MATCHING AND MAPPING OF REMOTE SENSING IMAGES:
Aspects of Methodology and Quality.

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Abstract.
The increasing importance of the integration of Remote Sensing data (RS) and Geographical Information Systems (GIS) has an increasing impact on the necessity of a proper methodology and quality analysis of image registration. After a brief review of methods of image to image or image to map registration by using mapping polynomials and control points, the paper emphasizes a judicious analysis of the quality parameters, both precision and reliability. Besides description of two testing variates the attention is especially focussed on the boundary value of each control point concerning the size and probability of remaining, undiscovered errors in the pointing of control points in image or map. An analysis of sources of pointing deviations is given in view of an *a priori* variance to be used in the testing procedure.

1. Introduction.

Recently there is an increasing need of remote sensing (RS-) image products which are geometrically corrected, so that its information content can be combined with the information stored in existing thematic maps at several scales and of diverging sources.
One of the most important reasons for this need of matching and mapping of RS-images is the increasing market flow of images of different sensor type, acquired at different time and at different height. By this the user of RS-images has to deal with the integration of latent information of a heterogeneous set of image data, together with other types of information and (fore-) knowledge. So there are two main aspects in the matching of data:

a. The combination of RS-images, being of the multispectral- or multivariable-, multisensor-, multistage-, multitemporal RS types, being especially important for the analysis of processes or change detection of objects on the earth surface.
b. The comparison of RS-images to existing data like topographic and thematic maps either in analogue form or digitally stored in geographical information systems (GIS), being especially important for the information flow in several disciplines and fields of application concerning land evaluation, land use, soil hydrology and vegetation monitoring (see e.g. Burrough, 1986).

Central issue in these image-to-image and image- to map-relations is the radiometric resp. thematic object data bound to a geographic location. So matching "image" and "map" by a geometric registration (geometric rearrangement of pixels or rastercells) and a resampling (attachment of derived pixel values to the new raster elements) is the cement of thematic information sources. So it is self-evident that a suitable and justified quality (both precision and reliability) of the geometric registration is of an eminent importance.

2. Differences between maps and images.

There are typical differences between maps and images, originating from the distinctions about the aim of data gathering, the acquisition technique, the data processing and the information representation to the user.
Because of most thematic data gathering is related to existing topographic maps, in table 1 the properties of topographic map will be compared to those of a remote sensing image in general.

Table 1:

TOPOGRAPHIC MAP	REMOTE SENSING IMAGE
- geometrical oriented	- radiometrical oriented
- emphasis on geometric boundaries of geographical units (concerning the object types points, lines and areas)	- emphasis on thematic aspect of area-elements (boundaries only deductable by segmentation or edge enhancement techniques)
- map of vector elements (after idealisation, topographical interpretation and generalisation)	- matrix of raster elements (except aerial photographs which are to be digitized first)
- map figuration with legend, having distinct thematic assignment	- image with a sliding (continuous) representation of radiometric variables (except when classification took place)

These differences continuously keeping in mind is essential for an accurate selection of control points at the geometric registration stage, for a proper judgement of radiometric and thematic values at the resampling stage, and for the quality analysis of the information layers after matching. In the next sections the resampling stage will be let out of consideration and emphasis is laid on the quality aspects of registration.

3. Methods for image registration using control points.

There are two streams of image to map registration procedures to be distinguished, the one being the parametric way (using all required flight parameters of the remote sensing platform in a mathematical 3D - restitution model), the other being the non-parametric way (using a set of control points for the geometric registration of 2D - data layers). In the latter case height information may be brought into account as a relief displacement correction for pixel positions.
This paper will be restricted to the non-parametric way, which is applicable alike to image-to-image registration. So it is more convenient to speak of the matching (geometric registration) of MASTER (X,Y) and SLAVE (x,y), being two coordinate systems (rasters) with respect to image (i) and map (m) in one of the four possible combinations at will of the user (i-i; i-m; m-i; m-m respectively). In the current RS-image processing systems on the market, control points are acquired by manually and directly pointing on the screen display. This paper deals with this method and discusses the factors determining the accuracy and limitations of it (section 4) in view of the differences between map and image as discussed in section 2.
Besides there are indirect methods of control point acquisition in development nowadays based on correlation algorithms or least squares techniques of pairs of candidate image windows. This will not be considered here; this paper pays attention to the background philosophy of pointing factors in general.

