THE ACCURACY POTENTIAL OF SELF-CALIBRATING AERIAL TRIANGULATION WITHOUT CONTROL

A. Gruen, Prof. Dr.
A. Runge, Dipl.-Ing.
Institute of Geodesy and Photogrammetry
Federal Institute of Technology
ETH -Hoenggerberg
CH - 8093 Zurich, Switzerland
Commission III

Abstract

GPS data, acquired in a dynamic measuring mode, is becoming increasingly available for photogrammetric triangulation projects. For high accuracy block adjustments self-calibration is mandatory. This paper examines, with the use of synthetic and real block data, to what extent additional parameters and the parameters of interior orientation can be recovered in a block with poor control or even no control at all. GPS data is simulated. The results show that in future with the integration of GPS measurements the use of ground control points can be reduced drastically and will only be necessary for the transformation of the photogrammetric block into the ground coordinate system and for the recovery of the interior orientation. Standard sets of additional parameters can be determined with no control at all, given highly accurate GPS data.

1. Introduction

Photogrammetric point positioning has achieved a high level of accuracy and is competetive economically with terrestrial surveying methods in many applications. However, a major cost factor in photogrammetric triangulation projects is the acquisition of control points. In addition, control point coordinates are often not very reliable and covariance matrices for them are rarely available.

Although not primarily photogrammetric in nature, these problems often hamper the acceptance of photogrammetric point positioning techniques in practice, although its efficiency has been demonstrated in a lot of theoretical investigations and practical projects (e.g.: *Grün, 1986; Grün, Runge, 1987*).

Recent developments in satellite geodesy show new aspects for point positioning. With the satellites of the Navstar Global Positioning System (GPS) relative point accuracies in the order of centimeters or even millimeters can be achieved. The execution of the measurements is possible at almost any time and place. Intervisibility between the points is not required. These measurements in the static mode are used more and more in practice. But also the dynamic mode is of great interest to photogrammetry.

First experiments concerning the determination of the perspective centers of the camera or of a reference point during the photo flight mission have been performed with satellite receivers on board of an airplane and on the ground (*Mader, Carter, Douglas, 1986*). The results are quite encouraging. Accuracies of better than a decimeter can be expected in the near future.

One aim of this paper is to show the accuracy behaviour of the bundle block adjustment with measured elements of exterior orientation (control orientation containing control position and control attitude) with respect to different project parameters. The average standard deviations are chosen as accuracy indicators. They are directly obtained from the inverse of the normal equation system.

Another aspect of the paper is to discuss the possibility of applying the method of self-calibration if the number of control points is drastically reduced. For this purpose the set of 12 orthogonal parameters of Ebner (*Ebner, 1976*) is chosen. When using no control points at all the block adjustment results might be distorted by even small errors in the elements of interior orientation. Therefore the set is expanded by three parameters for the camera constant and the coordinates of the principal point. It is shown under what conditions the method of self-calibration can still be used in this case for the elimination of systematic image errors.

The presented results point out that for small and medium scale projects it is possible without loss of accuracy to tremendously reduce the number of control points if the elements of exterior orientation have accuracies in the order of decimeters. The remaining control points are necessary for the transformation from the satellite coordinate system to the ground coordinate system and for the reconstruction of the elements of interior orientation.
The investigation concerning additional parameters shows that even in blocks with only one full control point the method of self-calibration including three elements of interior orientation, can be employed for the elimination of systematic image errors.

