Photogrammetric Treatment of Linear Features

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Abstract

Current analytical treatments in photogrammetry are based almost exclusively on point fields. While these have worked well for hard copy photographs and human operators, the introduction of digital imagery and the desire to apply automated techniques require new treatment based on other features. Object space features can be in the form of points, lines (straight and otherwise), and surfaces (planar and curved). Analytical photogrammetric techniques should be developed to treat the relationship between such object features and their projections in the image space. Both functional and stochastic models are required.

In this paper a rigorous mathematical development is presented for the photogrammetric observation condition equations of the straight linear feature and the conic sections. Consideration is given to various photogrammetric operations such as: space intersection, space resection, and triangulation by using these linear features.

1. Introduction

For the past several decades, photogrammetric activities dealt with hard copy photographic records, suitably designed equipment to accommodate the photographs, and the human observer to manipulate the instruments and extract the desired information. While this well established approach remains in much use, the past few years have brought significant advances which will potentially change the standard approach quite dramatically. The fundamental factor causing the change is embodied in the word digital; digital imagery, digital processing, and digital products.

We have seen significant increase in the acquisition of digital imagery, ranging from those taken from space to images acquired in the laboratory for industrial applications. Digital processing has actually been in existence for a relatively longer period, including batch processing in, for example, triangulation, and on-line reduction involved in analytical plotters and the like. Digital products followed logically from digital processing, where the restrictive graphical products were slowly replaced by the much more flexible digital form, which can readily be used to produce a variety of hardcopy graphics.

Of the three elements of advancement, digital imagery is likely to have the most profound impact on modernization of the photogrammetric tasks. It opens the door to the implementation of a multitude of digital image processing and analysis operations. Such operations will slowly take over tasks currently performed by the human operator.

Current photogrammetric reduction relies almost exclusively on "points", both in the object and their corresponding images. Such points are described by their 3D object, or 2D image coordinates; and the mathematical formulation of photogrammetry has essentially been based solely on point correspondence. With digital imagery, and digital image processing, points actually are much less attractive and usable than other features such as lines and areas. Remarkably, this may actually have also been the strength of the human observer. In order to rigorously accommodate the extraction and correspondence of these features, suitable photogrammetric formulation must be performed. It is critical to point out that in dealing with features, for example a line segment, we must be so general as to assume no point correspondence. Thus, a line feature on the ground may be represented by two different line segments, one on each of two images, where one represents an object segment totally different from that represented by the other.

One may come to realize that we are now entering an era of feature-based photogrammetry. Not only should the photogrammetrist develop the proper theory to deal with corresponding features and their reduction to object space, but he should lend support to solving the correspondence and matching problem. The insight and geometric knowledge provided by experience in photogrammetry, can be effectively used in seeking a better solution to this problem. Furthermore, such experience can provide support in a better approach to three-dimensional solid object modeling. Finally, with the extensive experience of the photogrammetrist in the treatment of stochastic properties of observation, a much needed treatment of linear and area features can be properly done.

Recognizing the extensive nature of the subject, this paper will be limited to treating some aspects of linear features. However, research is currently proceeding at Purdue University on various aspects of this topic.

A linear feature may be described as the path taken by a point as it travels through space. Examples are: the straight linear feature $\mathbb{L}$, the circular linear feature $\mathbb{C}$, the helix, and so on. Linear features are classified as two dimensional and three dimensional.

The image of a three dimensional linear feature is a linear feature in the two dimensional image plane. There are two main ways to utilize the image space information. The first method would be to use the geometric relationship due to the projection of the three dimensional linear feature. For example, the image of a straight linear feature $\mathbb{L}$ is projected as a straight linear feature, and the image of a circular linear feature $\mathbb{C}$ is generally an ellipse. However, besides these two examples the determination of the geometric description of the image of a linear feature becomes very difficult as the object space linear feature becomes complex. The second method would be to realize that the image of a linear feature is a collection of points recorded by a photogrammetric process in such a way that the directions of the light rays have been preserved. This second method is able to accommodate many different types of linear features whose geometric shape in the image would be difficult to determine. Furthermore, this method can be easily extended to other sensors such as the linear array and the optical-mechanical scanner. We will call this method: **Object Space Form Fitting of a Linear Feature** and it is the method detailed in this paper.

Typically a linear feature will be found in the image space of several cameras. The problem of determining the object space position of a linear feature would be greatly reduced if the images are brought into orientation and correspondence between the points in the images and the points in the object space has been assigned. This case would be a matter of form fitting in the object space only since the object space coordinates of the points on the linear feature would be known, see Mikhail and Mulawa (1985) for some techniques. However, this matching of conjugate image points is generally done by a human observer and furthermore the matching of conjugate image points is unnecessary for the determination of the object space linear feature. In this paper, we assume that the conjugate matching of image points has neither been attempted nor accomplished.

It is important to determine whether the photogrammetric observation of an arbitrary point on a linear feature provides redundancy. Note that in general, the photogrammetric recording of a ray or direction represents two degrees of freedom. In the case of frame photography this would be the photo coordinates. Each point on a fixed linear feature requires only one degree of freedom. For example, each point on a linear feature can be made to correspond to the arc length from a given point on the linear feature; thus requiring only one degree of freedom: arc length. Therefore we find that each photogrammetric ray affords a single redundancy, and this can be used in a redundancy budget to solve for treatments such as: Space Intersection, Space Resection, and Triangulation.

