AN APPROACH TO FORMING DTM WITH THE FIELD-SAMPLING POINTS

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Abstract

The field-sampling points are generally the feature points of the terrain which are relatively few in quantity.Although the possibility of forming DTM with the feature points was mentioned before, few conc-rete methods have been presented so far.An approach to forming DTM with the field-sampling points and the links among them is developed in this paper.The basic idea of the method is: 1)forming anirregular triangle network; 2)giving each triangle subarea in the network a proper function of plane co-ordinates according to its characteristics,and the curve surfaces defined by these functions should link relatively smoothly one another. 3)interpolating the heights of regular grid points to form DTM.The accuracy of the DTM obtained in this method is also briefly discussed.It is discovered from the experiment of a test area that the DTM's accuracy is relatively high with the maximum error 1.65m.As far as the applications are concerned,the large scale topographic maps and the perspective views can be obtained. The programmes involved in the procedure of forming and applying DTM are written in FORTRAN 77.The method discussed in this paper is also available to the feature points obtained photogrammetrically.

Introduction

Since the concept of Digital Terrain Model(DTM) was proposed in the late 1950s,a lot of studies have been made on the methods of creating DTM and its applications by scientists all over the world.So far,it is admitted that original data to create DTM can be mainly obtainedby three different ways:

1) sampling from the maps by digitizing the cotours;
2) sampling from the sterero-images photogrammetrically;
3) sampling directly from the terrain.

So far,the original data used in theoretical and appliable studies of DTM have been mainly obtained photogrammetrically.The examples of obtaining original data by field-sampling are hardly found. This is because the field-sampling instruments were not advanced enough to survey effectively in field,another reason is that the heights of grid points,which are conveniently managed and applied, can be directly obtained photogrammetrically,but this is impossible for field-samplingmethod.Even then,there are some deficiencies in the photogrammetricmethod.It is necessary for photogrammetric method to expirence all of the steps in ptotogrammetry,which are compratively complicate in order to create a DTM. This method takes relatively more time to cre-ate a DTM.Because of the limitation of accuracy,the DTM obtained photog-rammetrically is only used in the ocassion of small or middle scale applications.

In the past years,Intelligence Theomat became popular in many countries.This made it possible to get the 3-D co-ordinates of field points more rapidly and record them automatically.The method of field-sampling,therefore,became more effective than before,and the reality of field-sampling method to create DTM are generally realized.To a re-latively small area,field-sampl-ing is more rapid and accurate than any other sampling methods.So field-sampling method to createDTM is more available to the engineering items which require highaccuracy and large scale.On the other hand,many surveyors,presently,are concerning and studying on the methods of getting topographic maps automatically with field-sampling points with the help of computer.Undoubtly,the DTM created with field-sampling points gives an effec-tive way to get large scale topographic map automatically.

Procedure of Field-sampling

It is impossible for field-sampling to survey terrain points in a dense regular grid like photogrammetric method.A relatively effective sampling strategy for field-sampling is that only the feature pointsin the concerned area are sampled.In general,these feature points arerelatively few in quantity.So this sampling strategy is fit for field-sampling.The flow chart of field-sampling is showed in fig. 1.

In the process of sampling,it is necessary to record by hand the links among the sampling points,which include feature lines (ridge line or drinage line)and some indespensable link lines in or der to form irregular triangle network,besides recording the 3-D coordinates of sampling points automatically.For the sake of discrimination,feature lines are recorded with real-lines , and the un-feature lines arerecorded with lines of dashes.An example of recorded irregular tri-angle network is showed in fig.2.

In fact,to form an irregular triangle network is to divide the whole area into tringular subareas.The triangle network have some characteristics as follows:

1) All of the triangle vertices are sampling points and the link lines only intersect at sampling points.

2) The relation between any two triangles in the network is surely one of the following cases: only a pair of vertices coincide each other;two pairs of vertices (a pair of side) coincide each other;no part of two triangles coincide.

3) The whole area are composed of the divided triangular subareas.As a matter of fact,this kind of division is called real tringulation division in mathematics.

A Few Conclusions About Terrain

The shapes of terrain surface are greatly varied. In order to describethe terrain in each triangular subarea in the network with a interpolating function,it is necessary to find the common rules hidden in the varieties of terrain.By analysing the terrain and its contours,We discovered some rules about terrain which will be discussed in the following paragraphs.