Two main methods of non-parametric registration may be mentioned in short, the one being the method of trend modelling underlying the geometric fitting of the coordinate systems of master and slave; the other being the method of facet modelling. Section 5 deals with the trend model approach using a polynomial transformation and assuming that the residuals after a least squares trendmodel fitting may be considered to be stochastic (noise components) within the matching area as a whole, as in general is the case in satellite RS. Besenicar and Kengen, 1988, describe the facetmodel, to be used when image distortions are caused by locally fluctuating platform parameters, as in general is the case in aerial RS. In that situation a subdivision of the matching area in triangle-facets formed out of a well-chosen set of control points may solve the registration problem. Then each triangle has its own linear transformation model and no residuals remain.

As pointed out in section 1, a suitable and justified analysis of matching quality is required. Current image processing systems on the market have a lack of relevant testing parameters for the measurement of control points and the choice of a transformation model. The same can be said about modern textbooks like Jensen 1986, Richards 1986, Mather 1987. Only Haberäcker 1987, gives some indication. Section 5 of this paper will give an approach to fill this gap, offering tools for quality check.

4. Analysis of pointing quality.

4.1 Sources of pointing deviation of control points

As pointed out in section 2 there is a specific difference between map and image, originating from their scope and production. Nevertheless there are several common points of view in these two types of information layers, looking for the quality of pointing of control points.
Analysing the accuracy of a homological visual determination of a control point in both layers, one has to consider four questions:

a. What? In which way has the materialization of the control point been performed? What is the nature of display of the "point" e.g. is it a raster presented set of varying grey values or a set of line elements, color coding and symbols? How will it be seen, interpreted and identified in a mathematical abstract sense? This may be called the semantical aspect of the homological pointing of map point and image point, looking for the physical identity.
b. Where? What is the exact geometrical position of the interpreted control point and how can this be read out or digitized by an instrumental device? This may be called the spatial aspect of the pointing, being both a geometrical choice of position and an instrumental choice of coordinates.
c. When? Is there any difference in data acquisition time or in production date of the data source (image or map) questioning the possibility of occuring change of terrain features? So these topographical changes may disturb the identification process and the finding of the position of a homological pair of control point coordinates. This may be called the actuality aspect concerning the time correspondency of the data sources to be matched. The previous two questions are considering the representation of a control point, this third question is looking for the relevance of the data source (information layer) at which the control point may be picked out.
d. How? What is the nature of the information layer? Concerning maps, how has the map making and map compilation been done and which are the influences of differences of map scale (i.e. generalization, drawing process accuracy, displacements and exaggerations), at which the terrain objects representing the control points, may be displayed? Concerning images, which are the influences on (control) point representation by the sensor system properties like its spatial resolution, performance of radiometric values, its local geometrical distortions and (last but not least!) the influence of preprocessing including correction and resampling of grey values following the correction and rearrangement of geometrical positions. This is the source aspect of the pointing of control points.

It may be concluded that a "control point" is a concept relating a mathematical abstraction ("point") to a visualized materialization. Concerning maps the visualisation comprises at general an analogue representation of line and area elements besides isolated point

elements. Concerning digital remote sensing images on tape, they possess a latent nature, and have to be displayed by a color graphics system according to the specific (subjective) choice of screen performance, grey scaling, contrast enhancement, color coding and other image processing steps by the user. These factors may influence the visual identification process of a control point considerable. Using multispectral images, the optimal choice of spectral band is necessary for the best choice of materialization of the control points on the screen achieving the optimal pointing accuracy.
So an unique visualisation of a digital remote sensing image is out of question, only the digital numbers of the pixels recorded on tape or in the computer memory being unique!