2. Mathematical model

The basic functional model of bundle block adjustment is expanded by a parameter vector z for the set of additional parameters.

$$
\begin{aligned}
A_1x^N + A_2x^C + B_1t^N + B_2t^C + Cz &= l - e_B & D(l) &= \sigma_o^2\, P^{-1} \\
Ix^C &= l_c - e_xC & D(l_c) &= \sigma_o^2\, P_x^{-1} \\
It^C &= l_t - e_tC & D(l_t) &= \sigma_o^2\, P_t^{-1} \\
Iz &= l_z - e_z & D(l_z) &= \sigma_o^2\, P_z^{-1}
\end{aligned}
$$

The vectors of the conventional bundle parameters for object point coordinates and elements of exterior orientation are subdivided in order to be able to introduce control point coordinates (X^C, Y^C, Z^C) and control orientation (X_o^C, Y_o^C, Z_o^C, ω_o^C, ϕ_o^C, κ_o^C) as observed values.
With the assumption of uncorrelated observations the weight matrices are diagonal matrices. From the given standard deviations of the control points and the control orientations the individual weights can be derived. In this investigation the variance of unit weight is set to one and the standard deviation of the photo coordinates σ_o is assumed to be 3 microns. For the weight p_c of the control points we get

$$p_c = \sigma_o^2 / \sigma_c^2$$

and for the weight p_t of the control orientation

$$p_t = \sigma_o^2 / \sigma_t^2 \ .$$

For self-calibration the orthogonal 12-parameter set of Ebner is chosen which is expanded by three parameters (Δc, Δx_H, Δy_H) for the elements of interior orientation.

$$\Delta x = \frac{x}{c}\Delta c + \Delta x_H + xb_1 + yb_2 - 2kb_3 + xyb_4 + lb_5 + xlb_7 + ykb_9 + klb_{11}$$

$$\Delta y = \frac{y}{c}\Delta c + \Delta y_H - yb_1 + xb_2 + xyb_3 - 2lb_4 + kb_6 + ykb_8 + xlb_{10} + klb_{12}$$

with

$$k = x^2 - \frac{2}{3}b^2 \quad , \quad l = y^2 - \frac{2}{3}b^2 \qquad b = \text{photo base}$$

As in most cases there is no a-priori information about the additional parameters; they are introduced as free unknowns.

Written in a matrix equation the least squares solution finally yields

$$\begin{bmatrix} x^N \\ x^C \\ t^N \\ t^C \\ z \end{bmatrix} = \begin{bmatrix} A_1^TPA_1 & A_1^TPA_2 & A_1^TPB_1 & A_1^TPB_2 & A_1^TPC \\ A_2^TPA_1 & A_2^TPA_2+P_x & A_2^TPB_1 & A_2^TPB_2 & A_2^TPC \\ B_1^TPA_1 & B_1^TPA_2 & B_1^TPB_1+P_t & B_1^TPB_2 & B_1^TPC \\ B_2^TPA_1 & B_2^TPA_2 & B_2^TPB_1 & B_2^TPB_2 & B_2^TPC \\ C^TPA_1 & C^TPA_2 & C^TPB_1 & C^TPB_2 & C^TPC+P_z \end{bmatrix}^{-1} \begin{bmatrix} A_1^TPl \\ A_2^TPl+l_cP_x \\ B_1^TPl \\ B_2^TPl+l_tP_t \\ C^TPl+l_zP_z \end{bmatrix}.$$

From the diagonal of the inverse normal equation system we get the variances of the unknown parameters for the calculation of the average standard deviations.

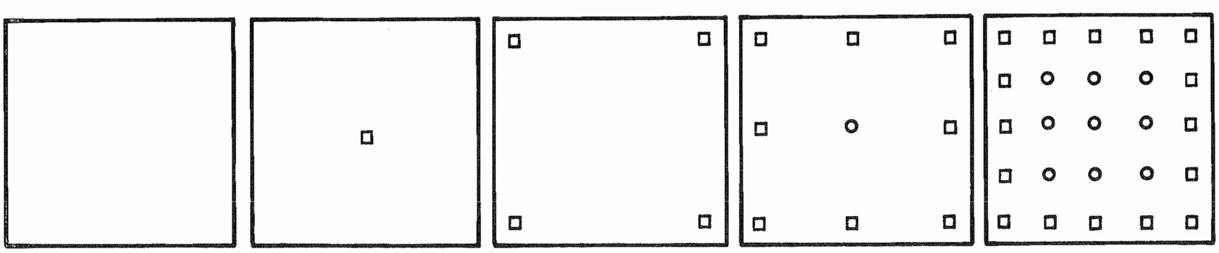
3. Investigations

For the investigations two different blocks are used. One is constructed from synthetic data ('SYBLOCK') and is therefore very regular, the other block ('HEINZENBERG') contains data from a practical aerotriangulation of an area with rather large height differences. Further details are given in Table 1.

