Bundle Adjustment with Additional Parameters
for SPOT Stereopairs
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ABSTRACT

This paper reports on investigations which were carried out to
use BASICC, (Bundle Adjustment for SPOT Imagery of CSRSR at
Central University) to determine the satellite orientation and
ground coordinates for points of interest.   A data snooping
scheme is implemented in the procedure. Accuracy and reliabili-
ty analyses are included. Ground coordinates for control and
check points were measured on 1/5,000 scale base maps with a
digitizer.   Image coordinates for each point were measured in
the following three ways : (1)direct locating pixel position on
a digital image processing system, (2)using image correlation,
and (3)using 2-D least squares image matching. The influences
of different functional models for orientation parameters with
respect to time, different types of additional parameters,
different numbers and configurations for ground control points,
and different ways to measure image coordinates were studied.
The stereopair used in the study comprises two level 1A
panchromatic digital images with about $30°$ convergent angle and
covers about 60 km x 40 km in the area near central Taiwan.

1. INTRODUCTION

SPOT has been successfully launched in Feb. 1986 and since then
its HRV system provides photogrammetrists and cartographers
with systematic stereo satellite data in digital formats at
spatial resolutions (10m, 20m) approaching those required for
mapping tasks. The multispectral mode, 20m resolution, supplies
for the application in environmental remote sensing. While its
panchromatic mode, 10m resolution, was designed for mapping.
Because the viewing direction of each HRV can be varied through
$\pm 27°$ relative to the nadir and to the orbit plane, the base-
height ratio could be larger than one.   Together with its high
resolution, it makes the topographic mapping up to a scale of
1/50,000 possible. [1,2]
Determination for orientation parameters, image rectification,
DTMs generation, and orthophoto production are the major tasks
in digital mapping. While determination for orientation parame-
ters is the fundamental and is a must for the three subsequent
processes [3,4]. Analytical plotters have been widely used in
photogrammetric mapping projects, now can also be used to
handle SPOT data after some modifications. For instance, BINGO
program SPOT-MODULE has been successfully implemented on ZEISS
C-100 series [2]. However, it is only for processing analogue
film.
This paper is to use digital approach to establish a mathemati-
cal model to determine the orientation parameters for SPOT
imagery. Base on that we develop a self-calibrated bundle block
adjustment program "BASICC". The program can handle a large
block but only a pair was used to investigate the performance.

Four methods were used to determine the image coordinates :
reading the line/pixel number on an image processing system,
using image correlation and two different types of least
squares matching method to sub-pixel level. The planimetric
coordinates for ground control and check points were digitized
from 1/5,000 scale photo base maps. The elevation is interpola-
ted in accordance with contour lines.
This paper will describe the geometric characteristics for SPOT
imagery first, then mathematical model for adjustment, and a
case study. BASICC program also includes reliability analysis,
data snooping, and robust estimation for gross error detection.
The study area is located in an area near central Taiwan.

2. GEOMETRIC CHARACTERISTICS FOR SPOT IMAGERY

Each of HRV1 and HRV2 CCD linear array sensors is composed of
6,000 detectors. The size of each detector is 13 μm x 13 μm
which approximately corresponds to a ground area of 10m x 10m
in vertical sampling. The focal length is 1,082 mm. A full
scene consists of 6,000 lines. The sampling interval for two
consecutive lines is 0.0015 seconds. Due to its pushbroom
scanning characteristics, imaging projection in sample dire-
ction is perspective and is approximately parallel in flight
direction. This is shown in Fig.1

