

QUALITY ANALYSIS OF
BUNDLE BLOCK ADJUSTMENT WITH NAVIGATION DATA

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[ABSTRACT]

Precision and reliability are two basic factors of quality analysis for any geodetic and photogrammetric adjustment system. In this paper, simulated studies are done on the precision and reliability of aerotriangulation with navigation data. With the newly developed program system for combined adjustment WUCAPS (abbreviated from Wuhan Combined Adjustment Program System), navigation data with various levels of precision are processed. This investigation is quite helpful to the practical use of aerotriangulation with navigation data such as GPS data and/or INS data.

1. INTRODUCTION

The aerotriangulation with air borne data may date back to about more than 30 years ago. Because of their poor precision and high expense, we did not pay enough attention to it and it was not widely used in practice.

With the fast development of navigation systems, such as GPS and INS, in 1980's photogrammetrists are becoming highly interested in aerotriangulation with navigation data. With the deletion of linear systematical errors in the raw navigation data, Prof. F.Ackermann reported about 0.5m planimetric precision of aerotriangulation /Ackermann, 1984/. In America, camera stations with the precision of 0.1m were simulated with two GPS carriers, of which one is put on the air plane and the other is at one geodetic station. The decimeter level precision, both in planimetry and in elevation, was obtained by bundle block adjustment without ground

control (Schwarz, 1984; Lucas, 1986). On the Symposium of Commission III 1986, in Rovaniemi, a thoroughly simulated study was presented on practical precision of aerotriangulation with navigation data (Ackermann, 1986; Friess, 1986).

Following on the above studies, the precision and reliability of bundle block adjustment with navigation data are more thoroughly and theoretically investigated with the aid of computation of Q_{xx} and $Q_{vv} \times P$. This investigation shows again the potential applicability of aerotriangulation with navigation data.

2. TEST DESIGN AND BRIEF DESCRIPTION OF WUCAPS

Tests are performed with simulated data. The block is made up of 10 strips with 21 photos in each strip and 9 image points per photo. Other parameters are listed below.

number of object points,	441
focal length f ,	152mm
photo scale,	1:60,000
average flying height H ,	approx. 9000m
forward overlap,	60%
side overlap,	20%
average difference in elevation,	approx. 150m
orientation angles,	approx. 0
format of photo,	23cm×23cm
root mean square error (RMSE) of image points,	5 μ m
RMSE of control points,	0.1m

As for the navigation data, various levels of precision are taken, in consideration of the matching between the precision of camera stations and of orientation angles, i.e.

$$\sigma_{\alpha} = \sigma_{\varphi, \omega, \kappa} = \frac{\sigma_s}{H} \cdot \rho'' = \frac{\sigma_{x_s, y_s, z_s}}{H} \cdot \rho'' \quad (1)$$

The different combinations of camera stations and orientation angles of various precision make up the test design (see Tab.1).

The distributions of ground control are shown in Fig.1, where version a has no control points. Version a and b are only for adjustment with navigation data, while version c and d are used to conventional bundle block adjustment without navigation data.

Tab.1 Precision of navigation data in simulated computation

Version σ_x / σ_y	0.1m	1.0m	3.0m	6.0m	10m	∞
2".3	E-1	E-2	B-1	E-3	E-4	D-1
22".9	F-1	F-2	B-2	F-3	F-4	D-2
68".8	A-1	A-2	A-3	A-4	A-5	A-6
137".5	G-1	G-2	B-3	G-3	G-4	D-3
229".2	H-1	H-2	B-4	H-3	H-4	D-4
∞	C-1	C-2	B-5	C-3	C-4	

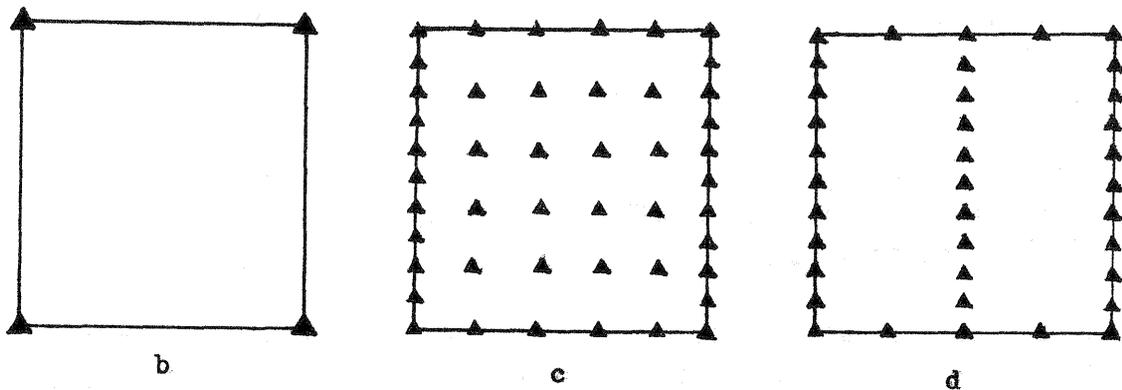


Fig.1 Distributions of ground control

The computation is performed with the newly developed program system for combined adjustment---WUCAPS in Siemens 7.570-C. The flowchart of this program system is shown in Fig.2.