1.Types of Terrain surface

In spite of variety of the terrain surface,the contours of the terrain between two feature lines are surely one of the four types which are showed in fig.3. The typical curves of the 4 types of contours are showed in Fig 4. The corresponding section curves are showed in fig.5.Obviously,4 types of the curves showed in Fig 5 are similar to corresponding curves of the functions as follows:

(i) $Z = A\sin(\pi x/s)$

(ii) $Z = -A\sin(\pi x/s)$

(iii) $Z = A\sin(2\pi x/s)$

(iv) $Z = -A\sin(2\pi x/s)$

where

S is distance between two ends of the curve;

X is abscissa (x co-ordinate) of a point on the curve;

A is amplitude of the Sine function and is positive.

2. Amplitude of the Terrain Surface

Besides the discrimination of terrain types, the variety of the amplitude of terrain surface is also an important reason for the variety of terrain. The amplitude is affected mainly by following factors:

1) the distance between two ends of the curve, S.A is directly proportional to S.

2) the slope of feature line, K.Generally,A is also directly proportional to K.

3) the angle between two adjacent triangle plane (See fig.6) , α. A varies with α . If α increases,A also increaes; If α decreases,A also decreases. α ranges from 0 to π, i.e.

$$0 < \alpha < \pi$$

It is regarded that A is directly proportional to Sin(/2) . Therefore, the formula which describe the relation between A and , K,S is

$$A = Kc \cdot S \cdot K\sin(\alpha/2) \qquad (2)$$

where Kc is a constant, which is relate to the characteristics of terrain, and generallly ranges from 0 to 0.5.i.e.

$$0 < Kc < 0.5$$

Formula (2) is for type 1 and type 2. Similarly, the formula for tpye 3 and type 4 is

$$A = 0.5Kc \cdot S \cdot K\sin(\alpha/2) \qquad (3)$$

The trianglular Subarea concerned generally links to another triangular plane on the each side of it. So there are two amplitudes for the concerned triangular area. See Fig 7.The amplitude for left side is signed Al, and the amplitude for right side is singed Ar, therefore,

$$Al = Kc \cdot S \cdot Kl \sin(\alpha l/2)$$
$$Ar = Kc \cdot S \cdot Kr \sin(\alpha r/2)$$

Where Al,Kl,αl are for left side,Ar,Kr,αr are for right side. The ultimate amplitude A is taken as

$$A = PlAl + PrAr$$

Where

$$Pl = 1 - (X/S)^n$$

$$Pr = (X/S)^n \qquad (n > 1)$$

Pl and Pr are two weight functions, Pl is for left and Pr is for right.

3. Concaveness and convexity of Terrain

The terrain surface on each side of the triangular subarea is surely convex or alternatively concave. In fig.8, we discuss the link line ab which is the intersect line of two adjacent triangular plane abc and abd. α is the angle between the two triangular plane. The terrainsurface is convex if $\alpha > \pi$. Particularly, the terrain surface is even if $\alpha = \pi$; On the contrast, the terrain Surface is concave if $\alpha < \pi$. Particularly, hte terrain Surface can be regarded as convex or concave with the amplitude A=0 when $\alpha = \pi$. According to the defini-tion of "convex" and "concave", the typical curves corresponding to the four types of terrain discussed above are respectively as follows.
Type 1: Convexo - Convex
Type 2: Concavo - Concave
Type 3: Convexo - Concave
Type 4: Concavo - Convex
The mehtod to judge the terrain to be "Convex" or "Concave" is showed as follows.

Suppose the 3-D co-odinates of a,b,c and d are respectively (Xa, Ya,Za),(Xb,Yb,Zb),(Xc,Yc,Zc) and (Xd,Yd,Zd), then the plane equation determined by a,b and c is

$$Z1 = A1X + B1Y + C1$$

and the equation deternined by a,b and d is

$$Z2 = A2X + B2Y + C2$$

where A1,B1,C1,A2,B2 and C2 are coefficients which are relate to the 3-D co-ordinates of a,b,c and d.

The discriminat to judge the terrain to be "Convex" or "Concave" is

$$\Delta 1 = A1Xd + B1Yd + C1 - Zd$$

If $\Delta 1 > 0$, the terrain at the side of ab is "Convex"; If $\Delta 1 < 0$, the terrain at the side of ab is "Concave".Equivalentely, another discriminant is

$$\Delta 2 = A2Xc + B2yc + C2 - Zc$$

4. Determination of the Curve Type

We can decide the Curve type on each side of the triangular sub-area with the convexity or concaveness on the side of the triangular sub-area. In the triangle abc showed in fig.9, side bc is taken as an ex-ample to decide the type of the curve which is determined by the "concaveness" or "convexity" of the side ab and ac. The possible cases are showed in Tab.1. Similarly, the curve types of the sides ab and ac can be decided.

