A PLANNING STRATEGY FOR COMBINED PHOTOGRAMMETRIC BLOCKS AND TERRESTRIAL NETWORKS

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ABSTRACT

The theory of quality control for landsurveying networks and photogrammetric blocks, which has been developed over the past decade gives clear directives for the planning for individual networks. The photogrammetrist faces the problem, however, that he has to deal with a combination of a photogrammetric block and a supporting terrestrial network. Planning strategies should be modified so that these combined set ups can be dealt with as well.

This paper gives an outline for such a modified strategy taking in to account criteria for precision and reliability of the final coordinates of newly determined points. By a backwards reasoning criteria for the individual block and network are derived from the criteria for the total system.
1 INTRODUCTION

Working group III/1 of ISPRS in the period 1984-1988 had two main tasks. The first task concerned the integration of different data types for point determination. The second task concerned the problem of formulating planning strategies for combined point determination systems. These planning strategies should be based on criteria for the quality of the output of such systems, i.e. the reliability and precision of the final coordinates of the determined points.

The concepts of the present theory for quality control have first been formulated by Baarda [1] [2] [3]. Related concepts especially for precision theory were formulated by Grafarend [13] [14] and Meissl [18]. These concepts and those of Baarda most of all have been developed into a rather complete theory for quality control. A review of this theory was given in [12] [22] [23] [9].

Strategies for network design have been formulated in the context of this theory [12] [22] [23]. These strategies were mainly formulated, however, for the case that networks are designed in one stage, whereas one type of network is considered per case, e.g. a terrestrial network with theodolite and distance measurements, or a leveling network or a photogrammetric block.

The photogrammetrist is, however, in the situation that he always has to deal with different types of networks, measured and designed at different stages. He connects his photogrammetric block to groundcontrol points, which are obtained from a terrestrial network. Nowadays there is the trend to replace the terrestrial data by GPS data. The experimental results published in other papers for this workinggroup are very promising, but it is not clear yet how much time it will take before the use of GPS data is common practice.

It is very likely that for many applications photogrammetrists will still have to rely on conventional groundcontrol points obtained from terrestrial measurements for the coming decades. That is why we use that case as an example for describing a planning strategy for multistage network design. The multistages will be restricted to two stages in this paper.

To develope a planning strategy the structure of the blockadjustment proces and the quality propagation through the different stages of this proces will be given. No formulae will be presented, because there is ample liter­ature available where these are given.

In this paper their actual structure is less important then the structure of the total information flow.

2 QUALITY ASPECTS OF TERRESTRIAL NETWORKS AND PHOTOGRAHMETRIC BLOCKS

Terrestrial networks

Consider the situation of a photogrammetric block supported by a terrestrial network. In the network theodolite observations r are made and distance measurements l. Their precision is expressed in a variance covariance matrix \( \sigma_r \).
These data and information about the network structure are given as input to a network adjustment program, which gives as an output the network coordinates \((X^c, Y^c, Z^c)\). Suffix \(c\) refers to control points for the photogrammetric block.

![Diagram](image)

**Fig. 1** Information flow for network adjustment

Modern adjustment programs give also information about the quality of the computed network in terms of precision and reliability of the output coordinates. This information can be provided before the observations \(r\) and \(l\) are available. For the evaluation of precision we only need information about the expected precision of the observations \(\sigma_{r,l}\) and about the network structure:

![Diagram](image)

**Fig. 2** Information flow for precision evaluation of network

For the evaluation of the network reliability this information is required too, but additional information is required about the error detection- or hypothesis testing techniques which will be applied.

![Diagram](image)

**Fig. 3** Information flow for network adjustment
In this figure $V_r, V_l$ are the boundary values of the observations. They give the internal reliability of the network [2]. $V_X^C, V_Y^C, V_Z^C$ are the boundary values of the coordinates, they give the external reliability of the network, they depend on $V_r, V_l$ and the network structure [2].

**Photogrammetric blocks**

The discussion in this paper refers to photogrammetric independent model blocks in which the $UW$ coordinates of model points have been observed. Their precision is given by a variance-covariance matrix $\sigma_{uvw}$. The structure of the computational process given here below is however, equally valid for bundle blocks.

