A Method of Gross Error Detection for Control Points in a Block Adjustment Program with Independent Models and Its Experiments

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Abstract

In this paper a fast method of gross error detection for control points in a block adjustment program DM-PG with independent models on micro-computer IBM PC/XT is presented. A weight function of sudden drop character is adopted. A series of experiments with gross errors in control points have been executed in a test block. It shows that the location rate of gross errors in sparse horizontal control distribution has reached more than 65% in four iteration computation. The method can successfully detect two or three gross errors in control points simultaneously.

1. Introduction

In a block adjustment program DM-PG with independent models a fast method of gross error detection for control points on a micro-computer IBM PC/XT has been designed. It is able to locate a small gross error only in four iterations. But gross errors of large magnitude must be eliminated before adjustment. Therefore the test of gross errors in control points is divided in two steps. The first step carried out in data preparation stage is to detect large gross errors in control points. The preparation also provides final block adjustment with good approximate values. Therefore, it gives a good basis for the adjustment and the detection of small gross errors in the second stage.
This paper will discuss only the fast method of small gross error detection in control points in the block adjustment stage and its experiments.

2. Basic principle of the method

The block adjustment with independent models is based on least square adjustment. Therefore, it is necessary to analyse the method of gross error detection in control points according to formula of residuals of observations based on least square adjustment. It is well known from Baarda's data-snooping theory

$$V=-(QvvP_{ll})\varepsilon_l \qquad (1)$$

This formula shows the relationship between observation errors ε_l and their residuals V. It tells us that the residual v_i of an observation l_i is influenced by all observation errors ε_l; the residual influence v_i^* of an observation error ε_{li} itself depends on corresponding redundancy number r_i, which is the corresponding diagnal element of the matrix $QvvP_{ll}$. It can be expressed by following expression

$$v_i^* = -r_i \varepsilon_{li} \quad (2)$$

The r_i shows how far an error ε_{li} is revealed in the residual v_i. Redundancy number r_i is defined by the expression

$$r_i = (QvvP_{ll})_{ii}, \quad 0 \leq r_i \leq 1 \quad (3)$$

r_i of a control point coordinate in block adjustment is far smaller than 1.0. Consequently, it is difficult to distinguish a small gross error from its residual.

Let's analyse the expression (3)

$$r_i = (QvvP_{ll})_{ii} = (E - AQxxA^T P_{ll})_{ii} \quad (4)$$

where

E -the unit matrix

A -the coefficient matrix of observation error equations

Qxx -the weight coefficient matrix of unknowns

P_{ll} -the weight matrix of observations.

As the matrix $AQxxA^T$ depends mainly on geometric configuration of observations, the redundancy number r_i will increase to 1.0, when the corresponding weight P_i decreases to 0. Therefore, the gross errors revealed in their residuals will increase, if their weights decrease. It is the main reason why the weight function methods find wide use in practice of gross error detection. This idea is adopted in current method as well.

3. Mathematic model of the method

In block adjustment program DM-PG following weight function is designed for the gross error detection in the control points

$$P_i = F(w_i, Po(I)) \quad (5)$$

i.e. the weight P_i of an observation is the function of standardized residual w_i of the observation and basic weight Po(I), value of which increases with increasing the iteration number I. The complete expression of the weight function is as follows

$$P = \begin{cases} Po(I), & \text{for } w \leq C(I) \text{ or } I=1 \\ Po(I)/w_i^{a_i}, & \text{for else} \end{cases} \quad (6)$$

where

$$Po(I) = P'/T^I \quad (7)$$

is a basic weight for a control point with no gross error, in which T=0.1, P'=0.001 for x and y of control points, and P'=0.01 for z of control points.

But Po(I) is not more than 20.

$$C(I) = Co + (I-2)C_1 \quad (8)$$

is a critical value of gross errors. This value increases with increasing the iteration number I too. In expression (8)

Co=3.0, C_1=2.0 for x and y

Co=2.5, C_1=1.5 for z.

The exponent a_i is calculated by following expression

$$a_i = \theta_i^2 + 7 \qquad (9)$$

where

$$\theta_i = w_i^{(I)} / w_i^{(I-1)}$$

expresses the ratio of the standardized residuals of adjacent iterations, taking w_i =3.0 for the 1st iteration.