3. MATHEMATICAL MODEL

For exterior orientation parameters, 6 elements per line, we
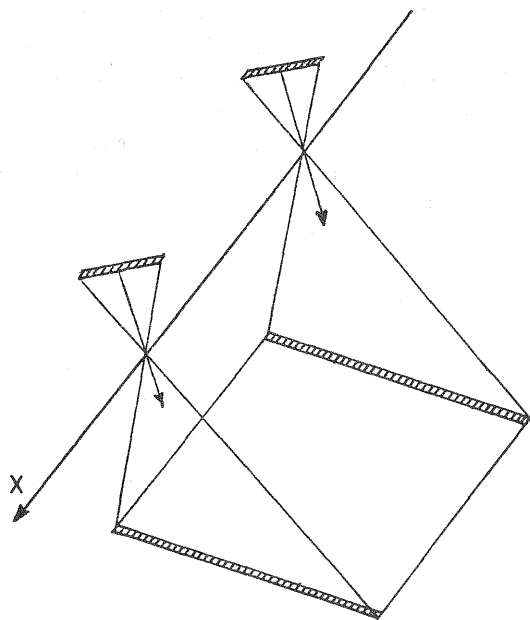have 36,000 elements. This large amount of parameters can not
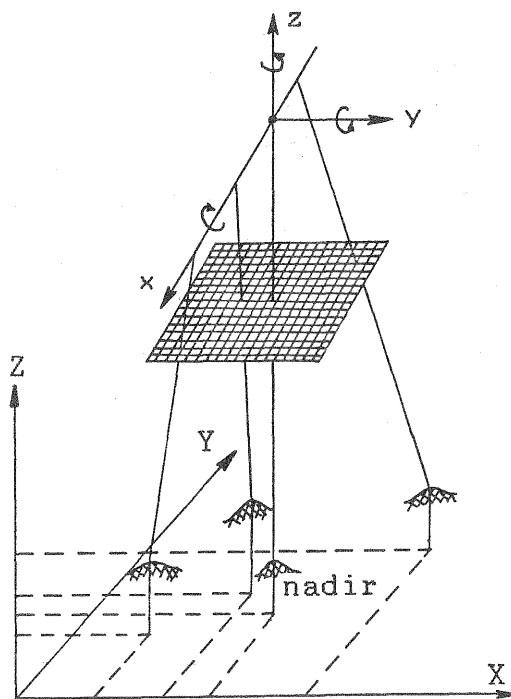
Figure.1 Imaging Geometry
for SPOT

Figure 2. The relationship
between object space and
image space for SPOT

be used directly and therefore need to be simplified. Due to stable and fast moving for the satellite, the parameters can be modeled as function of time. A lot of models have been used [2,5]. We select the functions up to the second order for orbit parameters $(\bar{X}t, \bar{Y}t, \bar{Z}t)$. After curve fitting for on-board attitude data, we found out that the third order polynomials gave a satisfactory result. Accordingly, we select the attitude function to the third order. We define the x-axis in flight direction and y-axis in sample direction for image coordinate system. The relationship between object space and image space is shown in Fig.2

3-1 Modified Collinearity Equation

For SPOT data, the collinearity equation are modified as:

$$\Delta x_i + o = -f \frac{m_{11t}(X_i - \bar{X}_t) + m_{12t}(Y_i - \bar{Y}_t) + m_{13t}(Z_i - \bar{Z}_t)}{m_{31t}(X_i - \bar{X}_t) + m_{32t}(Y_i - \bar{Y}_t) + m_{33t}(Z_i - \bar{Z}_t)}$$

$$\Delta y_i + y_i = -f \frac{m_{21t}(X_i - \bar{X}_t) + m_{22t}(Y_i - \bar{Y}_t) + m_{23t}(Z_i - \bar{Z}_t)}{m_{31t}(X_i - \bar{X}_t) + m_{32t}(Y_i - \bar{Y}_t) + m_{33t}(Z_i - \bar{Z}_t)}$$

(1)

where f : Focal length

t : Sampling time relative to the center line

$\bar{X}t, \bar{Y}t, \bar{Z}t$: Object space coordinates for projection center at time t