The central component in this stage of the computational process is a block adjustment program which requires as input data: the model coordinates and their precision, information about the blockstructure and the controlpoint configuration, the controlpoint coordinates and their precision. The output of this program are the XYZ coordinates of newly computed terrain points:

$$\sigma_{uvw} \xrightarrow{\text{block adjustment program}} (X,Y,Z)$$

**Fig. 4** Information flow in blockadjustment

The model coordinates and the controlpoints coordinates effect the block-adjustment in two ways: e.g. directly by their values and indirectly by the information they contain about the blockstructure.

An evaluation of the block quality can be made before the actual $(U,V,W)$ values and $(X^C, Y^C, Z^C)$ values are available.

For precision we need information about the blockstructure, controlpoint configuration and the expected precision of the modelpoint coordinates and controlpoint coordinates.

$$\sigma_{uvw} \xrightarrow{\text{block adjustment program}} \sigma_{xyz}$$

**Fig. 5** Information flow for evaluation of blockprecision

Instead of the actual $X^C, Y^C, Z^C$ values, approximated values can be used. $\sigma_{xyz}$ is the v.c. matrix of the final coordinates.
For the evaluation of the reliability we have to specify again an error detection technique. In this stage of the process distortions in the model coordinates and the controlpoint coordinates should be detected. So quality evaluation should result in boundary values \((V_U, V_V, V_W)\) and \((V_{bXc}, V_{byc}, V_{bZc})\) for the internal block reliability.

It may be though that undetected errors in the original terrestrial network slip through this phase too and therefore they have an effect on the external reliability of the final block coordinates. This latter error source may give systematic like distortions which often are very difficult to test for.

**Fig. 6** Information flow for reliability analysis of a block

Figure 2 and 5 give the complete information flow for the evaluation of the block precision, where fig. 2, 3 and 6 give the information flow for the evaluation of the reliability.

From these figures it is clear that the information flow for the precision evaluation is simpler than for reliability analysis. An additional fact which is not obvious from these figures is that the propagation laws for v.c. matrices are simple. This is also true for the propagation from one network stage to the next (see [6] [21]). Such simple rules do not exist yet for the global reliability measures such as \(\lambda_0\) defined by Baarda (= \(\delta_0\) defined by Förstner) see [4] [9]. The local reliability measures, the boundary values of individual hypotheses can easily be propagated in the case of errordetection, through different stages, but it would require a lot of computational effort to do this for all relevant hypotheses in a network. For these reasons we may expect that a multistage design strategy for precision is less difficult to formulate, we will first consider that case.

### 3 A DESIGN STRATEGY FOR THE PRECISION OF TWO STAGE NETWORKS

Two main approaches have been advocated in literature for the design of networks with respect to precision. The first one is the optimisation technique where the precision of an ideal network is represented by a criterion matrix \(h_{xyz}\).
A network is designed so that the actual v.c. matrix $\sigma_{XYZ}$ approximates $H_{XYZ}$ as good as possible. The second approach is the satisfisation technique where a criterion matrix $H_{XYZ}$ is used as an upperbound for precision. A network is designed so that the precision given by the actual $\sigma_{XYZ}$ is never worse than the precision given by $H_{XYZ}$.

For reasons given in [22] we will follow the satisfisation approach in this paper. The author knows of no examples given in literature where the optimisation technique was used for multistage networkdesign, whereas we will see that the satisfisation technique is, in that case, easy to handle.

The targetfunction for satisfisation of network precision is formulated by means of the general eigenvalue problem [3] [19] [22]:

$$| \sigma_{XYZ} - \lambda H_{XYZ} | = 0$$

$$\lambda_{max} \leq 1$$

Let us write this symbolically as:

$$\sigma_{XYZ} \leq H_{XYZ} \quad (1)$$

This targetfunction should be applied to the output of the blockadjustment. Analysis of fig. 2 and 5 tells us what steering parameters we have to reach this result.