Aglorithm

After the 3-D co-ordinates of the Sampling points and the triangular network are obtained in the sampling procedure, the most direct and the simplest method to compute the heights of the regular grid po-ints is to interpolate linearly in each triangular subarea in the triangle network. Suppose a(Xa,Ya,Za),b(Xb,Yb,Zb), C(Xc,Yc,Zc) are the vertices of a triangular subarea in the triangle network, The linearinterpolating function in this subarea is

$$Z = Ax + By + C \qquad (4)$$

$$\Delta = \begin{vmatrix} x_a & y_a & 1 \\ x_b & y_b & 1 \\ x_c & y_c & 1 \end{vmatrix} \qquad \Delta_a = \begin{vmatrix} z_a & y_a & 1 \\ z_b & y_b & 1 \\ z_c & y_c & 1 \end{vmatrix}$$

$$\Delta_b = \begin{vmatrix} x_a & z_a & 1 \\ x_b & z_b & 1 \\ x_c & z_c & 1 \end{vmatrix} \qquad \Delta_c = \begin{vmatrix} x_a & y_a & z_a \\ x_b & y_b & z_b \\ x_c & y_c & z_c \end{vmatrix}$$

Then $A = \Delta_a/\Delta \qquad B = \Delta_b/\Delta \qquad C = \Delta_c/\Delta$

Linear interpolating function can only assure the continuity of the surface on the whole area, but it cannot assure that the interpolating functions smoothly link one another on the sides of the triangles.Furthermore, the terrain undulation is not concerned in linear interpolating functions, and the accuracy of linear interpolating functionis comparatively low. In order to find a kind of interpolating function which can reflect the undulation of terrain, the method we use is to add a corrective function $\Delta Z(x,y)$ to the linear interpolatingfunction. Therefore, the ultimate interpolating function is

$$H(x,y) = Z(x,y) + \Delta Z(x,y) \qquad (5)$$

According to discussio above,the curve funcfion on each side of the triangle can be obtained.As long as the interpolating function is made to pass the three curve functions on the sides of the triangle,our goal is achieved.The interpolating function we will discuss in the following sections can meet this request.Because this interpolating method generally takes the standard triangle as the studying object,it is necessary to transfer an arbitrary triangle into a standard triangle.

In fig.10,abc is an arbitrary triangle in the space of xy,a b c is a standard triangle in the space of pq.Then the transfering formula from pq to xy is

$x = x(p,q) = x1 + (x3 - x1)p + (x2 - x1)q$

$y = y(p,q) = y1 + (y3 - y1)p + (y2 - y1)q$

The transfering formula from xy to pq is

$$p = p(x,y) = \frac{(y_2-y_1)(x-x_1)-(y-y_1)(x_2-x_1)}{(x_3-x_1)(y_2-y_1)-(x_2-x_1)(y_3-y_1)}$$

$$q = q(x,y) = \frac{(y-y_1)(x_3-x_1)-(y_3-y_1)(x-x_1)}{(x_3-x_1)(y_2-y_1)-(x_2-x_1)(y_3-y_1)}$$

1. BBG Interpolating Formula

In Fig 11, Suppose the functions on the three sides of the stand--ard triangle are respectively F(p,o) F(o,q), F(p,1-p) or F(1-q,q), then the projection operators PiF (i=1,2,3) are respectively as follows:

$$P1F(p,q) = \frac{1-p-q}{1-q} F(o,q) + \frac{p}{1-q} F(1-q,q)$$

$$P2F(p,q) = \frac{1-p-q}{1-p} F(p,0) + \frac{q}{1-p} F(p,1-p)$$

$$P3F(p,q) = \frac{p}{p+q} F(p+q,o) + \frac{q}{p+q} F(o,p+q)$$

The multiplying operator PiPj which is defined by Pi and Pj (i j) is

$$PiPjF = Pi[PjF]$$

for example,

$$P1P2F = (1-P-q)F(0,0) + \frac{1-p-q}{1-q} q F(o,1) + \frac{p}{1-q} F(1-q,q)$$

$$P2P1F = (1-P-q)F(0,0) + \frac{1-p-q}{p+} p F(1,o) + \frac{q}{1-p} F(p,1-p)$$

A Theorem: If F(p,q) is a continous function on the triangle are a T, then the Boole Sum of Pi and Pj

$(Pi \oplus Pj)F$

is also a continous function in T and it passes F(p,q) on the sides.