Table 1: Block parameters

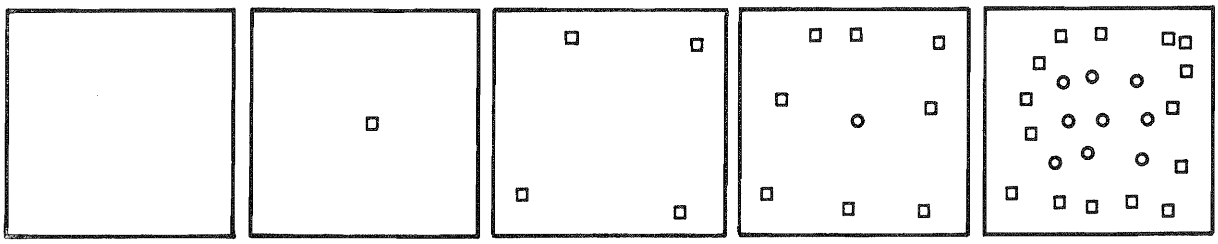
	SYBLOCK	HEINZENBERG
Point distribution	regular	irregular
Height differences in a model	flat terrain	up to 1,000m
in the block	flat terrain	up to 1,500m
Camera constant	150mm	153.18mm
Dimension of the block	48 km x 48 km	5.5 km x 5.5 km
Object points in a model	6	13 to 18
in the block (used)	99 (81)	131 (105)
Image scale	1 : 60,000	1 : 15,000
Forward / side overlap	60% / 60%	60% / 60%
Number of photographs	81	25

Figure 1 shows the control point distribution of the two blocks. Version P3 refers to a bridging distance of approximately two base lenghts. The control points are introduced as nearly errorfree with a standard deviation of 0.001m in planimetry and height.
For the perspective centers several assumptions for the standard deviations of the position are made. From these the accuracies of the orientation angles are derived by means of the flying height. The values range from errorfree to infinite and this value corresponds to the conventional form of block adjustment.



P0: No control **P01** : 1 FC **P1**: 4 FC **P2**: 8 FC, 1 HC **P3**: 16 FC, 9 HC

(a): SYBLOCK



P0: No control **P01** : 1 FC **P1**: 4 FC **P2**: 8 FC, 1 HC **P3**: 15 FC, 9 HC

(b): HEINZENBERG

Fig. 1: Distribution of control points

P0, ... ,P3 = Versions of control point distribution
FC = Full control point
HC = Height control point