From fig. 5 we see that the steering parameters in the second stage are the blockstructure including the controlpoint configuration, the precision of the photogrammetric data and the precision of the controlpoints. Ample literature has been published on the effect of blockstructure and controlpoint configuration and precision of photogrammetric data. Much less attention has been point to date to the effect of the precision of the controlpoints on the final blockprecision, see [21]. Therefore we will assume for the strategy proposed here that a blockstructure (overlap, tiepoint configuration, scale etc.) has been chosen according to recommendations given in literature. The remaining task is then to set a criterion for the precision of the controlpoints in relation to the controlpoint configuration.

This means that in fig. 5 we have to replace $\sigma_{XYZ}^c$ by a criterionmatrix $H_{XYZ}^C$:

![Diagram](image)

*Fig. 7* Computation of a pseudo v.c matrix for blockprecision
So instead of \( \sigma_{\text{xyz}} \) we get a pseudo v.c. matrix \( \Sigma_{\text{xyz}} \) for the precision of the output coordinates of the block. Instead of (1) we now get the target function:

\[
\Sigma_{\text{xyz}} \leq H_{\text{xyz}} \quad (2)
\]

When the block parameters have been fixed we only have the control point configuration and \( H_{\text{xyz}} \) left as steering parameters for the system to fulfill (2). The matrix \( H_{\text{xyz}} \) now serves as a criterion matrix for the control point network. If it is structured as proposed by Baarda [3] or Karadaidis [16] [5], see also [13], then the actual criterion set by the choice of the values for two or three parameters. The tuning of \( H_{\text{xyz}} \) by means of these parameters should be done so that (2) is just fulfilled. The resulting matrix is then given to the landsurveyor to design his network so that it fulfills the criterion.

\[
\sigma_{\text{xyz}}^{2} \leq H_{\text{xyz}}^{2} \quad (3)
\]

With these two steps (2) and (3) we see that a two-stage planning strategy for photogrammetric blocks and terrestrial networks is feasible. In fact the strategy can easily be adapted for more than two steps.

### 4 TWO STAGE DESIGN OF NETWORKS WITH RESPECT TO RELIABILITY

There is a basic difference between the analysis of the precision of a geodetic network and the analysis of its reliability. This difference is apparent from a comparison of figures 2 and 3. Precision analysis only requires information about the precision of the input data, i.e. measuring procedure and the network structure. For reliability analysis also information is required about the error detection technique which is applied. Hence reliability analysis always refers to a specific hypothesis or a group of hypotheses to be tested. So network design with respect to reliability can never give an overall optimal structure as does network design for precision.

So what has been advocated in literature to date is that network and block design is done with respect to the reliability of a special group of tests, i.e. those aiming at finding observational errors. Besides that several proposals have been made to optimise the detection of systematic deformations [11] [17].

The most common approach for network design assumes that error detection is done by the datesnooping technique of Baarda [2] and that the reliability of this technique is considered. This is justified by the fact that it is the theoretically based on a set of most powerfull tests, meaning that other techniques can in principle not be more sensitive although they might have a better computational efficiency in certain situations see [24] and ([9], Ch. 3.1 - 3.2).
During the two inter congress periods from 1976 until 1984 many papers and articles have been published on the reliability of photogrammetric blocks. In this literature attention has been paid to internal blockstructure, tie-point configurations and controlpoint configuration. See e.g. [19] [15] [9]. That means that in fig. 6 most attention has been paid for the analysis of the values $(V_U, V_V, V_W)$ and $(V_{bX}, V_{bY}, V_{bZ})$ and their effect on $(V_X, V_Y, V_Z)$. Less attention has been paid to the quality of the terrestrial networks and their reliability in terms of $V_r, V_l$ or $(V_{XC}, V_{YC}, V_{ZC})$. This problem has been discussed though in [6] and [21]. The examples in these latter two papers showed that observational errors in a groundcontrol network may lead to distortions in the coordinates of several groundcontrol points. Such distortions may have a pseudo systematic effect on the block-adjustment. These effects are hard to detect when only datasnooping is applied in the blockadjustment.

This problems being realised, then still leaves us with the difficulty of formulating a multistage strategy for network planning with respect to reliability. For precision a criterion could be formulated by means of an artificial v.c. matrix $H^{xyz}$, without reference to the actual networkstructure. As stated earlier, the fact that there are well defined laws for the propagation of v.c. matrices is essential for a multistage criterion theory. Insertion of criterionmatrices in these propagation laws makes it possible to propagate precision criteria from one stage to another.