Standardized residual w_i of observation is obtained using

$$w_i = |v_i| / \hat{\sigma}_0 \sqrt{q_i} = |v_i| / \hat{\sigma}_c k_i \qquad (10)$$

where k_i, simplified values of q_i, are the geometric constants for two groups of control points. k_i=0.86 has obtained from table 1 for control points located at corners of the block, while k_i=1.0 for control points located on the sides, assuming the redundancy numbers of these points are the same.

$\hat{\sigma}_c = (v_i v_i / n)^{1/2}$ is the estimated standard deviation of control point coordinates after having deleted the v_i, which is larger than $3\hat{\sigma}_0$; $\hat{\sigma}_c$ will be substituted by a priori value $\hat{\sigma}_0$, if it is smaller than $\hat{\sigma}_0$.

From above parameters we can see that the weight function consists of a flat part for normal control points and a steep part for control points with gross errors. Its steep part has a sudden drop character. A diagram is shown in figure 1

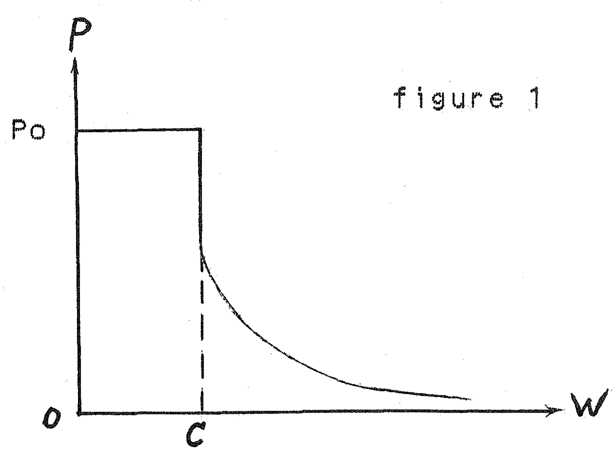

figure 1

Table 1. Redundancy numbers r_i for four locations of control points

point Location / redundancy nubber	1/4 side	1/2 side	3/4 side	corner
r_i	0.86	0.91	0.88	0.66

Table 1, obtained from experiments, presents the redundancy numbers r_i of four locations of horizontal control points after the first iteration, where the weights P_i of all these points were taken as P_i=0.01, while the weights P_i of the common points were taken as P_i=1.0. A critical value C shuld be properly selected, so that it makes the w_i of control points with no errors fall on the flat part of the weight function, and the w_i of control points with gross errors fall on its steep part. Thus the gross errors in control points will be located very fast. But a few control points, which have no gross errors, may also fall on the steep part at the second iteration, thus the rejected observations, which have no errors, called here temporary rejected true observations, will be caused. However, we can notice from tables 2, 3 and 4, that increase of w_i of the temporary rejected true observations is far less than that of control points with gross errors (see figures 3, 4). In order to avoid causing the rejected observation, which has no error, a set of control values C(I) are applied as a function of the iteration number I, i.e. the values of C increase with increasing of the iteration number.

In order to expand the weight differences between the control points with gross errors and normal points, falling simultaneously on the steep part of weight function, a term θ_i^2 is added to the exponent a_i (see expression (9)).

4. Experiments for locating gross errors in control points

The experiments were made using a real test block. It was flown with a wide-angle camera RC-10 and a photoscale of approximately 1:38000. The forward and side overlaps in strips are about 65% and 30% respectively. The area was covered by 8 strips with 16--17 photopairs each.

Six strips were used for the experiments. Two distribution schemes of horizontal control points were selected:6 and 10 horizontal control points, of which 3 and 5 points were located at each of the top and bottom sides of the block (see figure 2).