Since the form fitting will be done in the object space, we wish to develop the relationship between the object space and the image space. Recall the collinearity equations for point features:

$$\begin{bmatrix} x \\ y \\ -f \end{bmatrix} = k\,M\,(P-L) \tag{1.1}$$

where:				
x,y	are reduced photo coordinates	M	is a rotation matrix taking object into image
f	is the focal length	P	an arbitrary point in the object space
k	is the scale factor	L	the perspective center

We specify that (x,y) are the reduced photo coordinates for simplicity and require that systematic corrections such as: reduction to principal point, radial distortions, and atmospheric refraction have already been performed. Note that a direction vector ρ in the object space that corresponds to the observed image space vector is:

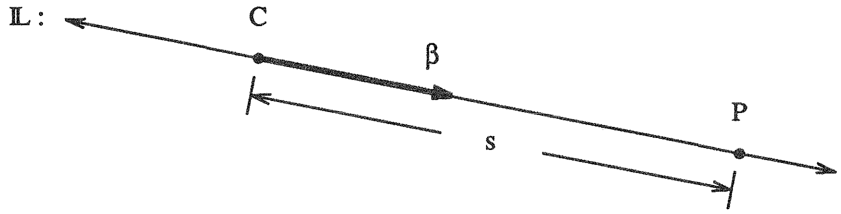
$$\rho = M^T \begin{bmatrix} x \\ y \\ -f \end{bmatrix} \tag{1.2}$$

As review, three vector products are very useful and their notation is presented here. Let: α, β, and γ be direction vectors. Then the dot product or scalar product is denoted as: $\alpha \circ \beta$, the cross product as: $\alpha \times \beta$, and the scalar triple product as: $\alpha \circ \beta \times \gamma$ or the determinant $|\ \alpha\ \ \beta\ \ \gamma\ |$. Also, the norm of a vector is denoted by: $\|\alpha\|$.

2. The Straight Linear Feature: $\mathbb{L}$

2.1 Standard Form of the Parametric Representation

We believe that the best way to describe the construction of a straight linear feature $\mathbb{L}$ is through the use of a fixed point on the line $\mathbb{L}$ and a vector in the direction of the line $\mathbb{L}$. The **Parametric Straight Line Equations** are a well known set of equations that can be used to describe a straight linear feature $\mathbb{L}$ in the three dimensional object space.



$$\mathbb{L}: \; P = C + s\beta \tag{2.1}$$

where:

	$P = [x_P \; y_P \; z_P]^T$	any point on the line $\mathbb{L}$
	$C = [x_C \; y_C \; z_C]^T$	a fixed point (center) of the line $\mathbb{L}$
	$s =$	the length scalar
	$\beta = [x_\beta \; y_\beta \; z_\beta]^T$	the direction of the line $\mathbb{L}$

The length scalar s is a nuisance parameter which is associated one to one with the point P on the line $\mathbb{L}$. The observation to underscore is that all the information needed to describe the straight linear feature $\mathbb{L}$ is contained in the direction vector β and the fixed point C. For this reason we will call the fixed point C and the direction vector β the **primary descriptors** of the straight linear feature $\mathbb{L}$. We state this as:

$$\mathbb{L}: \; \{C, \beta\} \tag{2.2}$$

Representation of the line $\mathbb{L}$ by a fixed point C on the line $\mathbb{L}$ and a vector β in the direction of the line $\mathbb{L}$ is not unique. Two more conditions are needed to ensure that the representation of the line $\mathbb{L}$ is unique. First, we require the direction vector β to have unit length.

$$\|\beta\| = 1 \tag{2.3}$$

or, equivalently,

$$\beta \circ \beta = 1^2 \tag{2.4}$$

The advantage of this is that we can interpret the length scalar s as the Euclidian distance or arc length from the fixed point C to the point P. For example:

$$|s| = \|P - C\| \tag{2.5}$$

There still exists one degree of freedom in selecting which point C is to be fixed on the line $\mathbb{L}$. We note that there exists a unique point on the line $\mathbb{L}$ which is closest to the origin O.

$$\|C\| \rightarrow \text{minimum!} \tag{2.6}$$

It can be seen from the figure below that for C to be the closest point on the line $\mathbb{L}$, then the vector $C - O$ is orthogonal to the direction β of the line $\mathbb{L}$. Thus we have:

$$\|C\| \rightarrow \text{minimum!} \; \leftrightarrow \; (C - O) \perp \beta \; \leftrightarrow \; C \perp \beta \; \leftrightarrow \; C \circ \beta = 0 \tag{2.7}$$