Following are basic functions that WUCAPS has.

- bundle block adjustment with self-calibration;
- combined adjustment with navigation data and geodetic observations;
- computation of weight cofactor matrix of unknown parameters Q_{xx} and reliability matrix $Q_{vv \times P}$ with the analysis of quality;

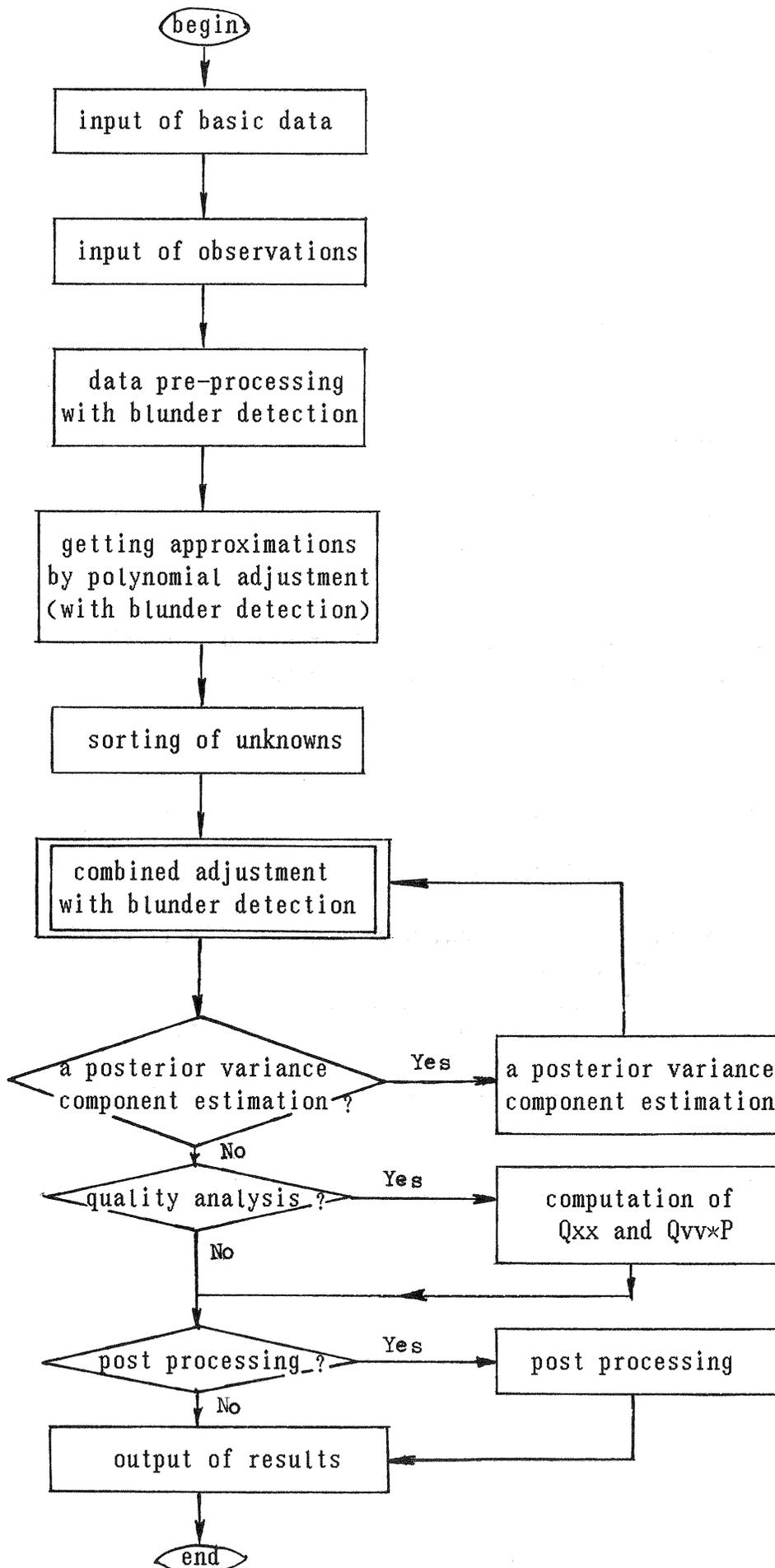


Fig.2 Flowchart of WUCAPS

- blunder detection in pre-processing and in the following (combined) adjustment;
- a posterior variance component estimation of various kinds of observations;
- post processing of residuals at control points with least squares collocation;
- adjustment of free net.

Quality analysis includes two aspects, i.e. precision analysis and reliability analysis. They will be discussed respectively below.

3. PRECISION ANALYSIS

Generally, the covariance matrix of object points

$$D(X) = \sigma_0^2 \cdot Q_{xx} \quad (2)$$

is taken as the measure of theoretical precision. And the theoretical precision of i -th object point is measured by

$$m_i = \sigma_0 \sqrt{(Q_{xx})_{ii}} \quad (3)$$

The average theoretical precision of n object points is

$$\bar{m} = \sigma_0 \sqrt{\text{tr}(Q_{xx})/3n} \quad (4)$$

For simulated data the practical precision of object points in average is represented by

$$\bar{\mu} = \sqrt{\sum(\Delta X^2 + \Delta Y^2 + \Delta Z^2)/3n} \quad (5)$$

where $\Delta X, \Delta Y, \Delta Z$ are differences between adjusted and simulated coordinates of an object point.