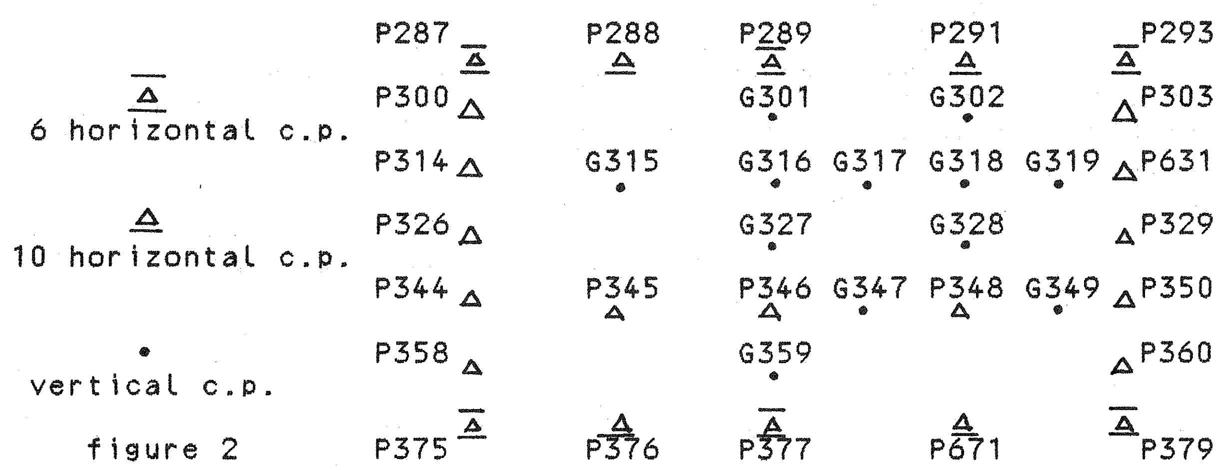

figure 2

In order to obtain the location rate, ±7m gross errors were added to x and y of all horizontal control points separately and ±6 m were down to the vertical control points.(see tables 2, 3 and 4). In addition four experiments with several gross errors were made in case of 10 control points, the values and signs of the added gross errors were the same as those, which had been located successfully in the experiments with one gross error (see table 5). Finaly two sets of experiments with sevaral gross errors larger than 20 $\hat{\sigma}_o$ were made (table 5).

In table 2, 3 and 4 part of experiments with gross errors,successfully located, and one of the temporary rejected true observations, if appeared, are listed. In every listed point the values w of corresponding iterations are illustrated.

Changes in the average values of w_i of gross errors ±7m and ±6m in control points and temporary rejected observations in four iterations are shown in figures 3 and 4.

5. Analysis of the test results

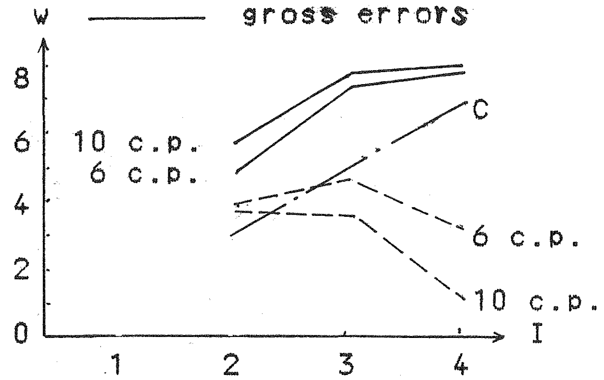

Figure 3 Changes in w of horizontal control points with gross errors and temporary rejected true observations in iterations

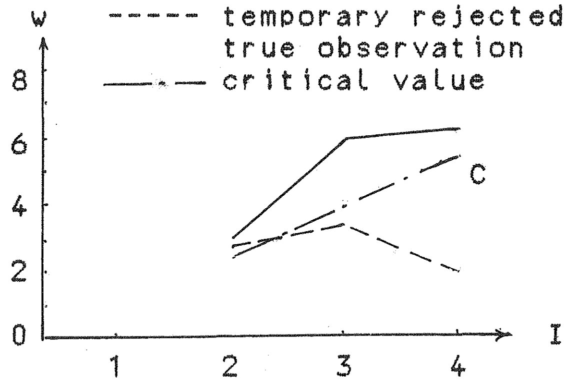

Figure 4 Changes in w of vertical control points with gross errors and temporary rejected true observations in four iterations

5.1 The table 1 shows, that the r_i of all horizontal control points located on perimeter of the block are larger than 0.65. It means, that the gross errors of these points may be revealed more than 65% in corresponding residuals of 2nd iteration. Obviously, it provides a beneficial condition for the gross error detection.

5.2 According to the estimation formula of variance $\hat{\sigma}_i^2 = v_i v_i / r_i$, following values of standard errors of the coordinates were obtained: $\hat{\sigma}_{x,y}$ =0.68m, $\hat{\sigma}_z$ =0.65m. Consequently, the values of gross errors +6--+7m, added to each control point, are about 9--10$\hat{\sigma}_0$, that belong to the small gross errors.
The results of experiments with small gross errors in control points, presented by rates of location and rejected true observation, are listed in table 6. It shows, that the location rates of the horizontal control points are equal to 67% and 78% corresponding to cases of 6 and 10 control points respectively, and that of the vertical control points is equal to 68%. Rejected true observations are absent for the horizontal control points and equal to 2% for the vertical control points.