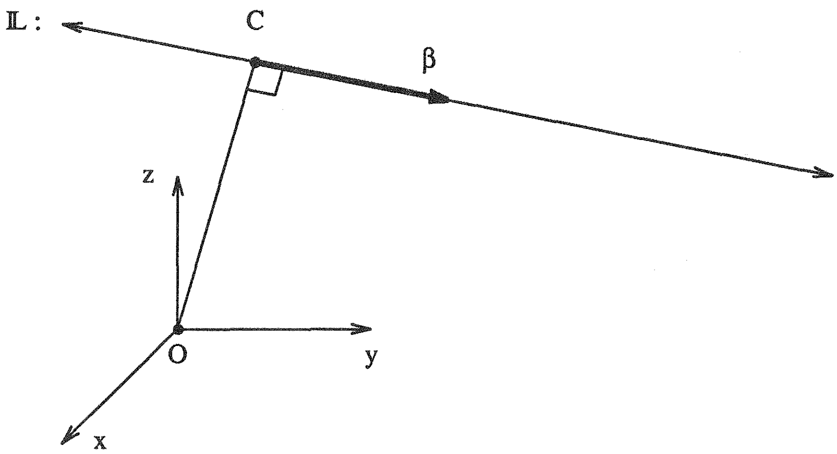
This gives us our second condition to be enforced:

$$C \circ \beta = 0 \tag{2.8}$$

This closest point C on the line $\mathbb{L}$ deserves a special name to distinguish it from all the points on the line and we will call this closest point the **center** C of the line $\mathbb{L}$.

There is a special case that occurs when the line $\mathbb{L}$ passes through the origin O, where the center C must also be the origin O. This special case will satisfy equation (2.8).

A review of equation $\mathbb{L} : P = C + s\beta$ shows that each point P on the line $\mathbb{L}$ is composed of a linear combination of two orthogonal components: C and β. In this linear combination one component C is fixed, but the other component $s\beta$ has a variable length equal to $|s|$.



There is another item to address. We may represent a line as:

$$\mathbb{L}: \ P = C + s\beta = C + (-s)(-\beta) \tag{2.9}$$

This indicates that the sign of the direction β is not resolved. In general, this must be a convention and the sign is not recoverable. However, the sign selection of the direction β only affects the sign of the length scalar s. For these reasons the sign of the direction vector β will not be a problem.

The line descriptors C and β represent 6 unknowns, which when taken together with the two constraints: $\|\beta\| = 1$ and $C \circ \beta = 0$, reveal that the straight linear feature $\mathbb{L}$ has $4 = 6-2$ independent unknowns. Determination of independent descriptors for the line $\mathbb{L}$ is highly desirable for adjustment techniques and analysis of the propagation of variances. However in using independent descriptors the geometric reasoning and interpretation is not as clearly presented as when using the center C and direction β as line descriptors. Some of our future research will be the investigation of possible independent descriptors.

The selected representation which enforces equations (2.3) and (2.6) is called the **standard form:**

$$\mathbb{L} \ \text{(standard form)}: \ \{C,\beta \ \mid \ \|\beta\| = 1, \|C\| \rightarrow \text{minimum!}\} \tag{2.10}$$

or equivalently:

$$\mathbb{L} \ \text{(standard form)}: \ \{C,\beta \ \mid \ \beta \circ \beta = 1^2, C \circ \beta = 0\} \tag{2.11}$$

The use of the center point C and the direction vector β as straight linear feature $\mathbb{L}$ descriptors offer much flexibility, freedom from degenerate cases, and a straight forward geometric reasoning.

2.2 Method of Additional Parameters

Recall that the recording of a photogrammetric ray of a linear feature represents a single redundancy. Suppose we wish to carry both the intersection point P and the scalar s from the parametric straight line equations (2.1) in our model. This would give four additional unknowns and when combined with the single redundancy we find that each arbitrary point observed on the line $\mathbb{L}$ requires five condition equations. For this model we suggest using two collinearity equations and three straight line $\mathbb{L}$ form fitting equations. Thus:

$$\begin{aligned} x &= -f\,\frac{m_{11}(x_P-x_L) + m_{12}(y_P-y_L) + m_{13}(z_P-z_L)}{m_{31}(x_P-x_L) + m_{32}(y_P-y_L) + m_{33}(z_P-z_L)} \\ y &= -f\,\frac{m_{21}(x_P-x_L) + m_{22}(y_P-y_L) + m_{23}(z_P-z_L)}{m_{31}(x_P-x_L) + m_{32}(y_P-y_L) + m_{33}(z_P-z_L)} \\ P &= C + s\beta \end{aligned} \tag{2.12}$$

Another similar method would be to eliminate the three straight line $\mathbb{L}$ fitting equations by substituting the parametric straight line equations into the collinearity equations (1.1) This leaves one additional unknown s and a single redundancy, thus giving two condition equations:

$$x = -f\,\frac{m_{11}(x_C+sx_\beta-x_L) + m_{12}(y_C+sy_\beta-y_L) + m_{13}(z_C+sz_\beta-z_L)}{m_{31}(x_C+sx_\beta-x_L) + m_{32}(y_C+sy_\beta-y_L) + m_{33}(z_C+sz_\beta-z_L)}$$