4.2 Estimation of pointing accuracy

Now the four questions mentioned above, are leading to three measures of accuracy (expressed as standard deviations) for the pointing precision:

IDENTIFICATION ACCURACY	$\sigma 1$ concerning	the semantic	aspect a. and
		the spatial	aspect b. (geometrical choice)
DIGITALISATION ACCURACY	$\sigma 2$ concerning	the spatial	aspect b. (instrumental choice)
SOURCE QUALITY ACCURACY	$\sigma 3$ concerning	the actuality	aspect c. and
		the source	aspect d.

Although some deviations in an information layer give evidence of a systematic error at local parts of image or map, nevertheless they may occur occasionally. A random nature may be contributed to them considering the whole area of image or map. They may be expressed in a standard deviation for the source in general.
In table 2 estimated values are given for $\sigma 1$, $\sigma 2$, and $\sigma 3$. They have been obtained by careful experiments, carried out on the matching of Landsat-Thematic Mapper images, 1984 and 1986, spectral band 4 (Near Infrared) and Dutch topographical maps (1977 to 1985, coloured, scales 1:50,000 and 1:25,000) in the centre of the Netherlands. These images are comprising several classes of land use, small land units in the agricultural parts, and less and more distinct boundaries, road features or water elements.

St. dev.	MAP	IMAGE
$\sigma 1$	0.2 mm	0.4 pixel
$\sigma 2$	0.15 mm	0.4 pixel
$\sigma 3$	0.1 mm	0.3 pixel
	σ_m = 0.27 mm ≃ 0.45 pixel (TM)	σ_i = 0.65 pixel (TM)
σ_t	0.80 pixel	

Table 2:

ESTIMATION OF POINTING PRECISION
σm = map precision; scale 1:50,000
σi = image precision; Landsat-TM, pixel dimensions 30m x 30m
σt = pointing precision in one of the axis directions
σi or $\sigma m = (\sigma 1^2 + \sigma 2^2 + \sigma 3^2)^{1/2}$
$\sigma t = (\sigma i^2 + \sigma m^2)^{1/2}$

Identification accuracy:
Usually the interpretation of a control point in the image takes place on the screen in an enlarged subimage (window) around the control point to be chosen. So each pixel itself (on tape) is represented by a distinct number of identical grey or colour coded screen elements. The materialization of the "point" consists of an undulating variety of grey (colour) screen values within the window out of which the mathematical point in mind has to be identified compared to the materialization on the map. The image materialization usually being

less or more fuzzy, the map materialization is mostly an interpreted, clear set of line elements in accordance to the map legend (compare section 2), leading to the values of $\sigma 1$ in table 2.

Digitalisation accuracy:

The visually identified control point on the map has to be read out by digitalisation with respect to two rectangular axes X and Y. The accuracy of this action is dependent on the digitalisation stepwidth of the instrumental encoder, but in general a $\sigma 2 = 0.15$ mm on map scale appears to be a proper estimation.

Concerning control points on the image, the cursor (cross or circle) will be moved until the desired identification has been reached. The cursor is moved with discrete steps, at some times jumping over from the one pixel to the other. This means that we have to do with a Bernouilli distribution for an individual control point, or a binomial probability distribution for all control points together.

When probabilities $P(X=1)=p$ and $P(X=0)=q=1-p$ for a random value X of which only two values are possible (the one pixel or the neighbour), the value $\sigma 2$ of an individual point is in practice lying between 0.3 and 0.5 pixel ($p=0.1$ and $q=0.9$ respectively $p=0.5$ and $q=0.5$). Besides for an uniform distribution the square root of variance is $(1/12)^{\frac{1}{2}}=0.29$. Dependent on the enlargement factor of the subimage and on the width of the cursor cross hairs, an estimation in general of $\sigma 2=0.4$ pixel for the image digitalisation accuracy seems to be justified.