5.3 One rejected true observation point P293 appeared in table 4, which is located just above the P303 with gross error -6m. It shows, that these two control points are strongly correlated.

5.4 The location rates of small gross errors in control points located at corners of the block equal to 68% and 81% corresponding to the cases of 6 and 10 control points are a little higher than that on sides. It makes clear that the geometric constants k_i of control points are basically correct.

5.5 The average ratios of w between 1--2 and 2--3 iteration intervals are calculated (see table 7). It shows that the average ratios θ_i of the points with gross errors are larger than that of the temporary rejected true observations in a same iteration intervals. Consequently, the term θ_i^2 in exponent (9) of the weight function is beneficial for restraining the rejected true observations and accelerating the location of gross errors in control points.

5.6 Figures 3 and 4 show that the w of gross errors corresponding to each iteration are larger significantly than the critical values of corresponding iterations. They rise fast between 2nd and 3rd iteration and slow after 3rd iteration. At the same time the w of temporary rejected true observations have a slow rise between 2nd and 3rd iteration and a fast drop after 3rd iteration. The w of temporary rejected true observations of 2nd iteration are smaller than corresponding critical values C.

6. Conclusion

6.1 The fast method of gross error detection is able to reveal all the part of gross errors in their residuals within 3 iterations, even if the block has a sparse distribution of the horizontal control points.

6.2 There are no rejected true observations appearing in the experiments of gross error detection of horizontal control points. It proved that the gross error detection of horizontal control points has reached high reliability. However, it is lower for vertical control points. In order to raise the reliability for gross error detection of vertical control points, the critical value C must be increased. But this will cause the decrease of the location rate of gross errors in vertical control points.

Table 2. The results of locating one gross error in 6 control points

test No:	gr.err. c. p. No:	tmp.rjct. tr.obs. p. No:	c. p. location	gr.err value (m)	w_i of iterations 2	3	4	residual (m)
1.1	P289		side	x= 7	6.43	8.97	8.74	7.68
1.4	P289			y=-7	8.49	10.18	10.19	-9.09
		P293	corner	y= 0	4.36	4.32	3.66	0.
2.2	P377		side	x=-7	5.83	8.79	8.75	-7.64
2.3	P377			y= 7	5.72	6.58	8.08	7.08
		P289	side	y= 0	3.46	3.50	<1.0	0.
2.4	P377			y=-7	6.04	7.37	7.19	-6.31
		P293	corner	y= 0	3.24	5.41	4.84	0.
3.2	P287		corner	x=-7	6.22	10.29	9.06	-6.58
		P293	corner	y= 0	5.10	5.79	4.38	0.
3.3	P287			y= 7	4.20	13.04	13.04	9.57
4.1	P375		corner	x= 7	4.56	8.04	8.28	5.99
		P293	corner	y= 0	3.27	5.47	4.49	0.
4.2	P375			x=-7	3.05	8.47	9.09	-5.95
4.3	P375			y= 7	3.65	7.12	9.07	6.66
		P293	corner	y= 0	3.94	4.90	3.94	0.
4.4	P375			y=-7	5.05	6.83	8.62	-7.24
		P293	corner	y= 0	3.12	4.18	3.79	0.
5.2	P293		corner	x=-7	5.35	7.95	12.54	-9.44
		P289	side	y= 0	3.56	3.82	<1.0	0.
5.3	P293			y= 7	7.95	11.30	13.29	9.75
		P289	side	y= 0	3.81	2.86	<1.0	0.
6.1	P379		corner	x= 7	7.02	8.75	9.33	8.64
		P293	corner	y= 0	4.65	6.32	4.43	0.
6.3	P379			y= 7	3.16	5.86	9.32	7.06
		P293	corner	x= 0	3.42	3.37	<1.0	0.
6.4	P379			y=-7	5.21	8.16	8.42	-6.50
		P293	corner	y= 0	3.42	5.24	4.32	0.