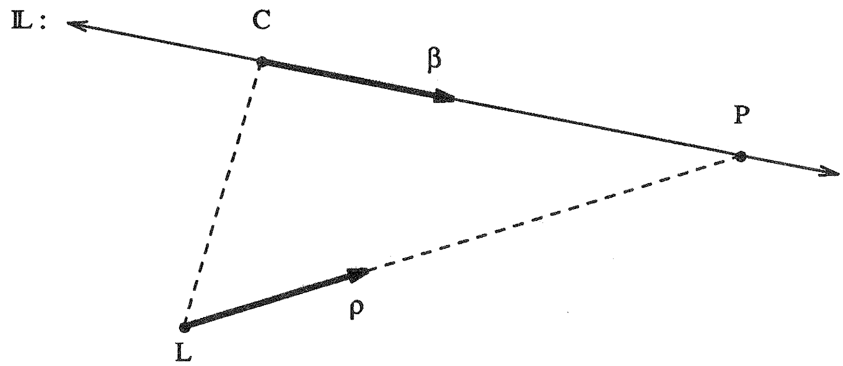
$$y = -f\,\frac{m_{21}(x_C+sx_\beta-x_L) + m_{22}(y_C+sy_\beta-y_L) + m_{23}(z_C+sz_\beta-z_L)}{m_{31}(x_C+sx_\beta-x_L) + m_{32}(y_C+sy_\beta-y_L) + m_{33}(z_C+sz_\beta-z_L)} \tag{2.13}$$

While both of the above models will work adequately they may be sensitive to initial approximations and require solving more equations and carrying more unknowns than is necessary. This stems from the fact that equations (2.12) and (2.13) are more of an algebraic solution than a geometric solution as is presented in the next section.

2.3 Coplanarity Relationship of the Straight Linear Feature: $\mathbb{L}$

Recall that recording a photogrammetric ray of a linear feature represents a single redundancy. Thus the photogrammetric relation of the observation of an arbitrary point P on a straight linear feature $\mathbb{L}$ can be represented by a single condition equation if no additional parameters are included.

Consider the object space geometry represented in the figure below.



where:

where:	C	the center of the line $\mathbb{L}$	P	an intersection point on the line $\mathbb{L}$
	β	the direction of the line $\mathbb{L}$	L	the perspective center
	ρ	the observed direction vector to the point P on the line $\mathbb{L}$		

The direction vector ρ of the photogrammetric ray, the direction vector β of the line $\mathbb{L}$, and the direction vector formed by the perspective center L and the center C of the line $\mathbb{L}$ must be **coplanar.**

$$|\ \rho\ \ \beta\ \ (C-L)\ | = 0 \tag{2.14}$$

Note that this representation is similar to the coplanarity equation used in photogrammetry for point features. In our case, the center C and the direction β act like a camera that can only look in one direction. This is a very convenient form to represent the redundant condition equation. The intersection point P is not in equation (2.14) because the coplanarity relationship captures the essence of the geometry without needing the nuisance parameters of the points P on the line $\mathbb{L}$. Equation (2.14) will allow for the determination of the primary descriptors $\mathbb{L}$: $\{C,\beta\}$ without the the need to determine the intersection points P on the line $\mathbb{L}$.

Since the coplanarity relation is written for a single photogrammetric ray, the following is an indexing system that will allow for several intersection points P on a line $\mathbb{L}$ as well as several cameras L and several lines $\mathbb{L}$.

Let: i be the camera number, j be the line number, k be the point number

Then the coplanarity relationship for the straight linear feature $\mathbb{L}_j$ as written for condition equation F_{ijk}:

$$F_{ijk} = |\ \rho_{ijk}\ \ \beta_j\ \ (C_j - L_i)\ | = 0 \tag{2.15}$$

Where:

$$\rho_{ijk} = M_i^T \begin{bmatrix} x_{ijk} \\ y_{ijk} \\ -f_i \end{bmatrix} \tag{2.16}$$

subject to the standard form representation of the straight linear feature $\mathbb{L}_j$:

$$\mathbb{L}_j \text{ (standard form): } \{C_j, \beta_j \mid \beta_j \circ \beta_j = 1^2, C_j \circ \beta_j = 0\} \tag{2.17}$$

2.4 Photogrammetric Treatments Using the Straight Linear Feature $\mathbb{L}$

There are three main photogrammetric treatments that can be applied to linear features: Space Intersection, Space Resection, and Triangulation.

Space Intersection can be used to determine the object space position of a straight linear feature $\mathbb{L}$ given two or more cameras each with known position and orientation. Caution must be exercised to avoid viewing the straight linear feature $\mathbb{L}$ solely as an epipolar view. For example, suppose that two cameras are used and the image of the line $\mathbb{L}$ falls on an epipolar line between the cameras. Then we may determine the plane in the object space that contains the line $\mathbb{L}$ but can not determine the location of the line $\mathbb{L}$ in that plane.

A straight linear feature $\mathbb{L}_j$: $\{ C_j, \beta_j \}$ may be uniquely determined by using two photogrammetric rays from each of two different cameras. This would give four condition equations of the form (2.15) which when combined with the two constraint equations given in (2.17), yield six equations. Since we have six unknowns $\{ C_j, \beta_j \}$, and the equations are independent this satisfies a unique solution for $\mathbb{L}_j$: $\{ C_j, \beta_j \}$.