Source quality accuracy:

About the source quality it may be said that in the experiments a systematic difference in map representation of a control point surrounding has been found concerning the topographical map scale 1:50,000 compared to 1:25,000, leading to differences in control point position of about 0.2 to 0.4 mm, occurring local. Estimated over the whole map area nevertheless a source quality accuracy $\sigma 3=0.1$ mm in X- or Y-direction has been found to be a proper standard deviation.

Concerning image sources occasionally local disturbance has been assumed to be present of about 2 pixel dimensions in maximum, with respect to actual or apparent local line shifts and crumbling representation of supposed line elements in the image by fast changing grey values. These effects are caused by data acquisition errors, remaining preprocessing errors or simply by the ordinary resampling process (e.g. cubic convolution) of the raw image data. Also the effect of spatial resolution is not to be underestimated, being evident when remote sensing images of the same sensortype but of different acquisition dates are compared with respect to the representation of continuous edges (to be interpreted as line elements or as crossings of them).

However, in general it may be said that TM-images proved to be considerably well constructed with respect to the configuration of map elements. In the experiments the map production dates were comparable to those of the remote sensing images.

By this the factors discussed in section 4.1 ("when" and "how") have been estimated to get the standard deviation $\sigma 3=0.3$ pixel both for X and Y direction (or respectively along track direction and scanning direction).

Of course for other map or image sources of information and for other places of the world than employed in the experiments underlying table 2, other estimations will be the result, but it seems that the given values are valid in general. Besides it may be stressed that the theoretical partitioning of the estimation of pointing precision in terms of $\sigma 1$, $\sigma 2$ and $\sigma 3$ as defined above, is essential in a general way for the practical implementation of a supervised image registration procedure.

Although the considerations mentioned above are concentrated at the matching of map and image, the same line of thought may be followed concerning the matching of one image and another image. In general any matching of MASTER and SLAVE, whatever they actual are, is

affected by the discussed aspects of pointing precision. It will be clear that in the case of matching of images acquired by the same sensor system (- as will be the case in processing multitemporal images -) the similarity of candidate control point materializations is to be expected and so the identification accuracy ($\sigma 1$) may be better.
Where the pointing of a control point instead of manually and visually, takes place at a sophisticated or an interactive automatical way, both the identification accuracy ($\sigma 1$) and the digitalisation accuracy ($\sigma 2$) may be better, supposed that conditions about certain approximated location and source representation within the search area are satisfied. In each matching situation the user itself has to decide which values of $\sigma 1$, $\sigma 2$ and $\sigma 3$ are to be applied as a base for testing the results of transformation.

5. Quality parameters and the testing procedure for geometric registration.

Assuming that a polynomial transformation of some chosen degree is underlying the matching (registration) of master and slave, one has to ask oneself two questions:

a. How suitable is the choice of transformation model satisfying the available data to be registered? This is the aspect of model evaluation.
b. Are there any mistakes present in the preceeding (assumed) homological determination of the control points? This is the aspect of error detection.

There is a mutuality in both questions: testing of control points is pre-supposing the relevance of the chosen transformation model. Again the judgement of the choice of transformation model is pre-supposing the relevance of the actual set of control points. However, the parameter for internal reliability to be presented can be computed independant of the actual data and is acting as a parameter for the choice of transformation model together with the number and the geometrical distribution of the control points in the matching area.
To meet both questions a testing procedure is proposed, consisting of the assessment of precision by means of two statistical testing variates and one parameter of internal reliability, based on the theory of Baarda, as founded in Baarda 1968 and elaborated by him and others in later years (see e.g. Staff TUD 1982; Molenaar 1982; Förstner 1985; Förstner and Molenaar 1986).