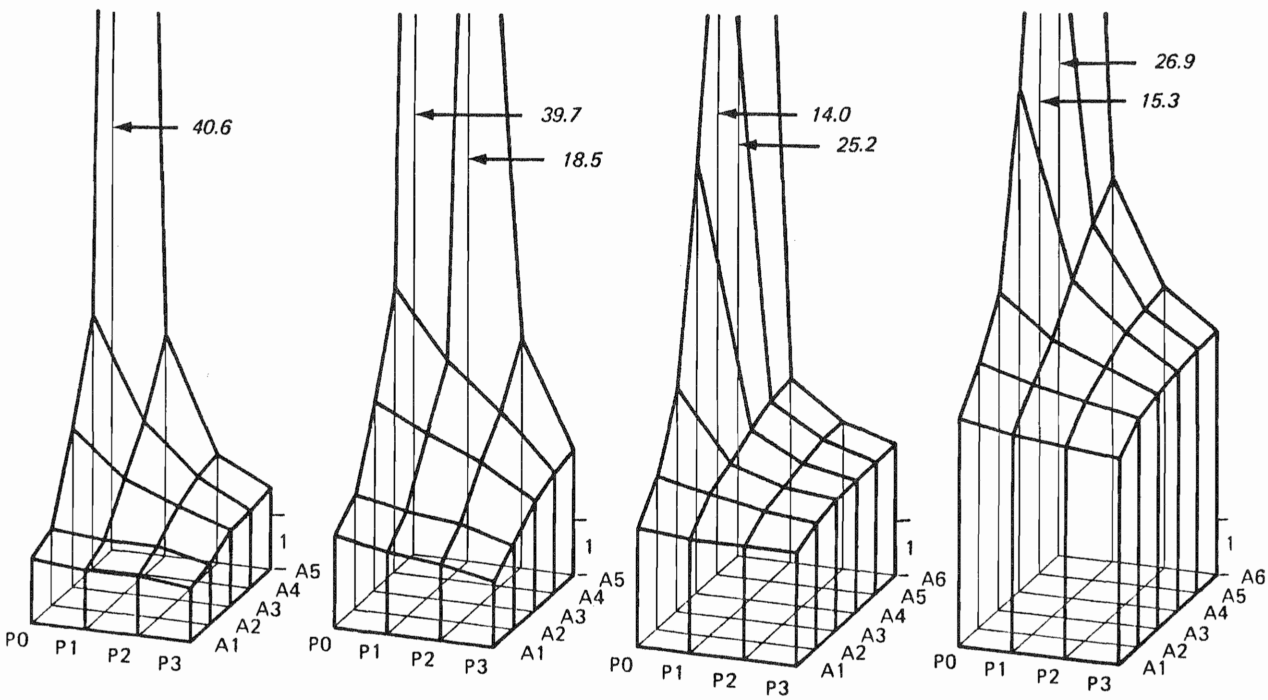
Table 2: Accuracy of exterior orientation

Control Orientation Version	SYBLOCK $\sigma_{Position}$ [m]	SYBLOCK $\sigma_{Attitude}$ [cc]	HEINZENBERG $\sigma_{Position}$ [m]	HEINZENBERG $\sigma_{Attitude}$ [cc]
A1	0	0	0	0
A2	0.1	7.1	0.045	12.7
A3	0.5	35.4	0.1	28.3
A4	1.0	71.0	0.25	71.0
A5	10.0	710.0	0.5	142.0
A6	∞	∞	1.0	283.0
A7			∞	∞

The elements of exterior orientation are introduced for all photographs of the block. When calculating the accuracy indicators, only the standard deviations of points which are situated inside the perimeter of the block are taken into account, because in general there is no interest in the coordinates of tie points outside the control perimeter. Finally in SYBLOCK 81 points and in HEINZENBERG 105 points are used, including some tie points inside the perimeter of the block.
The investigations are performed in two major steps. Firstly, different accuracies of the control orientation (A1, ... ,A7) for both blocks (SYBLOCK, HEINZENBERG) are investigated in combination with varying control point distributions (P0, ... ,P3). The calculations are performed with self-calibration based on the 12-parameter set.
Secondly, the expanded 15-parameter set is used with very few control points and varying accuracy of exterior orientation. In both cases the calculations are also performed without additional parameters in order to have reference versions.

4. Results

The results give a good insight into the accuracy structures of aerial triangulation systems with measured elements of exterior orientation and without control. For the visualisation of the accuracy indicators for the computed versions a three-dimensional dimetric illustration is selected. One axis represents the different accuracy levels of the elements of exterior orientation (A1, ... , A7), another axis shows the used control point distribution (P0, ... ,P3) and the vertical axis, the 'height', finally gives the average standard deviations of the adjusted object point coordinates. For better comparison all results are transformed into the image space.
Figure 2 shows the theoretic accuracy indicators σ_{xy} and σ_z for SYBLOCK and block HEINZENBERG. The values have been multiplied by an assumed a-posteriori standard error of unit weight of 3 microns. It is obvious that for poor control point versions (P0, P1) with weak control orientation the standard deviations increase very strongly. Denser control point distributions with a bridging distance of 2 to 4 base lengths give satisfactory results even without any control orientation. In order not to lose this accuracy level through a reduction of control points it is necessary to have more accurate values of the elements of exterior orientation. This will be explained in more detail by an example: With eight full control points and one height control point, which is equivalent to control point version P2, we get a standard deviation in planimetry in SYBLOCK of about 2 microns (Fig. 2(a)). This corresponds to 12 cm in the object space. The same result can be achieved without any control points but with elements of exterior orientation with standard deviations in the range of versions A2 to A3.