If more than the minimum amount of observations are available, then a least squares adjustment is used. A least squares adjustment program has been written to test the numerical stability and computational effort required to perform this space intersection. The numerical experiments indicate that the coplanarity relationship of the straight linear feature $\mathbb{L}$ is very stable and robust with respect to the initial approximations. This may be indicated by an examination of some of the partial derivatives. Dropping the subscripts from (2.15) gives:

$$\frac{\partial F}{\partial \rho} = \left[\beta \times (C - L) \right]^T \tag{2.18}$$

$$\frac{\partial F}{\partial \beta} = \left[(C - L) \times \rho \right]^T \tag{2.19}$$

$$\frac{\partial F}{\partial C} = \left[\rho \times \beta \right]^T \tag{2.20}$$

From inspection of the previous figure, it can be seen that each of these partial derivatives converges to a vector which is normal to the plane defined by line $\mathbb{L}$ and the perspective center L. These directions for the gradient of the observations and unknowns offer strong convergence and numerical stability. The aspect of computational effort was found to be quite modest. In particular, since in equation (2.15) the observation of each photogrammetric ray ρ_{ijk} appears in only one equation, the coefficient matrix for the observations will be a block diagonal. If the observations of all the photogrammetric rays are uncorrelated with the other photogrammetric rays then the covariance and cofactor matrix of the observations will also be block diagonal. This offers a suitable structure to allow for the accumulation of reduced normal equations by using direct summation techniques as is commonly used for the point feature photogrammetry. Furthermore, this block structure will allow the partitioning of the unknowns in such a way as to allow for the determination of each line $\mathbb{L}_j$ separately from the rest.

One item to address in this section is the analysis of the covariance or cofactor matrix of the estimated straight linear feature $\mathbb{L}_j$. The matrix for $\mathbb{L}_j$ will be a 6×6 matrix, however its rank will be equal to four due to the constraints (2.17). Possibly, the best way to analyze this rank deficient matrix is by using a spectral decomposition to produce the eigen values and eigen vectors. From this decomposition the degenerate ellipsoid can be analyzed with respect to principal dimensions and orientation.

Space Resection is the determination of the position of the perspective center and the orientation of a frame camera by using features in the object space as control. In this case we will use the straight linear feature $\mathbb{L}$ as control. Note that the image of a straight linear feature $\mathbb{L}$ is a straight line in the image plane provided that all distortions have been removed. Since this line in the image plane can be defined by two distinct points, it follows that each straight linear feature $\mathbb{L}$ that is observed offers two photogrammetric rays and thus two condition equations. Additional photogrammetric rays offer redundancy in the feature definition and hence they can increase the reliability, but the additional rays do not contain additional information needed to solve for the resection. Since the resection of a frame camera requires six variables, and each observed line $\mathbb{L}$ offers two condition equations, space resection then requires a minimum of three straight linear features $\mathbb{L}$. Caution must be exercised to avoid degenerate cases. For example, using three parallel lines $\mathbb{L}$ will not give a unique solution. Also if three lines $\mathbb{L}$ intersect at a common point, then this point can move along a line connecting it to the perspective center L, while its image remains unchanged. These degenerate cases, as well as others, do not discount the utility of this method but instead are given to advise the knowledgeable use of the method.

A least squares adjustment program was written to perform the space resection of the frame camera using linear features $\mathbb{L}$ and to allow for some numerical experiments. The results of the experiments indicate that the coplanarity relationship for the straight linear feature (2.15) is well suited to the task of resection. The robustness to initial approximations is not as good as in the case of space intersection, but the convergence was strong when given reasonable initial approximations.

Triangulation using straight linear features $\mathbb{L}$ as "pass features" or "tie features" is implied by the successful use of these features for the photogrammetric treatments of intersection and resection. However at the time of the writing of this paper, numerical experiments have not yet been performed to test the numerical stability and robustness of the method. These numerical experiments are part of our on-going research. One interesting note is that it is not necessary to have overlapping imagery to perform triangulation using linear features. This is particularly the case when given imagery from linear CCD array acquired on the push-broom principle. In this case each linear array image appears as a narrow frame image and the triangulation of straight linear features $\mathbb{L}$ written as additional condition equations may increase the stability of the determination of the along track positions and orientations. In other words, the lines $\mathbb{L}$ are used to help "stitch" together the adjacent array images. Numerical experiments are planned to determine the increase, if any, in stability. Note further, that by using the coplanarity relationship (2.15) on the individual pixels of the images that the form fitting for the line $\mathbb{L}$ takes place in the object space where it belongs.

3. The Conic Sections: $\mathbb{C},\mathbb{E},\mathbb{H},\mathbb{P}$

Several commonly used linear features are known as the conic sections. Specifically these linear features are: the Circular linear feature $\mathbb{C}$, the Ellipse $\mathbb{E}$, the Parabola $\mathbb{P}$, and the Hyperbola $\mathbb{H}$. In this section we offer condition equations suitable for the photogrammetric observation of these conic sections. Since the nature of the derivation is similar from one conic section to the other we will present the circular linear feature $\mathbb{C}$ in some detail and the others in a briefer version. The treatments of space intersection, space resection, and triangulation can be applied to the conic sections (and any linear feature in general) by applying techniques as shown for the straight linear feature $\mathbb{L}$, but space only permits the stating of condition equations in this section.