The computing and testing procedure in the case of a trend transformation (section 3) will be as follows:

5.1 Transformation model

$f(X,Y) = x-u$ where X,Y = master coordinates
$g(X,Y) = y-v$ $\quad x,y$ = slave coordinates
$\quad u,v$ = least squares residuals

or combined and written in full for transformation polynomials up to the third degree, using a number of n control points:

$$
\begin{vmatrix}
1 & X_1 & Y_1 & X_1^2 & X_1Y_1 & Y_1^2 & X_1^3 & X_1^2Y_1 & X_1Y_1^2 & Y_1^3 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
1 & X_i & Y_i & X_i^2 & X_iY_i & Y_i^2 & X_i^3 & X_i^2Y_i & X_iY_i^2 & Y_i^3 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
1 & X_n & Y_n & X_n^2 & X_nY_n & Y_n^2 & X_n^3 & X_n^2Y_n & X_nY_n^2 & Y_n^3
\end{vmatrix}
\times
\begin{vmatrix} a_1 & b_1 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ a_{10} & b_{10} \end{vmatrix}
=
\begin{vmatrix} x_1 & y_1 \\ \cdot & \cdot \\ \cdot & \cdot \\ x_i & y_i \\ \cdot & \cdot \\ x_n & y_n \end{vmatrix}
-
\begin{vmatrix} u_1 & v_1 \\ \cdot & \cdot \\ \cdot & \cdot \\ u_i & v_i \\ \cdot & \cdot \\ u_n & v_n \end{vmatrix}
$$

> (first block) >> (second block) >>> (third block)

A *C1* *C2* *B1* *B2* *R1* *R2*

< first degree with n>3 >
< second degree with n>6 >>
< third degree with n>10 >>>

or: $A.C_k = B_k - R_k$ for column k=1 respectively k=2.

where A is the design matrix, C is the matrix of transformation coefficients, B is the matrix of known data and R is the matrix of residuals.

Assuming for convenience (as is the custom in image processing systems) that the variables are uncorrelated and having the identity matrix I as a weight coefficients matrix, the least squares solution is yielding:

$$C_k = (A^T A)^{-1} A^T B_k \quad \text{and} \quad R_k = B_k - A(A^T A)^{-1} A^T B_k \quad \text{for } k=1,2$$

T=transposed

5.2 Variance-ratio test

Compute the estimated variances (denoted by ^) in x- and y-directions from the residuals u and v:

$$(\hat{\sigma}_x)^2 = (R_1^T R_1)/r \quad \text{and} \quad (\hat{\sigma}_y)^2 = (R_2^T R_2)/r$$

with redundancy r=n-p (where p=3,6,10 respectively). These estimated variances will be tested against an a priori variance σ_o^2 (to be taken from the discussion in section 4.2) by applying the variance-ratio test with confidence region $(1-\alpha)$:

If $(\hat{\sigma}^2/\sigma_o^2) > F_{1-\alpha;r,\infty}$ reject H_o,

otherwise accept H_o, in which H_o is the null hypothesis that the transformation model is relevant (well chosen) and/or no mistakes are present in the pointing of the set of n control points. Accepting H_o it implies that the square root of the estimated variances ($\hat{\sigma}_x$ and $\hat{\sigma}_y$ respectively) are measures of precision of the matching of slave and master. Note that these standard deviations have been expressed in the x,y-pixel-dimensions of the slave. Because of the presupposition about the weight coefficients matrix of the coordinates, the coordinate axes are processed independently and the computation of residuals u and v is quite similar.

Explanation figures 2 and 3: When FVAR $= \hat{\sigma}^2/\sigma_o^2$ and for $\sigma_o = 1.0$ then:

Figure 2: FVAR (x) = 0.79<1.54, H_o is accepted

FVAR (y) = 0.50<1.54, H_o is accepted

Figure 3: FVAR (x) = 1.48<3.84, H_o is accepted

FVAR (y) = 4.68>3.84, H_o is rejected

For $\sigma_o = 0.8$ a similar analysis may be done by interpolation between the results for σ_o=0.5 and for σ_o=1.0.