(a) Planimetry **(b) Height** **(c) Planimetry** **(d) Height**

SYBLOCK **HEINZENBERG**

Fig. 2: Average standard deviations in image space in micron, displayed in dependence of control point distribution (P0, ... ,P3) and accuracy of control orientation (A1, ... ,A6); $\sigma_0 = 3$ µm.

Whereas with satellite measurements the definition of the position will soon be possible fairly well, there are still problems to measure the attitude of the camera with a sufficient accuracy, for example by an inertial measuring system. Also the idea of deriving the rotation angles from the measurements of, for example, three satellite receivers in the airplane (*Hartl, Wehr, 1986*), will not yield results of sufficient accuracy at present. But nevertheless, theoretical studies which are not shown in detail here indicate that an omission of the rotation angles as observations results in only one and a half fold poorer results.

In general we get the same results for the data of HEINZENBERG (Fig. 2: (c),(d)) as for SYBLOCK. It is possible not to use control points at all if the elements of exterior orientation of the camera are given with a sufficient accuracy and if a small loss of accuracy of the object points is acceptable.
Figure 3 shows the standard deviations of object points in a block without control points. The x-axis is non-linear and gives the assumed accuracy of the elements of exterior orientation.
The parallels to the x-axis show the accuracies which are obtained in practice by a conventional photogrammetric point positioning method with a dense control point distribution, refering to σ_{XY} = 2.5μm and σ_Z = 4.5μm (*Grün, 1982*). In SYBLOCK, with an image scale of 1 : 60,000, elements of exterior orientation with an accuracy of .25m to .5m may already render control points unnecessary. For the height, accuracies of less than .5m are sufficient because of the homogeneous information for the whole block.
The accuarcy demands of 0.1m to 0.5m have up to now been realized in test projects for the position coordinates. It therefore seems to be possible to achieve these results in practical projects in the near future too.

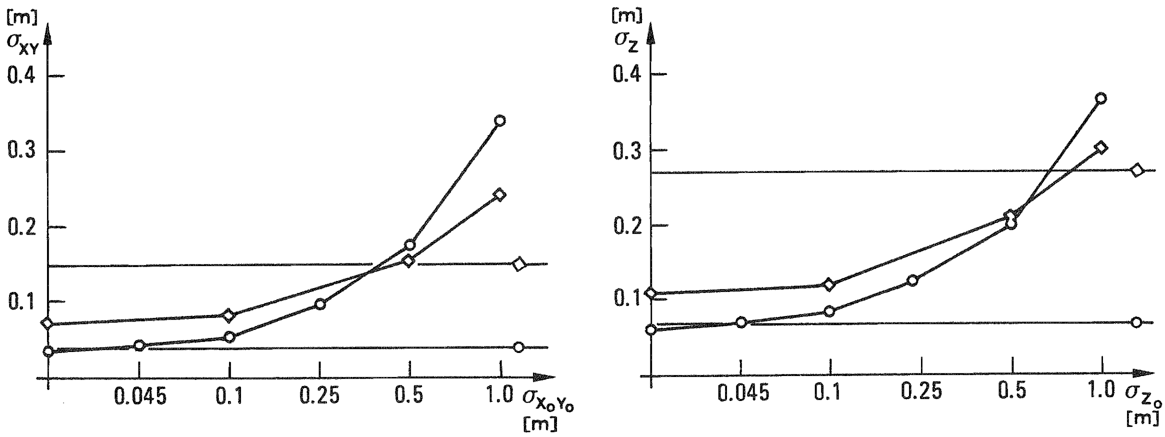
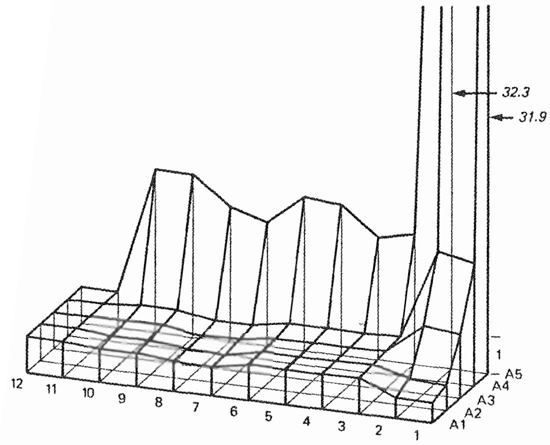
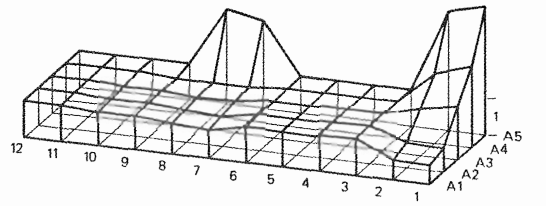