X_i, Y_i, Z_i : Object space coordinates for point i

$$\bar{X}_t = X_0 + X_1 t + X_2 t^2 \qquad \omega_t = \omega_0 + \omega_1 t + \omega_2 t^2 + \omega_3 t^3$$
$$\bar{Y}_t = Y_0 + Y_1 t + Y_2 t^2 \qquad \phi_t = \phi_0 + \phi_1 t + \phi_2 t^2 + \phi_3 t^3$$
$$\bar{Z}_t = Z_0 + Z_1 t + Z_2 t^2 \qquad K_t = K_0 + K_1 t + K_2 t^2 + K_3 t^3$$

(2)

m_{11t} to m_{33t} : Elements of rotation matrix at time t

y_i = [pixel number − (6000+1)/2] * 0.013 mm

$\Delta x_i, \Delta y_i$: Systematic correction for image coordinates

In this situation, the 36,000 elements are reduced to 21. In order to compensate for systematic errors, we introduced Δx and Δy which are functions of additional parameters to correct for earth curvature and affinity.

3-2 BASICC

The observation equations for collinearity condition can be obtained by linearizing eq. 1. Combining which with observation equations for ground coordinates and orientation parameters gives a complete mathematical model of the photogrammetric problem, i.e ; [6]

$$\begin{vmatrix} V \\ \dot{V} \\ \ddot{V} \end{vmatrix} + \begin{vmatrix} \dot{B} & \ddot{B} \\ -I & 0 \\ I & -I \end{vmatrix} \begin{vmatrix} \dot{\Delta} \\ \ddot{\Delta} \end{vmatrix} = \begin{vmatrix} \varepsilon \\ \dot{C} \\ \ddot{C} \end{vmatrix}$$

(3)

i.e;

$$\bar{V} + \bar{B}\bar{\Delta} = \bar{C}$$

(4)

Hence, the least square solution will be

$$\bar{\Delta} = (\bar{B}^T \bar{W}\bar{B})^{-1} \bar{B}^T \bar{W}\bar{C}$$

(5)

3-3 Reliability and Blunder Detection
- Data Snooping [6]

Var-Cov matrix of residual is $Qvv = Q11 - AQxxA^T$.
Internal Reliability index is $ri = (QvvP11)ii$.
Assuming that the standardized residual Wi of the observation

$$w_i = \frac{v_i}{s_{vi}} = \frac{v_i}{s_0} \cdot \sqrt{\frac{p_i}{r_i}} \tag{6}$$

is a standard normally distributed variable, the null
hypothesis $H_\circ$, that no gross error exists in observation
li is rejected if $|Wi| > K$, where K is a critical value.

- Robust Estimation [7]

Weight function is

$$P = \begin{array}{ll} 1 & \text{when } |Vi| < 2\sigma_0 \\ \text{EXP}[-0.05(|Vi|/\sigma_0)**4.4] & \text{when } |Vi| > 2\sigma_0 \text{ and } \\ & \text{for the first 3 iter.} \\ \text{EXP}[-0.05(|Vi|/\sigma_0)**3.0] & \text{when } |Vi| > 2\sigma_0 \text{ and } \\ & \text{for futher iter.} \end{array} \tag{7}$$

3-4 Image Coordinate Measurement

Pointing the line/pixel number on an image processing system
manually can reach the reading at one pixel level. In order to
measure the image coordinates to sub-pixel level, we use two
kinds of image matching methods. [9,10]

- Normalized Cross Correlation [NCC]

The correlation coefficient is computed by

$$r = \frac{\Sigma\Sigma(T-\bar{T})(S-\bar{S})}{[\Sigma\Sigma(T-\bar{T})^2]^{\frac{1}{2}} \cdot [\Sigma\Sigma(S-\bar{S})^2]^{\frac{1}{2}}} \tag{8}$$

where

r : Correlation Coefficient
T,S : Gray values for target and search windows
$\bar{T},\bar{S}$: Average gray values for target and search windows

then the sub-pixel interpolation for maximum r is

$$\Delta x = \frac{r(x+1) - r(x-1)}{4r(x) - 2r(x+1) - 2r(x-1)} \tag{9}$$

- Least Square Image Matching
  (a) Without considering geometric distortion (LS1)