Table 3. The results of locating one gross error in 10 horizontal control points

test No:	gr.err. c. p. No:	tmp.rjct. tr.obs. p. No:	c. p. loca-tion	gr.err. value (m)	w_i of iterations 2	3	4	resi-dual (m)
1.1	P287		corner	x= 7	6.03	5.07	8.16	10.16
		P288	side	x= 0	4.31	5.03	2.26	0.
1.3	P287			y= 7	4.61	9.82	10.61	7.79
2.2	P288		side	x=-7	7.39	11.09	10.47	-8.96
		P293	corner	y= 0	3.44	3.47	<1.0	0.
2.4	P288			y=-7	8.81	9.99	10.40	-9.04
		P293		y= 0	4.31	4.89	2.66	0.
3.1	P289		side	x= 7	5.40	7.19	8.34	7.12
3.2	P289			x=-7	4.24	6.08	7.80	-6.88
		P293		y= 0	3.14	3.93	<1.0	0.
3.4	P289			y=-7	8.91	8.89	9.13	-7.95
		P293		y= 0	4.36	5.08	2.74	0.
4.1	P291		side	x= 7	4.67	9.32	9.99	8.76
4.3	P291			y= 7	8.04	8.11	8.91	7.69
		P288		x= 0	3.44	3.96	<1.0	0.
5.1	P293		corner	x= 7	5.21	8.02	9.53	7.29
		P288		x= 0	3.21	3.01	1.02	0.
5.2	P293			x=-7	4.31	9.76	9.07	-6.50
5.3	P293			y= 7	8.89	10.79	12.35	9.10
		P288		x= 0	3.41	3.62	<1.0	0.
6.1	P375		corner	x= 7	6.42	9.24	10.39	7.79
		P293		y= 0	3.64	3.22	<1.0	0.
6.2	P375			x=-7	3.90	7.68	8.11	-7.32
6.3	P375			y= 7	5.67	6.13	8.17	5.99
		P288		y= 0	3.25	3.19	<1.0	0.
6.4	P375			y=-7	5.25	9.50	10.30	-7.76
		P293		y= 0	3.02	3.49	<1.0	0.
7.1	P376		side	x= 7	7.04	8.27	9.02	7.92
		P288		x= 0	3.02	3.50	<1.0	0.
7.2	P376			x=-7	4.17	7.22	7.00	-6.01
7.3	P376			y= 7	4.58	7.92	8.29	7.27
7.4	P376			y=-7	6.40	7.80	7.80	-6.72
		P288		x= 0	3.16	3.12	<1.0	0.
8.1	P377		side	x= 7	7.25	7.82	8.65	7.60
		P293		y= 0	3.66	3.13	<1.0	0.
8.2	P377			x=-7	4.40	7.42	7.45	-6.70
		P293		y= 0	3.15	3.93	<1.0	0.
8.3	P377			y= 7	6.32	6.70	8.09	7.09
		P293		y= 0	3.33	3.02	<1.0	0.
8.4	P377			y=-7	5.96	6.84	7.88	-6.88
		P293		y= 0	4.25	5.84	2.87	0.
9.1	P671		side	x= 7	6.76	7.33	7.85	6.67
		P293		y= 0	4.37	3.53	<1.0	0.
9.2	P671			x=-7	4.23	6.85	8.19	-7.29
9.3	P671			y= 7	7.66	8.30	9.05	7.94
		P288		y= 0	3.09	3.47	<1.0	0.
10.1	P379		corner	x= 7	7.57	10.04	10.88	8.37
		P293		y= 0	5.76	4.55	2.38	0.
10 2	P379			x=-7	4.05	5.92	7.04	-5.38
10.3	P379			y= 7	5.46	8.20	9.01	6.73
10.4	P379			y=-7	7.56	8.77	9.48	-6.96
		P293		y= 0	4.06	3.33	<1.0	0.