3.1 The Circular Linear Feature: $\mathbb{C}$

In the three dimensional space a circular linear feature $\mathbb{C}$ can be described as the intersection of a sphere and a plane containing the center of the sphere. Thus each point P contained in a circular linear feature $\mathbb{C}$ must satisfy these two condition equations:

$$\|P-C\| = r \tag{3.1}$$

$$(P-C)\circ\eta = 0 \tag{3.2}$$

where:
- P is any point on the circle $\mathbb{C}$
- C is the center of the circle $\mathbb{C}$
- η is a normal to the plane of the circle $\mathbb{C}$
- r is the radius of the circle $\mathbb{C}$

Note that the length of the normal η to the plane of the circle $\mathbb{C}$ does not play any part in the condition equations. Thus we select the standard form of the circle $\mathbb{C}$ by requiring that the normal η have unit length.

$$\|\eta\| = 1 \tag{3.3}$$

$$\mathbb{C}\text{ (standard form)}: \ \{\, C,\eta,r \ \mid \ \|\eta\| = 1 \,\} \tag{3.4}$$

Note that $\mathbb{C}$: { C, η, r } represents 3+3+1=7 unknowns. But the constraint equation (3.3) implies that 7-1=6 independent unknowns are found in a circular linear feature $\mathbb{C}$. However, we prefer to use the primary descriptors as given in statement (3.4). At this point we note that it is possible to follow section 2.2, Method of Additional Parameters, as was done for the straight linear feature $\mathbb{L}$. In the circular case we may include three additional parameters P per photogrammetric observation, and when combined with the single redundancy offered by the observation, we would write four condition equations. Taking two collinearity equations and the two circular linear feature equations to obtain:

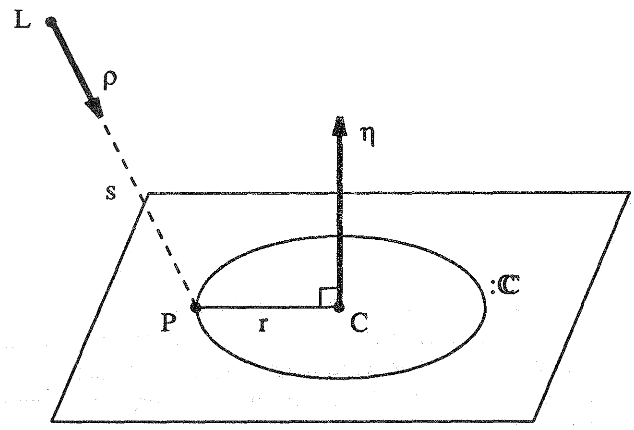
$$x = -f\,\frac{m_{11}(x_P-x_L) + m_{12}(y_P-y_L) + m_{13}(z_P-z_L)}{m_{31}(x_P-x_L) + m_{32}(y_P-y_L) + m_{33}(z_P-z_L)}$$

$$y = -f\,\frac{m_{21}(x_P-x_L) + m_{22}(y_P-y_L) + m_{23}(z_P-z_L)}{m_{31}(x_P-x_L) + m_{32}(y_P-y_L) + m_{33}(z_P-z_L)}$$

$$\|P - C\| = r \tag{3.5}$$

$$(P - C)\circ\eta = 0$$

This method can be very powerful, but unless it is necessary to carry the intersection points P for some other additional constraints the intersection points P should be left out of the model to reduce the number of equations and unknowns. This leads to a single condition equation as follows.



Referring to the figure we see that the collinearity relationship for point features is given as:

$$P = L + s\rho \tag{3.6}$$

Note that in this expression the scalar $s = 1/k$, where k is the scale factor of equation (1.1). Equations (3.1), (3.2), and (3.6) represent five condition equations containing four additional parameters $\{ P, s \}$. We will derive a single condition equation by determining where the photogrammetric ray (3.6) intersects the plane (3.2) and then use the remaining sphere equation (3.1). Thus substituting (3.6) into (3.2) we obtain:

$$((L + s\rho) - C)\circ\eta = 0 \tag{3.7}$$

Expanding:

$$(L - C)\circ\eta + s\rho\circ\eta = 0 \tag{3.8}$$

Solving for s:

$$s = \frac{-(L - C)\circ\eta}{\rho\circ\eta} \tag{3.9}$$

Note that if the photogrammetric ray ρ and the normal η of the plane are orthogonal, then the case degenerates to an edge view of the plane and hence $\rho\circ\eta = 0$ and $(L - C)\circ\eta = 0$ thus, s is undefined. Substituting (3.9) into the collinearity relation (3.6) we obtain:

$$P = L + \frac{-(L - C)\circ\eta}{\rho\circ\eta}\,\rho \tag{3.10}$$

Finally substituting (3.10) into the sphere condition (3.1) and rearranging we obtain:

$$\left\| (L - C) - \frac{(L - C)\circ\eta}{\rho\circ\eta}\,\rho \right\| = r \tag{3.11}$$

This represents the photogrammetric relation of the circular linear feature ℂ.