$$\Delta x = \frac{\Sigma\Sigma(I'_x \cdot \Delta I) \cdot \Sigma\Sigma(I'_y)^2 - \Sigma\Sigma(I'_y \cdot \Delta I) \cdot \Sigma\Sigma(I'_x \cdot I'_y)}{\Sigma\Sigma(I'_x)^2 \cdot \Sigma\Sigma(I'_y)^2 - (\Sigma\Sigma I'_x \cdot I'_y)^2}$$

$$\Delta y = \frac{\Sigma\Sigma(I'_y \cdot \Delta I) \cdot \Sigma\Sigma(I'_x)^2 - \Sigma\Sigma(I'_x \cdot \Delta I) \cdot \Sigma\Sigma(I'_y \cdot I'_x)}{\Sigma\Sigma(I'_x)^2 \cdot \Sigma\Sigma(I'_y)^2 - (\Sigma\Sigma I'_x \cdot I'_y)^2} \tag{10}$$

where
$$I'_x = (S(x+1,y) - S(x-1,y))/2$$
$$I'_y = (S(x,y+1) - S(x,y-1)/2$$
$$\Delta I = S(x,y) - T(x,y)$$

(b)Considering geometric distortion (LS2)

$$T(x,y) = h1 + h2 \cdot S(x,y;A) \qquad (11)$$

where
    A = Coefficients of affine transformation
    h1 = Shift parameter of gray values
    h2 = Scale parameter of gray values
We use bilinear interpolation in resampling.

4. CASE STUDY

The stereopair used in the study is shown in Fig.3 , The CSRSR
IDs for the pair are SPC049 and SPC050. The base-height ratio
is 0.57. The other related descriptions are shown in table 1.
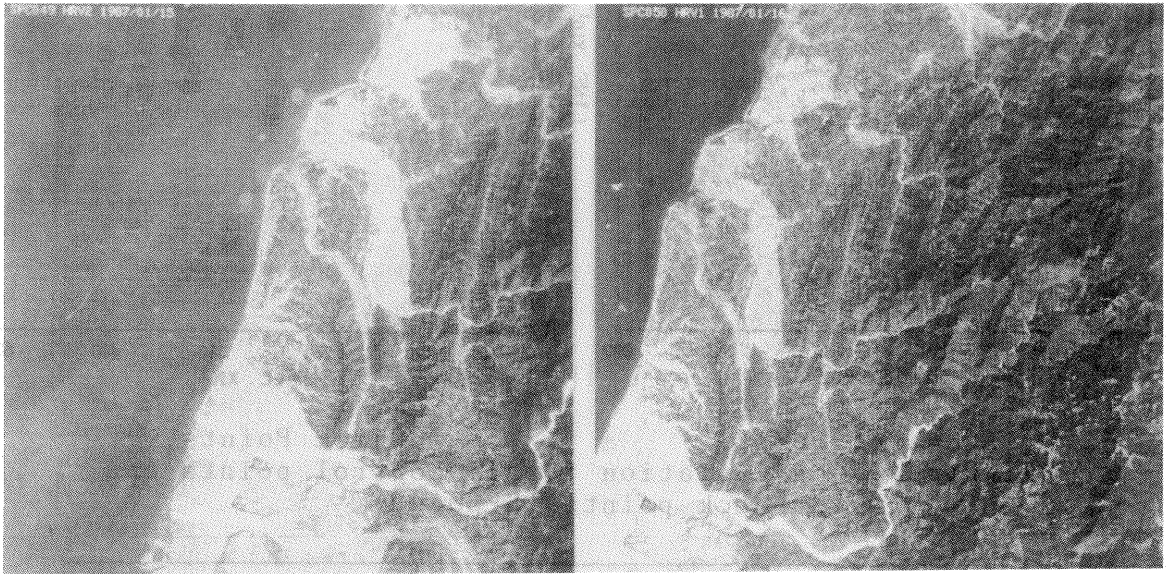
Figure 3. SPOT Stereo pair for case study