Fig. 3: Average standard deviations in object space (Version P0); σ_0 = 3 μm.
O HEINZENBERG
◇ SYBLOCK

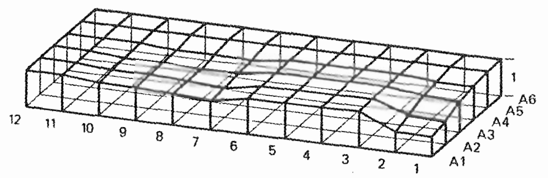
The results of block HEINZENBERG and SYBLOCK do not differ significantly. Therefore no further distinctions are made. Looking at the results of dense control point distributions (Fig. 4,5: (d)) no real improvement of the standard deviations of the additional parameters in relation to the reference version can be recognized. This fact is a hint that the choice of the reference version for the adjustment of the two blocks is acceptable. For versions with sparse control point distributions changes of the standard deviations are obvious only for poor control orientation (A4, ... ,A7) and without control points (P0). Refering to an acceptable deterioration factor of 3 the control orientation must be given in the range of versions A2 to A4 if no control points are introduced. These results are confirmed when using the standard deviations of the object points instead of the standard deviations of the additional parameters as a basis for analysis.
The conclusion of this first part of the investigation is that even in blocks without any control points the additional parameters are still determinable. It is therefore possible to apply the bundle adjustment with self-calibration to compensate systematic image errors, if GPS data is available with a reasonable accuracy.
The second part of the analysis refers to the influence of the elements of interior orientation on the adjusted object point coordinates. Table 3 shows the block adjustment accuracies separated in planimetry and height for computational versions without additional parameters (0), with 12 and 15 additional parameters (12, 15). The version 15 includes the three parameters for interior orientation. As control point versions P1 (four full control points in the block corners) and P01 (one full control point in the block center) are used. For the accuracy of the exterior orientation elements version A4 is applied.