5.3 Data-snooping test

When the variance-ratio test will reject the null hypothesis, a next statistical test may be applied. It should be possible to formulate a test about the simultaneous presence of a specific set of pointing errors in the determination of the control points. For simplification of the test procedure now only one error in one of the (slave-)directions of coordinate axes of a control point is assumed successively. This data-snooping is a special application of the more general w-test (references see above).

So for each control point i two variates $(W_x)_i$ and $(W_y)_i$ are computed from the respective residual divided by its standard deviation:

$$(W_x)_i = | u_i/\sigma_{ui} | = | u_i | / (\sigma_o \sqrt{N_{ii}})$$

$$(W_y)_i = | v_i/\sigma_{vi} | = | v_i | / (\sigma_o \sqrt{N_{ii}})$$

in which the dimensionless number $N_{ii} = (I - A(A^T A)^{-1} A^T)$ row i. column i.

Introducing the alternative hypothesis H_a that (only) one observation is affected by an error, a data snooping test is applied sequentially 2n times according to:

If $|W|>(F_{1-\alpha_o;1,\infty})^{1/2}$, check the observation (whether master or slave in both axes). In order to give the variance-ratio test and the data-snooping test equal power, the confidence level $(1-\alpha_o)$ should be in balance to $(1-\alpha)$ such that concerning the statistical power functions it will hold that the testing level $\lambda_o = \lambda(\alpha,\beta_o,r,\infty)$ should be approximately equal to $\lambda(\alpha_o,\beta_o,1,\infty)$, in which for the power β_o the value 0.80 may be chosen. This choice of equal values for λ and β in both tests means that a certain error is detected with the same probability.

In table 3 a set of λ_o and α_o values is given for increasing redundancy (r), taking $\alpha= 0.05$ and $\beta_o = 0.80$. By testing so, again an error detection is performed presupposing the transformation model is well chosen. This matter set bounds to the interpretation of the alternative hypothesis underlying the data snooping. In the case no errors are detected in fact although the variance-ratio test is not satisfying the a priori variance, several actions are possible:
- The a priori variance should be reconsidered for both tests.
- The degree of the polynomial transformation should be altered (increased in general) where some kind of systematic non-linear distortions may be assumed in master and/or slave.
- Another kind of alternative hypothesis in the data-snooping test should be seriously considered, e.g. some set coordinate variables should be tested together.
- The area to be matched should be subdivided, so decreasing the influence of locally situated distortions upon the total matching area, putting up with the drawback of the requirement of more control points to be determined.

Explanation figures 2 and 3: When u_i=VREST(x) and v_i=VREST(y), then no residual outliers are present in both figures.
Figure 2: For each control point, H_a is rejected, e.g.: $(W_x)_1=1.54<3.80$ and $(W_y)_1=0.62<3.80$.
Figure 3: To show the sensibility of the w-test, the y_4 value was changed with 12 pixels; the result is: $(W_x)_i=1.21<1.96$, H_a is rejected.
$(W_y)_i=2.16>1.96$, H_a is accepted.

5.4 Analysis of boundary values

Apart from the foregoing statistical tests, it is worthwhile to compute for each control point the marginally detectable error or boundary value of each control point in both directions of (slave-)coordinate axes, given a distinct probability β for detecting the error (references see above). Error values less than the boundary value perhaps may be detected but with a lower probability. The smaller the marginally detectable error, the better the reliability according to an alternative hypothesis. The boundary value may be computed without any actual pointing of control points and depends only on:

a. the a priori standard deviation σ_o.
b. the previously chosen testing level value λ_o (see table 3).
c. the proposed spatial (geometric) structure of the positions of the control points to be chosen in the area to be matched.
d. the proposed degree of transformation polynomial.