It is known from this theorem that functions

$$W(p,q) = \sum_{ij} P_{ij} (P_i \oplus P_j) F$$

are continous function on the triangle area T and pass F(p,q) on thesides of T, if Pij(p ,q) meet the condition

$$\sum_{ij} P_{ij} (p,q) = 1$$

$(i \neq j)$

where 6 non-negative weight functions Pij(p,q) are arbitrary. Particularly, if Pij = 1/6 , then

$$W(p,q) = \frac{1}{2}\Big\{ \Big[\frac{1-p-q}{1-q} F(o,q) + \frac{p}{1-q} F(1-q,q) \Big] + \Big[\frac{1-p-q}{1-p} F(p,o) + \frac{q}{1-p} F(p,1-p) \Big] + \Big[\frac{p}{p+q} F(p+q,o) + \frac{q}{p+q} F(o,p+q) \Big] - \big[(1-p-q) F(o,o) + pF(1,o) + q F(o,1) \big] \Big\}$$

2.Radial Direction Projection Formula

The interpolating directions of the three operators of radial direction formula, which is different from BBG interpolating formula, are showed in fig.12.QiF(i =1,2,3) are defined respectively as follows:

$$Q1F = qF(0,1) + (1-q)F(\frac{p}{1-q},0)$$

$$Q2F = pF(1,0) + (1-p)F(0,\frac{q}{1-p})$$

$$Q3F = (1-p-q)F(0,0) + (p+q)F(\frac{p}{p+q},\frac{q}{p+q})$$

Then the interpolating formula is
$QF = (Q1 \oplus Q2 \oplus Q3)F$
Qi and Qj ($i \neq j$) meets followig formula
$QiQj = QjQi$

In the triangle network,there are 4 possible cases for each side of the triangle, which are showed in Fig 13.
i) All of the three sides is "real-line";
ii) Two out of the three sides are "real -lines";
iii) One side is "real-line";
iv) No side is "real-line",or all of the three sides is line of dashes.
We take the Curve function on the "real-line" of a triangle as zero,and on the line of dashes,we take the function as one of the 4 curve types discussed above. By using BBG interpolating formula or radial direction projection formula to interpolate in an arbitrary triangle, the height correction!' w' Z of any regular grid point can be obtained, and the DTM can be obtained by adding Z's to the heights of regulargrid points.

Experiment and DTM's Accuracy

In a test area which is 800m long and 600m wide,124 feature points (sampling points) are surveyed with the Intelligence Theomat Tc2000 made by Wild Company, Switzland, and 205 triangles are formed from these sampling points and the link relations among them. The heights ofregular grid points can be interpolate by using radial direction projection formula, the interval between two adjacent grid points can be given arbitrarily. 50 check points are surveyed to examine the DTM's accuracy. The data showed in Tab.2 is the differences between sur-veyed heights and the computed ones of these 50 points. i.e. $\Delta h = hs - hc$.

These 50 check points are selected arbitrarily in the test area. As showed in Tab. 2, the maximum error w ·hmax = 1.65m, So the accuracy is relatively high.

Application

After creating DTM of the test area, we can get the contours and perspective views of the area with the help of computer,fig.14 is a contour map and fig.15 is a perspective view of the area.

References
1. Lu Ue et.al. 1981. Cartography Automation, Beijing.
2.Xu Li zhi et.al. 1985. On Approximation, Beijing.

bc \ ab / ac	convex	concave
convex	Type 1	Type 4
concave	Type 3	Type 2

Tab.1 Determining curve type on side bc

0.50	-0.15	-0.30	-0.49	0.65	0.12	1.08	-0.01	0.35	-0.23
-0.62	0.69	0.46	0.48	1.00	0.49	-0.25	-0.45	0.01	-0.83
1.54	0.47	-0.83	0.79	-0.30	0.70	0.50	-1.01	0.71	-0.11
0.67	-0.07	0.70	0.67	1.49	1.12	0.23	0.61	0.56	-1.04
0.79	0.78	0.59	-0.25	-1.13	-0.58	0.46	0.90	-0.17	1.48