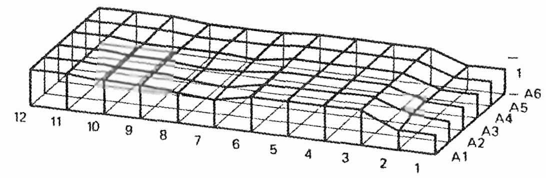
(a): P0: Without control



(b): P1: 4FC

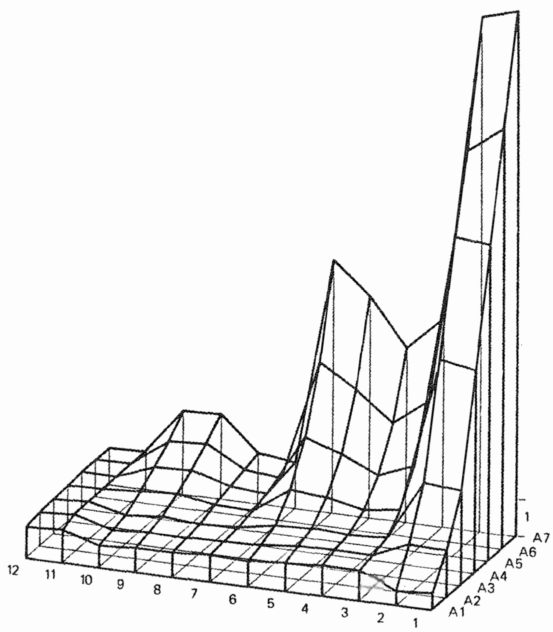


(c): P2: 8FC, 1HC

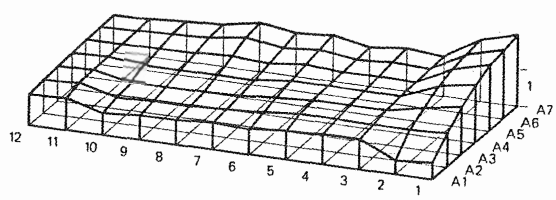


(d): P3: 16FC, 9HC

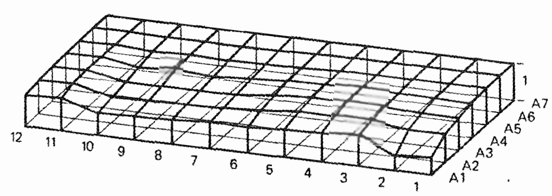
Fig. 4: SYBLOCK. Relative standard deviations of additional parameters



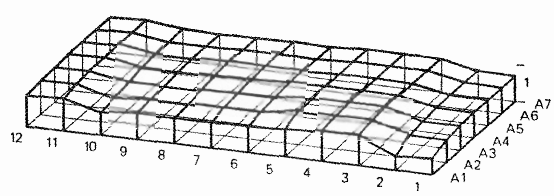
(a): P0: Without control



(b): P1: 4 FC



(c): P2: 8FC, 1HC



(d): P3: 15FC, 9HC

Fig. 5: HEINZENBERG. Relative standard deviations of additional parameters

The mean standard deviations of the object points indicate that with the use of four full control points the deterioration caused by both additional parameter versions is minor. Even with only one full control point the corrections to the coordinates of the principal point and the camera constant can be determined fairly well. However, one has to realize that the use of only one control point is a risky undertaking. Reliability is ensured only with 2 to 3 independently determined control points.

Table 3: Average standard deviations of object point coordinates, given in image space in micron, obtained with different additional parameter sets and exterior orientation version A4.

Version	Block	Planimetry			Height		
		0	12	15	0	12	15
P1: 4FC	SYBLOCK	2.5	2.8	3.2	4.0	4.0	5.1
	HEINZENBERG	2.9	3.1	3.2	5.7	5.9	6.2
P01: 1FC	SYBLOCK	3.5	4.3	4.4	4.7	4.9	5.7
	HEINZENBERG	4.1	6.0	6.5	6.2	7.2	7.5

5. Conclusions

New point positioning methods in geodesy by satellites in the dynamic mode will influence the way photogrammetry is executed today. The results of this paper show that the number of control points can be reduced down to a bare minimum of one if the elements of exterior orientation are measured with sufficient accuracy even in the case if a self-calibrating bundle adjustment with a standard additional parameter set including three elements of interior orientation is used. This control point is required for the reconstruction of the elements of interior orientation. In order to ensure sufficient reliability and for the transformation of the GPS coordinate system into a geodetic reference the use of 3 control points is recommended.
The presented values are to be regarded as a theoretical basis for further investigations. The indicated potential of point positioning has to be proved by practical projects.

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