3.2 The Ellipse: $\mathbb{E}$

An ellipse $\mathbb{E}$ may be described through the use of two foci $\{ F_1,F_2 \}$, the length of the semi-major axis a, and the plane $\{ C,\eta \}$ that contains the ellipse $\mathbb{E}$. The sum of the distances from any point P to the foci $\{ F_1,F_2 \}$ must be a constant equal to 2a. For the fixed point C in the plane we choose the center of the ellipse.

$$\| P-F_1 \| + \| P-F_2 \| = 2a \tag{3.12}$$

$$\left[P-\frac{F_1+F_2}{2} \right] \circ \eta = 0 \tag{3.13}$$

Solving for the intersection of the plane and the photogrammetric ray by using (3.6) gives:

$$s = \frac{-\left[L-\frac{F_1+F_2}{2} \right] \circ \eta}{\rho \circ \eta} \tag{3.14}$$

Then substituting into (3.6), and (3.12) and rearranging gives:

$$\left\| (L-F_1) - \frac{\left[L-\frac{F_1+F_2}{2} \right] \circ \eta}{\rho \circ \eta} \rho \right\| + \left\| (L-F_2) - \frac{\left[L-\frac{F_1+F_2}{2} \right] \circ \eta}{\rho \circ \eta} \rho \right\| = 2a \tag{3.15}$$

$$\mathbb{E} \text{ (standard form): } \{ F_1,F_2,\eta,a \mid \|\eta\|=1, (F_1-F_2)\circ\eta=0 \} \tag{3.16}$$

Apparently the ellipse $\mathbb{E}$ has 3+3+3+1-2=8 degrees of freedom.

3.3 The Hyperbola: $\mathbb{H}$

A hyperbola $\mathbb{H}$ may be described through the use of two foci $\{ F_1,F_2 \}$, a constant 2a, and the plane $\{ C,\eta \}$ that contains the hyperbola $\mathbb{H}$. The difference of the distances from any point P to the foci $\{ F_1,F_2 \}$ must be constant 2a. For the fixed point C in the plane we choose the center of the hyperbola.

$$\| P-F_1 \| - \| P-F_2 \| = 2a \tag{3.17}$$

$$\left[P-\frac{F_1+F_2}{2} \right] \circ \eta = 0 \tag{3.18}$$

Solving for the intersection of the plane and the photogrammetric ray by using (3.6) gives equation (3.14). Then substituting into (3.6), and (3.17) and rearranging gives:

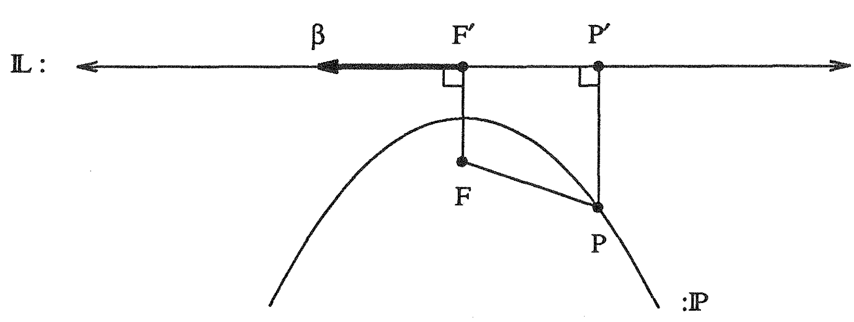
$$\left\| (L-F_1) - \frac{\left[L-\frac{F_1+F_2}{2} \right] \circ \eta}{\rho \circ \eta} \rho \right\| - \left\| (L-F_2) - \frac{\left[L-\frac{F_1+F_2}{2} \right] \circ \eta}{\rho \circ \eta} \rho \right\| = 2a \tag{3.19}$$

$$\mathbb{H} \text{ (standard form): } \{ F_1,F_2,\eta,a \mid \|\eta\|=1, (F_1-F_2)\circ\eta=0 \} \tag{3.20}$$

Apparently the hyperbola $\mathbb{H}$ has 3+3+3+1-2=8 degrees of freedom.

3.4 The Parabola: $\mathbb{P}$

The parabola $\mathbb{P}$ can be constructed by the use of a primary focus F, a conjugate focus F′ and a directrix line $\mathbb{L}$:{ F′,β }. Note that the line $\mathbb{L}$ and focus F define the plane of the parabola. Any point P on a parabola $\mathbb{P}$ is equidistant from the primary focus F and the directrix line $\mathbb{L}$.



$$\| P-F \| = \| P-P' \| \tag{3.21}$$

$$| (P-F)\ (F-F')\ \beta\ | = 0 \tag{3.22}$$

Note that:

$$\| P-P' \| = (P-F') \circ \frac{(F-F')}{\| F-F' \|} \tag{3.23}$$

Now using the equation of the plane (3.22) and the collinearity equation (3.6) we obtain:

$$s = -\frac{| (L-F)\ (F-F')\ \beta\ |}{| \rho\ (F-F')\ \beta\ |} \tag{3.24}$$

So combining equations (3.24), (3.6), and (3.23).

$$\left\| (L-F) - \frac{| (L-F)\ (F-F')\ \beta\ |}{| \rho\ (F-F')\ \beta\ |} \rho \right\| \ \| F-F' \| = \left[(L-F') - \frac{| (L-F)\ (F-F')\ \beta\ |}{| \rho\ (F-F')\ \beta\ |} \rho \right] \circ (F-F') \tag{3.25}$$

$$\mathbb{H} \text{ (standard form): } \{ F, F', \beta \mid \|\beta\| = 1, (F-F')\circ\beta = 0 \} \tag{3.26}$$

Apparently the parabola $\mathbb{P}$ has 3+3+3-2=7 degrees of freedom.