Scene ID	Sampling Date	Sensor	Incidence	Scene center
SPC049	1987.01.15	HRV2	L 10.4	N0242958 E1204303
SPC050	1987.01.16	HRV1	R 24.1	N0242958 E1205846

Table 1. General Descriptions for SPC049 and SPC050

Fifty six uniformly distributed and easily recognized points in
the stereomodel was selected as control points and check points
,then digitize it from 1/5,000 scale base map sheets with a
Calcomp 6000 digitizer. In image coordinate measurements, we
use IDIMS image processing system to read the line/pixel number
to pixel level and use image matching methods to calculate to
sub-pixel level. After deleting six bad points through data
snooping and robust estimation, fifty remaining points were
used in computation and analysis. Fig.4 shows the distribution
for 28 control points and 22 check points.

Table 2 and 3 compares the performances for different kinds of image coordinate measurements provided 28 control points and 22 check points. Which include (1) direct reading on IDIMS (2) NCC, (3)LS1, and (4)LS2. The sizes for different target windows are also shown in the tables. Where table 2 used the orbit parameter to the first order, while table 3 to the second order.

Control Points   Check Points

Figure 4. Configuration for 28 control points and
22 check points (SPC049)

Case	Method/ Window	Residual(RMS) line	pixel	Check Points(RMS) X(m)	Y(m)	Z(m)
1	IDIMS	0.335	0.394	7.454	4.105	9.103
2	NCC 13x13	0.338	0.410	7.198	4.127	9.570
3	NCC 19x19	0.335	0.406	7.755	4.684	8.839
4	NCC 25x25	0.355	0.468	7.953	5.132	10.639
5	LS1 13x13	0.318	0.404	7.812	4.183	8.489
6	LS1 19x19	0.321	0.405	7.647	4.627	8.292
7	LS1 25x25	0.347	0.441	8.045	5.153	10.928
8	LS2 13x13	0.296	0.417	7.564	3.901	8.759
9	LS2 19x19	0.289	0.421	7.402	3.945	8.629
10	LS2 25x25	0.293	0.427	7.583	3.941	8.799

Table 2. Performances of different ways of image coordinate
measurements (First order orbital model)

Case	Method/ Window	Residual(RMS) line	pixel	Check Points(RMS) X(m)	Y(m)	Z(m)
1	IDIMS	0.334	0.358	7.484	3.961	8.878
2	NCC 13x13	0.339	0.375	7.948	4.118	8.268
3	NCC 19x19	0.335	0.369	7.779	4.624	8.114
4	NCC 25x25	0.355	0.428	7.974	5.090	9.865
5	LS1 13x13	0.318	0.370	7.896	4.129	7.754
6	LS1 19x19	0.321	0.369	7.669	4.566	7.606
7	LS1 25x25	0.347	0.403	8.067	5.117	10.200
8	LS2 13x13	0.296	0.383	7.583	3.870	7.913
9	LS2 19x19	0.289	0.387	7.422	3.914	7.710
10	LS2 25x25	0.293	0.388	7.602	3.923	7.632

Table 3. Performances of different ways of image coordinate
measurements (Second order orbital model)

From table 2 and table 3, one can find that using LS2 with 19 x
19 target window using second order orbital model gives the
best results. The RMS in X,Y,Z are 7.4m, 3.9m, and 7.7m respec-
tively. Table 4 shows the values for orientation parameters in
the case. Fig. 5 and Fig. 6 show the error vectors of 22 check
points in X-Y, and Z.