Tab.2 Differences between hs and hc on check points

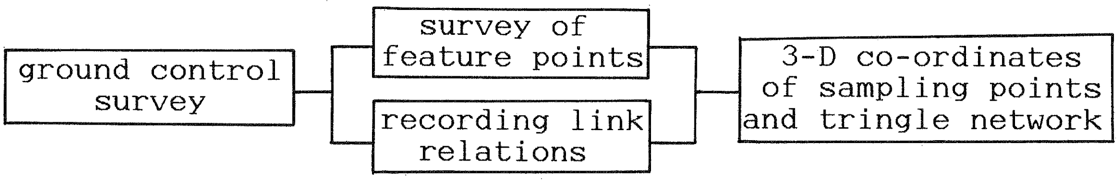

Fig.1 Flow chart of field-sampling procedure

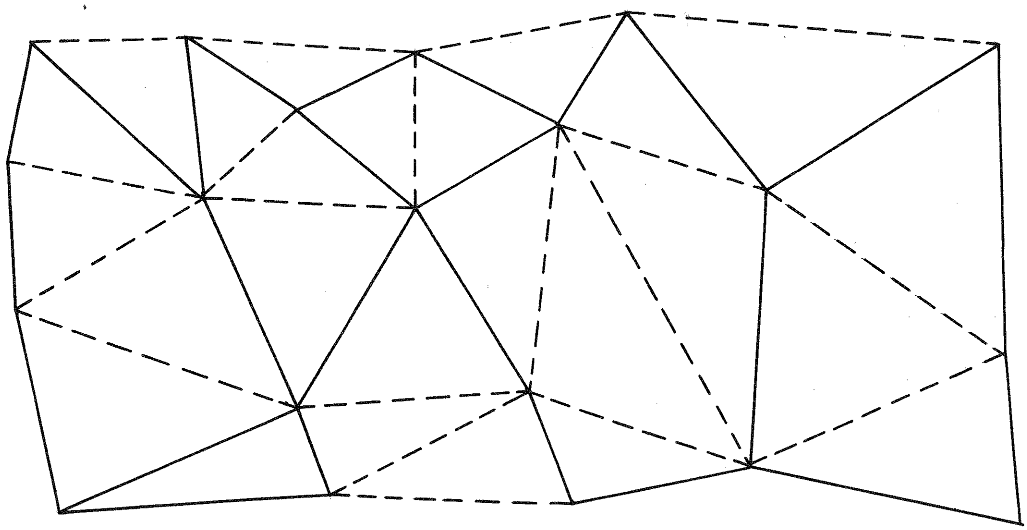

Fig.2 An example of triangle network

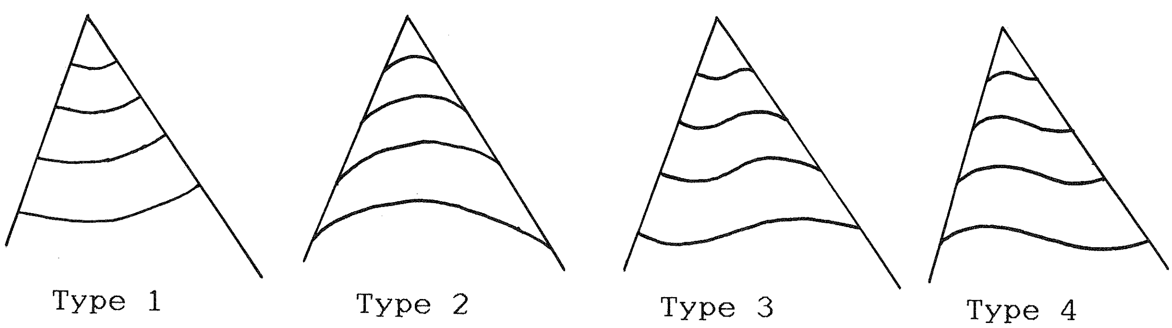

Fig.3 Four types of terrain surfaces

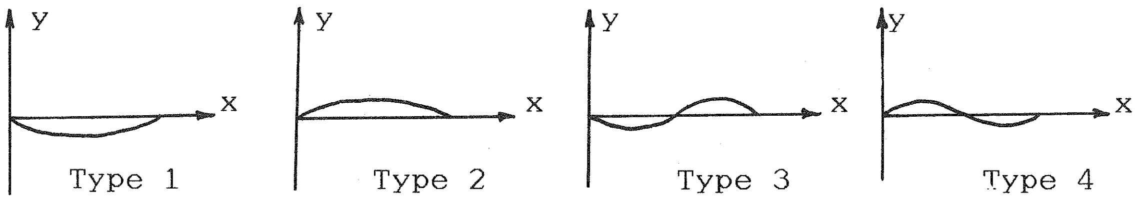

Fig.4 Typical curves of terrain contours