4. Parametric Linear Features

In sections 2. and 3. it was shown how to develop a single condition equation representing the photogrammetric observation of a linear feature. However, in the cases presented in these previous sections, this was possible because the linear feature was described in terms of geometric concepts. This is not always practical or possible for all varieties of linear features. In some cases the method suggested in section 2.2 may be the most practical. Suppose it is possible to describe the linear feature using parametric equations. Note that only one parameter may be used because a linear feature has only one degree of freedom such as arc length. However, many other feature descriptors may be used as needed. In general, let the linear feature be described parametrically by:

$$P(s) = \begin{bmatrix} x_P(s) \\ y_P(s) \\ z_P(s) \end{bmatrix} \tag{4.1}$$

Where $P(s)$, $x_P(s)$, $y_P(s)$, $z_P(s)$ are functions of the parameter s. If we choose to include four additional unknowns $\{ P, s \}$ per photogrammetric observation then, including the single redundancy, we must write five condition equations. This could be two collinearity equations and three linear feature fitting equations:

$$\begin{aligned} x &= -f\,\frac{m_{11}(x_P-x_L) + m_{12}(y_P-y_L) + m_{13}(z_P-z_L)}{m_{31}(x_P-x_L) + m_{32}(y_P-y_L) + m_{33}(z_P-z_L)} \\ y &= -f\,\frac{m_{21}(x_P-x_L) + m_{22}(y_P-y_L) + m_{23}(z_P-z_L)}{m_{31}(x_P-x_L) + m_{32}(y_P-y_L) + m_{33}(z_P-z_L)} \\ P &= P(s) \end{aligned} \tag{4.2}$$

Or the intersection point P coordinates may be eliminated and only carry the parameter s. This would thus require two condition equations and for this we may use the collinearity equations in the form of:

$$\begin{aligned} x &= -f\,\frac{m_{11}(x_P(s)-x_L) + m_{12}(y_P(s)-y_L) + m_{13}(z_P(s)-z_L)}{m_{31}(x_P(s)-x_L) + m_{32}(y_P(s)-y_L) + m_{33}(z_P(s)-z_L)} \\ y &= -f\,\frac{m_{21}(x_P(s)-x_L) + m_{22}(y_P(s)-y_L) + m_{23}(z_P(s)-z_L)}{m_{31}(x_P(s)-x_L) + m_{32}(y_P(s)-y_L) + m_{33}(z_P(s)-z_L)} \end{aligned} \tag{4.3}$$

If any constraints exist among the linear feature descriptors they must be enforced at the time of inversion of the normal equations.

One of the main opportunities of using the parametric linear features is in the application of a spline to describe an arbitrarily shaped linear feature. The spline is a mathematical model used to approximate the workings of the draftman's spline and should only be used with linear features that are relatively smooth. Typical splines are formed as cubic polynomials done in segments between knots and applying boundary conditions of continuity, slope and curvature at the knots. Only an outline of how the spline may be constructed will be presented here. Suppose that we had an ordered set of three dimensional points $\{K_1, K_2, \cdots, K_n\}$ that lie on a linear feature. Then three separate one-dimensional splines may be determined, one spline for the x-coordinate, one for the y-coordinate, and one for the z-coordinate, by using the points K_i as knots and arc length as the parameter. However, in the photogrammetric case the knots are not available as three dimensional information unless a correspondence has been made between conjugate image points to obtain space intersection. Thus, we must use fictitious knots determined from the spline itself. And at these fictitious knots the boundary conditions of the segments are enforced. The information that is available consists of the photogrammetric observation of the linear spline. Each point observed offers a single redundancy and if several of these intersection points are collected in a segment between the fictitious knots then enough information can be acquired to solve for the spline. This would indicate the necessity of a least squares model combined with a simultaneous solution of the spline parameters, the boundary conditions, the fictitious knots, and the intersection points.

5. Concluding Remarks

We have presented a rigorous treatment of linear features for photogrammetric applications. The standard form is selected as the most suitable formulation for direct implementation. Alternate forms, where added parameters are included, are also given in case geometric constraints requiring such parameters are to be imposed on the solution. Finally, a parametric approach, which would allow extension to the general case of splines, is sketched. Various computer programs have been written to demonstrate the theory developed and several experiments were performed.

Recognizing the extent and diversity of this area, research is proceeding along several directions. One is in the area of constructions using linear features, such as: two coplanar linear features, intersecting linear features, parallel linear features, corners made of three straight linear features $\mathbb{L}$ intersecting at right angles at a common point, etc. Another area involves the use of linear features to perform absolute orientation using the three dimensional linear conformal (or similarity) transformation.

Furthermore, while the frame imagery is mentioned, the developed condition equations: (2.14), (3.11), (3.15), (3.19) and (3.25) are general enough to be used with other photogrammetric systems such as the linear CCD array and the optical-mechanical scanner.

Another significant direction of our research is to incorporate feature-based photogrammetric techniques into the solutions for correspondence and matching, and for improved solid object modeling. This will allow for proper functional and stochastic treatment of combined metric and symbolic data.

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