For reliability there are also parameters which can be used as general criteria without reference to the actual networkstructure. Such parameters are Baarda's $\lambda_0$ (see [4] [23]) or Förstner $\delta_0$ (see [9]). Recently the idea of minimum risk tests as formulated in [29] has been generalised by Bouloucos and incorporated into a planning strategy [7] [8].

For all these reliability criteria there is however a restriction that they are only useful for the design of singlestage networks. There are no propagation laws which make it possible to transfer these criteria from one stage to another.

This propagation can only be done for the boundary values of individual hypotheses. Therefore the only solution which we have now is that groundcontrol networks are simulated and that the effect of errors in individual observations on the final blockcoordinates are computed. The computational effort for this task may be considerable though, because for each proposed netstructure one should in principle compute the external reliability for each observation and then propagate that to the final blockresults.

However, the exercises made to date give some indications how controlnetworks should be designed. Their structure should be so that the effect of undetected observational errors on the networkcoordinates is local and relatively small. This means that the network should have relatively large local redundancies which can be obtained by building it up with closed loops of not more than ca 15 sides.

As an example in [6] a weak peripheral network was given of 40 sides of 900 m (fig. 8a) fully measured with $\sigma_r = 1$ mgon and $\sigma_d = 1.5$ cm/km, it supplied controlpoints at the perimeter of a block. In such a network boundary values of the coordinates up to 1.5 meter occur ($a = 0.001$ and $b = 80\%$).
As soon as there is an internal structure in the network (see fig. 8-b) the reliability improves so that boundary values reduce to 30% of their original values. Errors which stay undetected have a much less serious effect now on the final block results.

In [23] similar conclusions were found for closed traverse networks. There it is advocated that traverse loops at the periphery of the network should contain no more than 6 or 7 points and loops inside the network should contain no more than 14 points. In that case errors in the observations with the magnitude of the boundary values have an effect on the coordinates and functions thereof which does not exceed 10 times their standard deviation ($\sqrt{\lambda_o} < 10$ Baarda's notation, $\delta_o < 10$ Förstner notation).

The example in [6] and [21] made clear that the standard error detection procedures (datasnooping) are not sufficient to detect net distortions due to observational errors, in the stage of block to groundcontrol connection. During the workshop of wg III.1 in Delft March 1987, Förstner from Stuttgart University stated that a two step testing procedure might help in that case. The first step is the connection of the block to groundcontrol by a robust adjustment. The results of this step may indicate where net distortions occur. A checking of the actual network structure can then be used to formulate a special alternative hypothesis which can be tested according to Baarda's method [2]. It appeared that net distortions which were overlooked by the datasnooping technique could be found with this strategy.

5 FINAL REMARKS

The previous sections made clear that to date we have been only partly successful in designing a planning strategy for combined photogrammetric blocks and terrestrial networks. The planning is basically a two stage problem. For quality requirements this means that starting from the quality requirements of the overall system, requirements for the individual stages should be derived.
This is possible for precision by means of two criterion matrices. First a criterion matrix for the overall system is designed, then a blockstructure is chosen which fulfills the requirements for precision. By backchaining one can then derive a criterion matrix for the groundcontrol points. The landsurveyor should then design his network so that the actual precision of the groundcontrol points is not worse than the precision given by this criterion matrix.

For reliability we have been less successful. Planning strategies for one stage networks have been given in literature but no theoretical tools are available yet to generalise these to strategies for multistage networks. Network simulations are required to see which networkstructures are safe in the sense that they give reliable groundcontrol for photogrammetric blocks. This analysis is based on a forward propagation of boundary values of individual networkobservations.

No backchaining of criteria is possible yet for reliability. This means that requirements for the reliability of groundcontrol networks cannot get a clear theoretical base. Photogrammetrists have to learn by experience what requirements they should formulate. This is possible and it is also done in practice.

One should be aware, however, that by relying on experience or "Fingerspritzgefühl" these requirements might come out too strictly or too relaxed. In the first case the networks will be too expensive, in the second case the network will be cheap, but distortions may hamper blockadjustment and become expensive after all. This is not a rare case in practice.

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