Table 4. The results of locating one gross error in vertical control points

test No:	gr.err. c. p. No:	tmp.rjct. tr.obs. p. No:	c. p. location	gr.err. value (m)	w_i of iterations 2	3	4	residual (m)
1	P287		corner	z= 6	3.05	7.66	8.17	7.10
		P293	corner	z= 0	2.51	2.60	<1.0	
2	P293		corner	z= 6	4.96	9.39	8.88	7.67
3	P289		side	z=-6	4.04	9.19	9.96	-8.67
4	P288		side	z=-6	2.60	3.30	5.52	-4.87
5	P377		side	z=-6	2.50	5.91	6.08	-4.30
6	p379		corner	z= 6	3.98	11.40	11.24	7.65
		P293		z= 0	2.68	2.80	<1.0	0.
7	P291		side	z= 6	2.81	7.19	7.92	6.87
8	P375		corner	z= 6	3.97	10.94	10.75	8.09
9	P671		side	z=-6	3.28	6.74	6.48	-6.16
		P293		z= 0	2.56	2.64	<1.0	
10	P631		side	z= 6	3.45	6.42	7.39	6.32
11	P329		side	z= 6	3.07	6.03	6.09	6.40
12	G328		inside	z=-6	4.21	7.40	7.27	-6.02
13	G317		inside	z= 6	3.02	8.80	9.07	7.86
14	P346		inside	z=-6	3.64	6.99	6.80	-5.25
15	P292		side	z=6	2.65	6.64	7.33	6.33
16	G318		inside	z=-6	3.72	7.47	7.47	-6.54
17	G301		inside	z=-6	3.34	5.32	6.60	-6.17
18	P290		side	z= 6	2.64	6.26	8.40	7.33
19	P360		side	z= 6	3.45	5.72	5.56	5.05
		P293	corner	z= 0	2.66	2.70	<1.0	0.
20	G302		inside	z=-6	4.35	8.31	8.01	-6.65
21	P303		side	z=-6	1.24	<1.0	<1.0	0.
		P293		z= 0	3.42	7.07	6.80	6.09
22	G327		inside	z=-6	3.83	7.25	7.19	-6.20
23	P344		side	z= 6	2.65	5.93	5.75	4.87
24	P319		inside	z=-6	4.14	8.84	8.49	-7.23
25	G315		inside	z=-6	3.78	7.49	7.81	-6.51
26	P314		side	z= 6	3.72	7.99	7.82	6.47
27	P348		inside	z=-6	3.41	6.26	6.29	-5.41
28	P326		side	z= 6	3.62	7.61	7.80	6.80

Table 6. The rates of location of gross errors and rejected true observations in control points

gross err. / tst schem.	±6m test number	location number	location rate	rejected true obs. nmbr	rejected true obs. rate	±7m test number	location number	location rate	rejecting true obs. nmbr	rejecting true obs. rate
6 c.p.	/	/	/	/	/	24	16	0.67	0	0.0
10 c.p.	/	/	/	/	/	40	31	0.78	0	0.0
v.c.p.	41	28	0.68	1	0.02	/	/	/	/	/

Table 5. The results of locating several gross errors in 10 horizontal control points

gr. err. nmb.	gr.err. c. p. No:	tmp.rjct. tr.obs. p. No:	c. p. location	gr.err. value (m)	w_i of iterations 2	3	4	residual (m)
2	P287	no	corner	x= 7	4.56	7.80	13.44	10.26
	P376		side	x=-7	4.68	6.50	6.82	0.
2	P275	no	corner	x=-7	3.50	7.81	8.20	-5.99
	P293		corner	y= 7	5.96	12.29	12.11	9.03
3	P376		side	y=-7	3.49	7.89	7.77	-6.68
3	P289	no	side	x= 7	4.90	8.54	8.34	7.11
	P379		corner	y=-7	4.09	9.38	9.43	-7.00
	P376		side	x= 7	3.06	8.51	9.15	8.07
3	P377	no	side	y=-7	3.51	7.87	8.40	-7.33
	P671		side	x= 7	3.43	7.46	7.79	6.60
	P287		corner	y= 20	15.17	14.25	23.58	21.17
	P377		side	x=-30	22.85	20.35	33.54	-29.26
3	G315		inside	z= 20	2.50	6.17	21.61	19.24
		P288	side	y= 0	5.05	2.45	<1.0	0.
		P379	corner	x= 0	5.76	4.19	<1.0	0.
	P376		side	y= 20	8.16	19.14	23.19	20.36
	P289		side	y= 20	6.77	18.93	22.27	19.44
4	P293		corner	x=-20	6.79	19.25	20.91	-18.44
	G317		inside	z= 20	3.05	23.21	24.46	21.62
		P375	corner	y= 0	4.50	4.47	<1.0	0.
		P288	side	y= 0	4.51	3.38	<1.0	0.

Table 7. The average ratios of w between adjacent iterations

types of control points \ iteration interval	6 control points 1--2	2--3	10 control points 1--2	2--3
with gross errors	1.80	1.47	2.01	1.21
temporary rejecting true observations	1.30	1.25	1.22	1.02

6.3 This method is also effective for locating several gross errors simultaneously and such gross errors, values of which are larger than $20\hat{\sigma}_0$ (see table 5).

6.4 From the critical value C(4) of gross errors we can generaly say, that this method can't detect such gross errors, values of which are less than $7\hat{\sigma}_0$ and $5.5\hat{\sigma}_0$ respectively.

The fast method of gross error detection works good, even if the geometry is weak (in case of 6 control points). But one can see from figure 3, that danger of causing rejected true observation in case of 6 control points is far larger than that of 10 points. Therefore,the poor geometry like the case of 6 horizontal control points should be avoided in practice.

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