The set of boundary values for a given choice of control points is a measure both for the model evaluation and for the possibility anyhow of detecting gross errors to some extent. Besides, where the candidate control points are regularly spaced, the boundary values tend to be equal (Baarda, 1968, chapter 11); see figure 2. Inversely notable differences between the boundary values are pointing out to a bad design of the spatial distribution of the candidate control points (see figure 3), to a polynomial degree chosen too high, or to a lack of redundant variables (value r=n-p). The remedy should be then a better distribution of control points over the area to be matched, an intensification of the control point density and a justified choice of the degree of transformation model.

Because of the mentioned assumption about the weight coefficients matrix of the observational variates the boundary value in x- respectively y-direction is equal for any control point. Of course there is some roughness in this, but in general this simplification satisfies. The boundary value for a control point i is computed from (see figure 1 for the meaning of λ_o):

$$BV = \sigma_o \cdot (\lambda_o)^{1/2} / (N_{ii})^{1/2}$$

Explanation figures 2 and 3: If the control points are equally spaced in the matching area, then (under the condition that the weight coefficient matrix of coordinate variates is a identity matrix) $N_{ii}= r/n$ (compare Förstner 1985). In that case is: $BV=(\sigma_o \sqrt{\lambda_o})/\sqrt{r/n}$.
When $\sigma_o=1$, then: Figure 2: r=22, n=25, $\lambda_o=21.7 \rightarrow$ BV (equal) = 4.96.
Figure 3: r=1, n=4, $\lambda_o=9.8 \rightarrow$ BV (equal) = 5.59.
Conclusion: in figure 2 there is a balanced spacing of control points. In figure 3 there is a bad spacing; the redundancy is very low; point 4 has a weak geometry concerning gross errors!

It should be noted, that the value BV only refers to the internal reliability of the transformation and its set of control points. It is possible to express the influence of undetected errors of the size BV on the final results ("external reliability"), being matched point-, line- and area elements of slave and image in raster mode. After registration and resampling the results of whether good or wrong superposition are perceptible, under the condition that the picture contains a distinct variety of spatial object structures which enables the comparison of slave data and master data. By lack of these terrain elements undetected errors may have their free play. In that case the matched data are not leading to reliable "information" when image interpretation takes place (e.g. in a GIS-structured environment or by change detection). Wrong information may be more detrimental than no information. So a previous analysis of boundary values plays an important role in a proper thematical analysis of remote sensing image data in matching them with other sources of information. Continuation of research about matching (both registration and resampling), precision and reliability is (to my opinion) required for the future development of GIS-activities.

6. Conclusion.

There is an increasing need for matching of information acquired from different sources of images and maps. The pivot of any thematic information is its geometric position in each of the data layers. Much care should be given to a suitable and accurate geometric registration of data. The testing variables described in this paper (variance-ratio test, data-snooping test and boundary value analysis) are considered to be proper tools in order to overcome undetected errors in matching.

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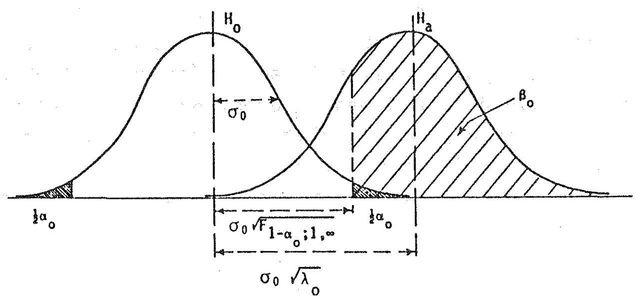
TABLE 3 :

with $\beta_0 = 80\%$, $\alpha = 5\%$

r	λ_0	$100\alpha_0$	$\sqrt{F_{1-\alpha_0;1,\infty}}$
1	7.8	5.00	1.96
2	9.6	2.50	2.24
3	10.9	1.40	2.47
4	11.9	0.92	2.60
6	13.6	0.43	2.86
8	15.0	0.26	3.02
10	16.2	0.15	3.17
12	17.3	0.09	3.32
14	18.3	0.06	3.44
16	19.2	0.04	3.54
18	20.1	0.029	3.63
20	20.9	0.021	3.70
22	21.7	0.015	3.80
24	22.4	0.010	3.90
26	23.2	0.007	3.98
28	23.9	0.005	4.04
30	24.5	0.004	4.10