*** Sensor Station's Orientation Parameters ***

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Omega-0     Phi-0     Kappa-0     XL            YL            ZL
Omega-1     Phi-1     Kappa-1     X1            Y1            Z1
Omega-2     Phi-2     Kappa-2     X2            Y2            Z2
Omega-3     Phi-3     Kappa-3     (m)           (m)           (m)
(units in deg, deg/t,....)
-------------------------------------------------------------------
SPC049
   8.80865  -2.00925  -10.18740  -34912.12  -239087.42  827927.38
   -.02395    .05678    -.02412    7328.06    -1251.82       46.18
    .00035   -.00009     .00447      -2.06      -10.40       28.73
    .00001   -.00001     .00162
   Scale Factor :   1.00330481

SPC050
 -21.32187   2.61362   -7.59898   34912.23   239089.23  827877.03
   -.00751    .05094    -.00917    7329.33    -1240.86       -4.89
    .00052    .00000    -.00068      -2.21      -16.77      -18.30
    .00001    .00004     .00236
   Scale Factor :   1.02873528
-------------------------------------------------------------------
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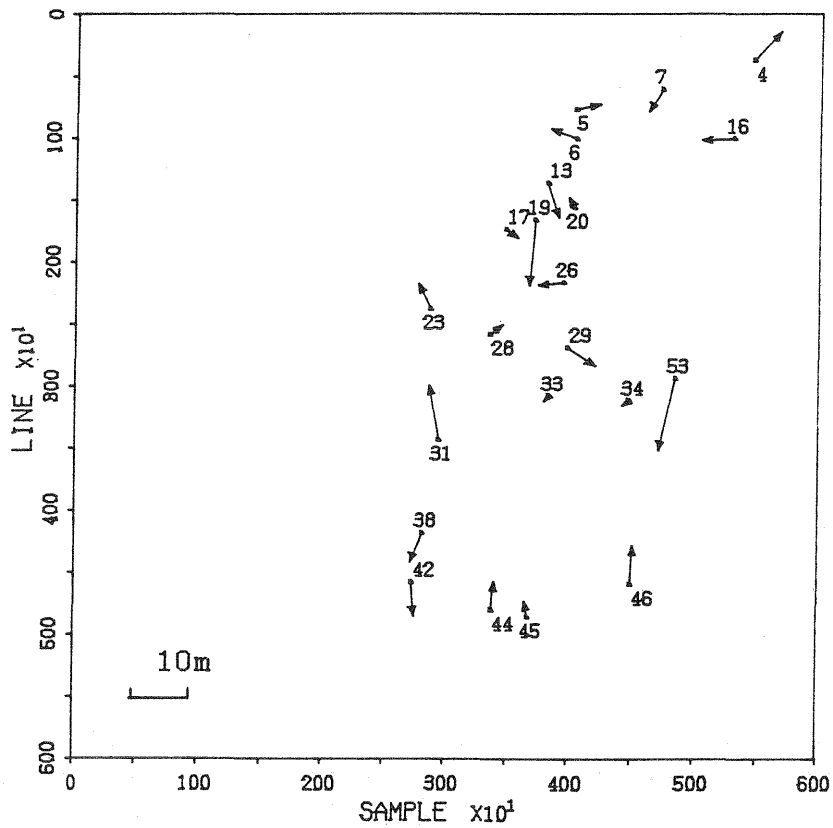Table 4. Orientation parameters for the stereo pair
(Case 9 of Table 3)
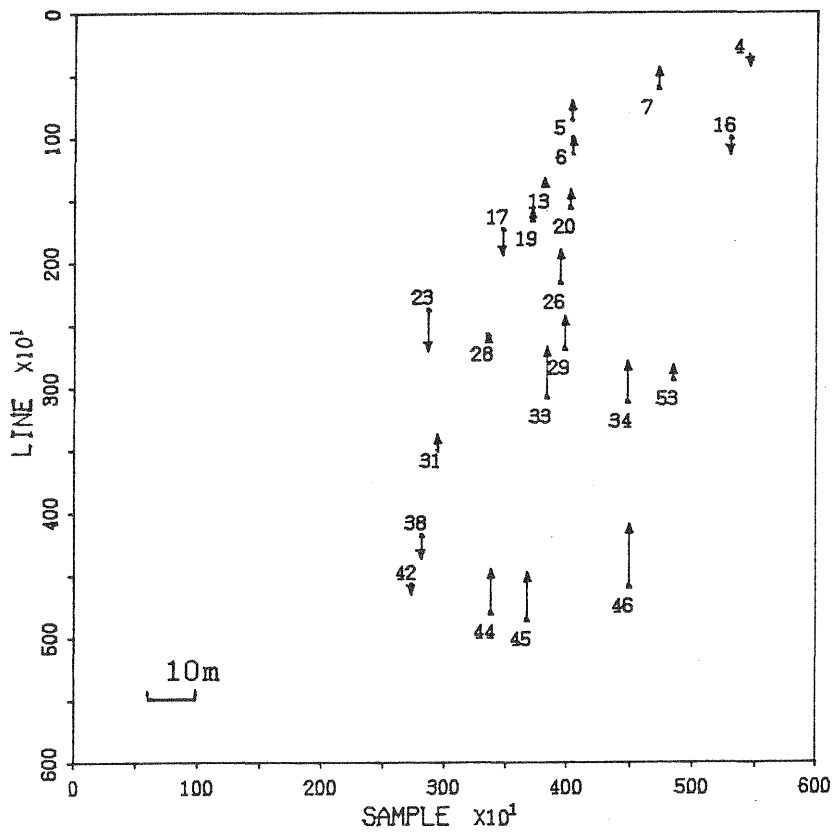
Figure 5. Planmetric error vectors for check points