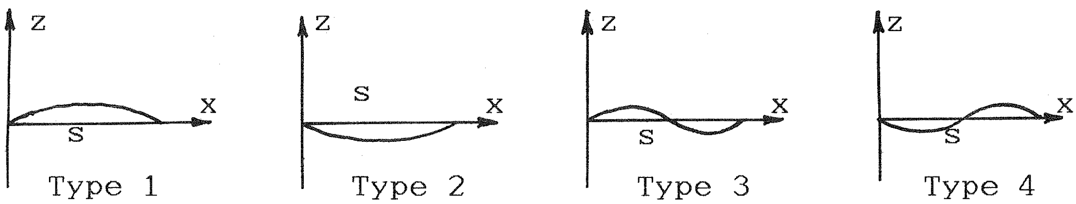

Fig.5 Section curves corresponding to typical curves

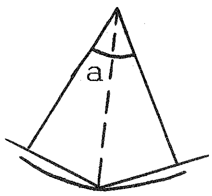

Fig.6

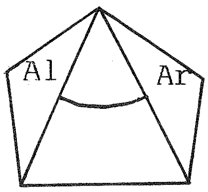

Fig.7

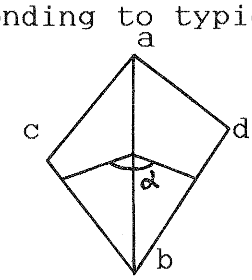

Fig.8

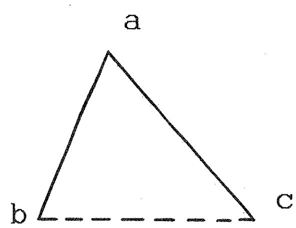

Fig.9

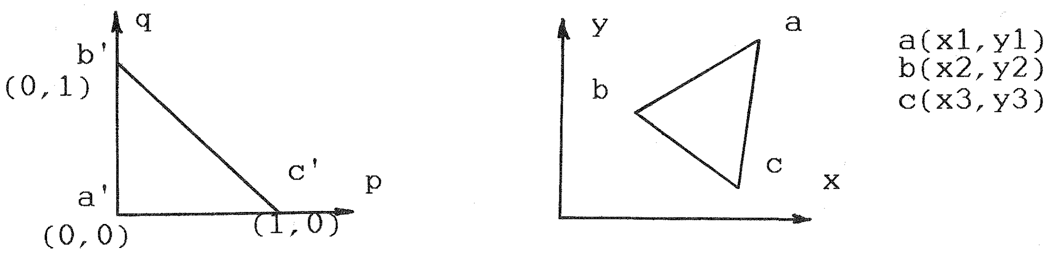

Fig.10

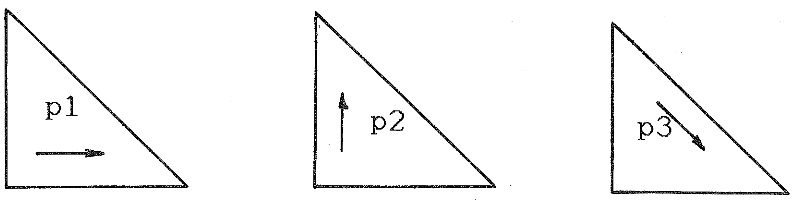