FIGURE 1 :



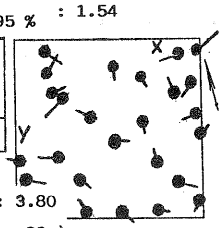
DEGREE OF TRANSFORMATION = 1 { WG 84.4 M, WG 86.4 S

ESTIMATED ST.DEV. IN X :	0.89	*
ESTIMATED ST.DEV. IN Y :	0.71	*

* : *expressed in (slave)pixel-dimensions*

VARIANCE-RATIO TEST : $F_{TEST\ 95\ \%}$: 1.54

$F_{VAR\ X}$ S_0= 0.5	$F_{VAR\ Y}$ S_0=0.5	$F_{VAR\ X}$ S_0=1.0	$F_{VAR\ Y}$ S_0=1.0
3.14	2.02	0.79	0.50



DATA-SNOOPING : F_{TEST} SQRT : 3.80
(REDUNDANCY = 22)

NR.	$V_{REST}X$ *	$V_{REST}Y$ *	$W_{TEST}X$ S_0=1.0	$W_{TEST}Y$ S_0=1.0	BOUND.VAL. S_0=1.0 *
1	- 1.42	+0.57	1.54	0.62	5.05
2	+ 0.01	+0.10	0.01	0.11	4.90
3	+ 0.82	-1.40	0.89	1.52	5.06
:					
24	+ 1.74	-1.47	1.93	1.63	5.16
25	- 0.13	+0.29	0.14	0.32	5.18

FIGURE 2 :
RESULTS OF TESTING VARIABLES, test area Wageningen, registration of TM-images 1984(master),1986(slave), with 25 control points. Read σ0 for S0 (σ0 =0.8 to be chosen). Note the balance of the boundary values !

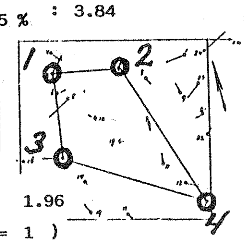
DEGREE OF TRANSFORMATION = 1 { WG 84.4 M, WG 86.4 S

ESTIMATED ST.DEV. IN X :	1.21	*
ESTIMATED ST.DEV. IN Y :	2.16	*

* : *expressed in (slave)pixel-dimensions*

VARIANCE-RATIO TEST : $F_{TEST\ 95\ \%}$: 3.84

$F_{VAR\ X}$ S_0= 0.5	$F_{VAR\ Y}$ S_0=0.5	$F_{VAR\ X}$ S_0=1.0	$F_{VAR\ Y}$ S_0=1.0
5.90	18.74	1.48	4.68



DATA-SNOOPING : F_{TEST} SQRT : 1.96
(REDUNDANCY = 1)

NR.	$V_{REST}X$ *	$V_{REST}Y$ *	$W_{TEST}X$ S_0=1.0	$W_{TEST}Y$ S_0=1.0	BOUND.VAL. S_0=1.0 *
1	+ 0.84	+ 1.50	1.21	2.16	4.04
2	- 0.63	- 1.13	1.21	2.16	5.36
3	- 0.52	- 0.93	1.21	2.16	6.52
4	+ 0.31	+ 0.56	1.21	2.16	10.80

N.B.
MISTAKE Y_4
+ 12 PIXELS

FIGURE 3 :
RESULTS OF TESTING VARIABLES, selection of 4 control points , where point 4 is dominant at the determination of the first degree transformation coefficients. Point 4 is bad controlled by the other points. Note the great differences of boundary values. A mistake of 12 (sic !) pixels in Y4 can hardly be detected (BV = 10.8 at σ0 =1)