Figure 6. Height error vectors for check points

Table 5 compares the influences of the systematic corrections for earth curvature and affinity. The condition is corresponding to case 9 of table 3. Table 6 compares the influence for the different number of control points.
As far as the computation efficiency is concerned, it only needs 10 seconds on a VAX-8650 computer or 4 minutes on an IBM PC/AT to execute the program with 5 iterations.

Affine Scale	Earth Curvature	Residual(RMS) line	pixel	Check Points(RMS) X(m)	Y(m)	Z(m)
no	no	0.349	9.198	14.752	71.658	210.356
no	yes	0.348	9.186	14.707	71.237	208.573
yes	no	0.334	0.363	7.483	3.954	9.325
yes	yes	0.289	0.387	7.422	3.914	7.710

Table 5. The influences of the systematic correction for earth curvature and affinity

Control Points	Check Points	Residual(RMS) line	pixel	Check Points(RMS) X(m)	Y(m)	Z(m)
15	35	0.189	0.241	8.091	5.268	9.239
20	30	0.225	0.337	7.657	4.453	8.136
28	22	0.289	0.387	7.422	3.914	7.710
35	15	0.309	0.371	7.416	4.103	7.705

Table 6. The influences for the different number of control points

5. CONCLUSION

1. The study indicates that using LS2 with 19x19 target window and second order orbital model can reach the best results with RMS in the level of 7.4m, 3.9m, and 7.7m. Although the study only refers to a single case study, the results allow general conclusions as to the high potential of using SPOT data in the small to middle scale mapping.
2. Introducing additional parameters to correct for earth curvature and affinity can indeed improve the accuracy in the adjustment. As for the orbital functional model, second order is better than first order.
3. The number of control points influences the accuracy significantly. Some 20 to 25 control points could be optimal in the adjustment.
4. Theoretically, using image matching can yield better results than direct reading. Some of our results support this but some do not. For example, LS2 with 19x19 target window has the best results. However, some of the other cases show contradictions on the argument. Before making a conclusion about this, one

should notice that the differences are marginal which are less than 1 meter. More importantly, error has been introduced in registration between the base map and the stereomodel for control and check points. This error could overshadow the differences between different kinds of image coordinate measuring methods. We have an ongoing project to investigate this.
5. Data snooping and robust estimation function well in blunder detection, it needs 4 to 5 iterations in robust estimation.

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