions for an operational system. On November 21, 1987, NASA declared that the Satellite lost after a failure in the power subsystem which occurred on October 9, 1987.

Seaset-1 was operating at an altitude of about 800 km in a near-polar orbit, and has carried five different sensors, one of these was Synthetic - Aperture Radar (SAR).

The SAR sensor is a single-channel Synthetic-Aperture with nominal wavelength of 25 cm. It was designed to provide images of ocean waves, sea and fresh water, ice and snow cover and of land surfaces. Its aim was to demonstrate the environmental monitoring capability under day, night and all weather conditions.
SAR was working as one-side looking (SLR) covering a 100KM swath from 200 to 300 km on the north side of the satellite track. This paper summarizes the SLR geometry and analyses geometric distortions and geometric accuracy tests in order to assess the possibility of using SAR imagery for very small-scale mapping.

**SLR GEOMETRY**

Ground co-ordinates of a target point 'P' are as shown in Fig.1. Image co-ordinates could be defined so that x-axis is in the direction of the spacecraft movement and y-axis is perpendicular and represents the timing of the receiving signals. In case of slant-range reading, y co-ordinate of an image-point "P" is proportional to the distance between the object-point "P" and the electrical centre of the Radar's antenna. Ground-range representation reduces slant-range into ground-range distance which is proportional to the orthogonal projection of the range vector. The X co-ordinates of an image point 'P' is a function of the time at which the object point "P" was imaged.

Geometric distortions of the SLR imagery caused mainly by; Earth curvature and rotation, relief and change in the platform's attitude. Other sources such as squint, interior orientation parameters and processing errors may cause additional geometric distortion.

**ANALYSIS OF GEOMETRIC DISTORTIONS**

Figures 2 and 3 show the effect of Earth curvature and relief displacement on ground-range where the combined distortion resulted from these two sources may be written as:

\[ Y = Y \left(1 + \frac{H}{(R+H)}\right) + Y\left(1 + \frac{h}{Y^2} \left(2H-h\right)\right)^{1/2} - 2Y \]  

For high altitude orbiting satellites such as Seaset and landsat, each rotation during platform movement will cause the real-flight path to be skewed by an Earth rotation angle (He); its value will depend upon the geographic position and the angular velocity of the orbiting platform. For a small section of SAR swath, the Earth rotation angle (He) could be assumed constant, so that its effect will be a displacement in the range-direction of the real-track which may be expressed as:

\[ dY = \text{He} \cdot X \]  

Where X is the co-ordinate in azimuth direction. Change in the platform attitude along the flight path will cause image distortion. Image displacement due to changes (errors) in the platform's exterior orientation elements (dX, dX, dZ, dw, dØ, dk) have been analysed by Leberl (1972) and can be expressed as:-
Fig. (1) SLAR co-ordinates system

Fig. (2) Earth curvature - $R = $ Earth radius; $S = $ slant range to point 'P'; $H = $ satellite altitude

Fig. (3) Relief displacement
\[ dx = a_0 Y + dk \cdot Y - d\phi \cdot H \]  
\[ dy = dZ H/S - dY Y/S \]  
where:

\( X, Y \) are ground co-ordinates of any target point,  
\( S \) is the slant-range,  
\( H \) is the platform's altitude, and  
\( a_0 \) is a constant represents changes (errors) in the platform speed.

**DERIVATION OF SAR RECTIFICATION POLYNOMIALS**

The collected effects of erroneous exterior elements on SAR imagery, Equations 3 and 4, together with the Earth curvature and rotation and relief, Equations 1 and 2, may be expressed as:

\[ dx = a_0 Y + dk \cdot Y - d\phi \cdot H \]  
\[ dy = dZ H/S - dY Y/S - 2Y + Y\{1 + H/(H+R)\}^{1/2} + Y\{1+h/Y^2(2H-h)\}^{1/2} \]  
(5)

Since the changes in the exterior orientation elements are not measured, they could be expressed as a function of the azimuth and applying this to Equations (5) with some approximations, one can obtains:

\[ dx = a_0 + a_1 X + a_2 Y + a_3 X^2 + a_4 XY + a_5 X^2 Y \]  
\[ dy = b_0 + b_1 X + b_2 Y + b_3 Y + b_4 X/Y + b_5 X^2/Y \]  
(6)

where:

\( dx, dy \) represent distortion in image co-ordinates and \( X,Y \) are ground co-ordinated of any target point.

**DATA AND OBSERVATION**

A film of SAR swath number 762 was used in this experiment. The full swath is divided into four subswathes 70 mm each with total swath width of about 100 km. Each subswath covers about 49 km width ground strip with about 33\% side overlap with the adjacent subswath. A section of that swath which covers the area of East Anglia in the UK was chosen to carry out this investigation. No geometric correction have been applied during the production of this SAR film.

About 100 ground control points were identified in both the SAR image and the 1:50,000 topographic maps. Coast lines, lakes, big canals and boundaries of woodlands were easy to recognize.
Image co-ordinates were measured on a stereocomparator with standard deviation of ± 20 Um image scale. Ground co-ordinates were derived from the 1:50,000 topographic maps with an accuracy of about ±15 M.

**GEOMETRIC ACCURACY TEST**

Geometric accuracy of SAR images was carried out by fitting image co-ordinates to ground co-ordinates by means of different mathematical models. Similarity and affine transformations in addition to general and derived (proposed) polynomials were used.

Geometric transformation was applied first for each zone individually. A block was formed by joining the four subswathes (zones) together by affine transformation using 7-Common points between each to adjacent zones. Image co-ordinates in the formed block, then, fitted into ground co-ordinates in the same way as in the first experiment.

Least squares method was used to determine transformation parameters and then residuals and root mean square errors (RMSE) at the check points were calculated and residual vectors were plotted.

**THE RESULTS**

Tables 1 and 2 show the obtained results of geometric accuracy tests as described in the previous paragraph. In these tables, 'N' is the total number of control points and 'K' is the number of points used for transformation.

Results indicated that geometric distortion in SAR images could be corrected successfully by polynomials. Ground control up to 10 points may be enough for SAR rectification.

Resulted average RMS error vector of about ±50 m after applying 7-elements polynomial is quite reasonable, bearing in mind accuracy of measured image and map co-ordinates.

The proposed polynomial gave results comparable to those obtained by the 7-elements polynomial however this polynomial requires less ground control points. The obtained accuracy is comparable to other SAR geometric accuracy tests carried out by other investigators.

**CONCLUSIONS**

The main conclusion is that increasing the number of ground control points has no significant advantage, once the right mathematical model have been applied.
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<th>Zone</th>
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TABLE (1):
ROOT MEAN SQUARE ERRORS IN METRES COMPUTED FROM RESIDUALS AT CHECK POINTS
AFTER FITTING IMAGE CO-ORDINATES IN EACH ZONE TO MAP CO-ORDINATES
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**Table (z)**

Root mean square errors in metres derived from residuals at check points after fitting image co-ordinates of the formed block to map co-ordinates.
Fig. (4) Residual distortion vectors in the SAR formed block after similarity and affine transformations.
Fig. (5) Residual errors at the formed SAR block after the proposed (theoretical) polynomials.
Due to the day and night and all weather operation capabilities of Imaging Radar Systems, SAR imageries offer a good solution for mapping areas in the third world with poor map coverage and where a weather problems exist.

Finally, it can be concluded, regarding obtained geometric accuracy of Seasat SAR imagery, that these accuracies meet the geometric accuracy requirements of planimetric mapping with scales of 1,150,000 and smaller.

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