Fig.11 Directions of BBG project operaters

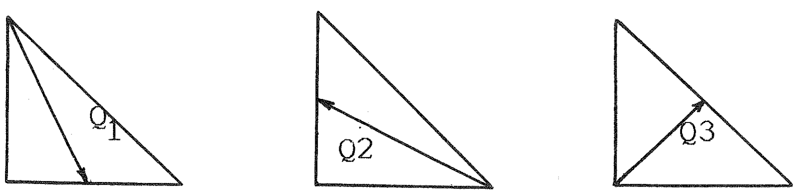

Fig.12 Directions of radial project operaters

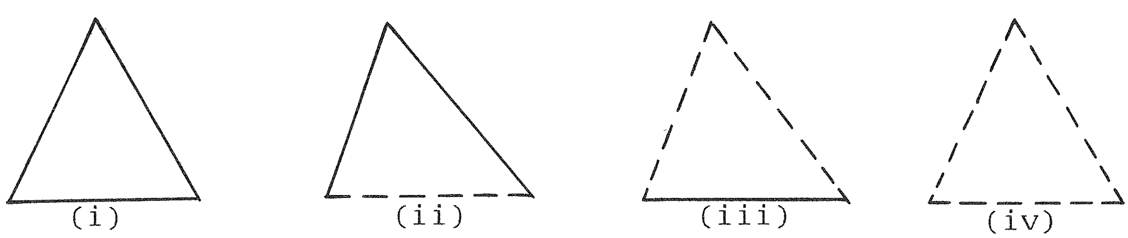

Fig.13 Four possible cases of triangle subareas

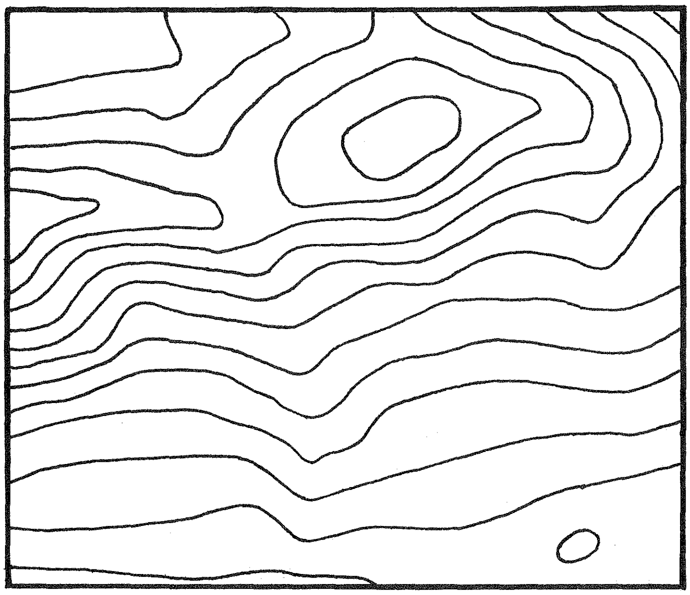

Fig.14 Part of the contours of the test area

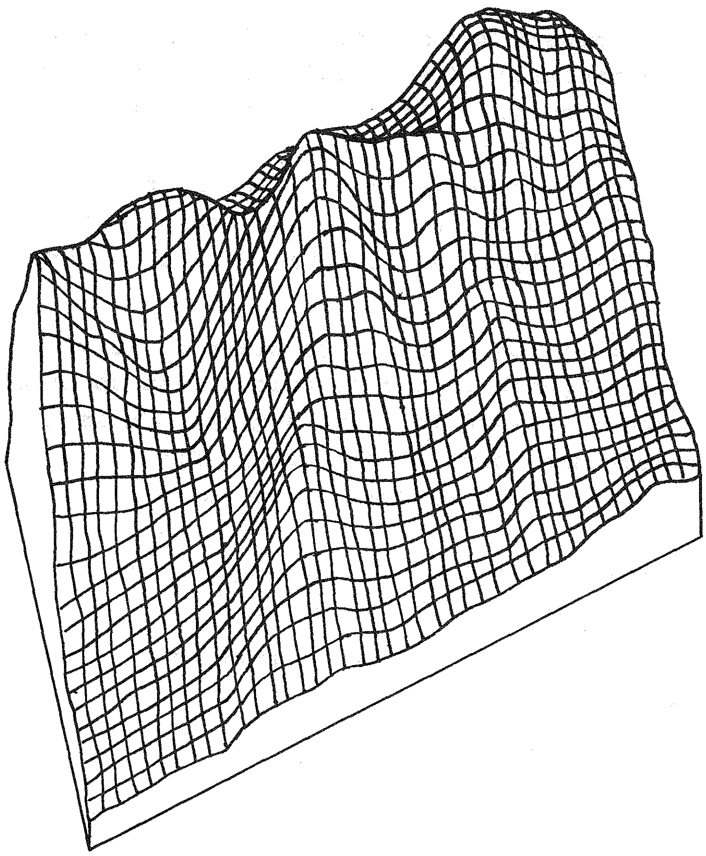

Fig.15 A perspective view of the test area