The results of the average theoretical and practical precision are listed in Tab.2, where line a and b refer respectively to the version without and with ground control as shown in Fig.1. The figures at the lower right corner of Tab.2 are for bundle block adjustment without navigation data.

Following conclusions for precision can be obtained with the analysis of Tab.2.

Tab.2 Average precision of adjustment with navigation data
 (μ/\bar{m} , ‰ as unit)

precision σ_x (")	σ_x (m)	0.1			1.0			3.0			6.0			10.0			∞		
		X,Y	Z		X,Y	Z		X,Y	Z		X,Y	Z		X,Y	Z		X,Y	Z	
2.3	a	0.6 0.6	1.1 1.1	1.1	0.8 0.8	1.2 1.3		1.0 1.0	1.3 1.5	1.5	1.0	1.5		2.2 3.0	1.2 2.7		2.2	1.2 1.8	1.9
	b	0.6 0.6	1.1 1.1	1.1	0.8	1.2		1.0	1.3 1.4	1.4	1.0	1.5		1.0	1.7		1.0	1.2 1.8	1.9
22.9	a	0.8	1.2		1.0 1.2	1.6 1.8		1.3 1.8	1.9 2.4	2.4	2.0	1.9		2.6	1.9		2.6	1.9	1.9
	b	0.8	1.2		1.0	1.6		1.4	1.8	1.8	2.0	1.9		2.6	1.9		2.6	1.9	1.9
68.8	a	0.8 0.8	1.2 1.3	1.3	1.2 1.3	1.8 2.0		1.6 2.2	2.8 3.1	3.1	1.6 2.2	2.8 3.1		2.9 4.4	3.7 5.3		2.9 4.4	3.7 5.3	3.6
	b	0.8 0.8	1.2 1.3	1.3	1.1	1.8		1.5 1.8	2.8 2.9	2.9	1.5 1.8	2.8 2.9		3.4 3.0	3.3 4.6		3.4 3.0	3.3 4.6	3.6
137.5	a	0.8	1.2		1.1	1.8		1.4	3.1	3.1	2.2	4.0		3.6	4.3		3.6	4.3	6.5
	b	0.8	1.2		1.1	1.8		1.4	3.1	3.1	2.2	4.0		3.6	4.3		3.6	4.3	6.5
229.2	a	0.8	1.2		1.1	1.8		1.5 1.8	3.3 3.1	3.1	1.5 1.8	3.3 3.1		3.2 4.8	6.1 7.0		3.2 4.8	6.1 7.0	14.0
	b	0.8	1.2		1.1	1.8		1.5 1.8	3.3 3.1	3.1	1.5 1.8	3.3 3.1		3.2 4.8	6.1 7.0		3.2 4.8	6.1 7.0	14.0
∞	a	1.0 0.9	1.3 1.4	1.4	1.2 1.3	1.9 2.0		1.9 2.2	3.4 3.4	3.4	1.9 2.2	3.4 3.4		3.2 5.0	7.8 7.7		3.2 5.0	7.8 7.7	2.0
	b	0.8 0.8	1.2 1.3	1.3	1.1	1.9		1.5	3.4	3.4	2.3	5.0		3.8 3.2	6.8 6.6		3.8 3.2	6.8 6.6	2.5

1) If the precision of camera stations is 1.0m or within 1.0m, the adjusted coordinates, especially the elevation, are better than those from conventional bundle block adjustment, no matter whether or not the ground control and/or orientation angles are available. The planimetry and elevation precision are $0.6--1.1\sigma_0$ and $1.1--1.9\sigma_0$ respectively. If $\sigma_0 = 5\mu\text{m}$ and photo scale 1:60,000 are chosen, the corresponding errors in ground are 0.18--0.33m and 0.33--0.57m respectively. It is very interesting that object points are more precise than the navigation data themselves with which the adjustment is performed, if the precision of camera stations is around 1.0m. Therefore the precision of point determination may reach the level of sub-meter, if only GPS can supply stations with the RMSE less than 1.0m. This would fulfill the requirement of international mapping of 1:100,000 to 1:10,000.

2) The same precision as 1) could be obtained with orientation angles, only when the orientation angles can reach very high precision of several seconds. This shows that the camera stations are dominant for bundle block adjustment with navigation data.

3) In general cases, the orientation angles could improve the planimetric precision quite limitedly, but they are very efficient to the precision improvement in elevation. Only when the precision of camera stations is rather poor, the planimetric precision can be raised with the aid of highly precise orientation angles. From this point of view, the introduction of orientation angles in aerotriangulation is helpful mainly to the precision in elevation.

4) The precision distribution of object point field in bundle block adjustment with navigation data is demonstrated in Fig.3, from which we may conclude the following remarks.

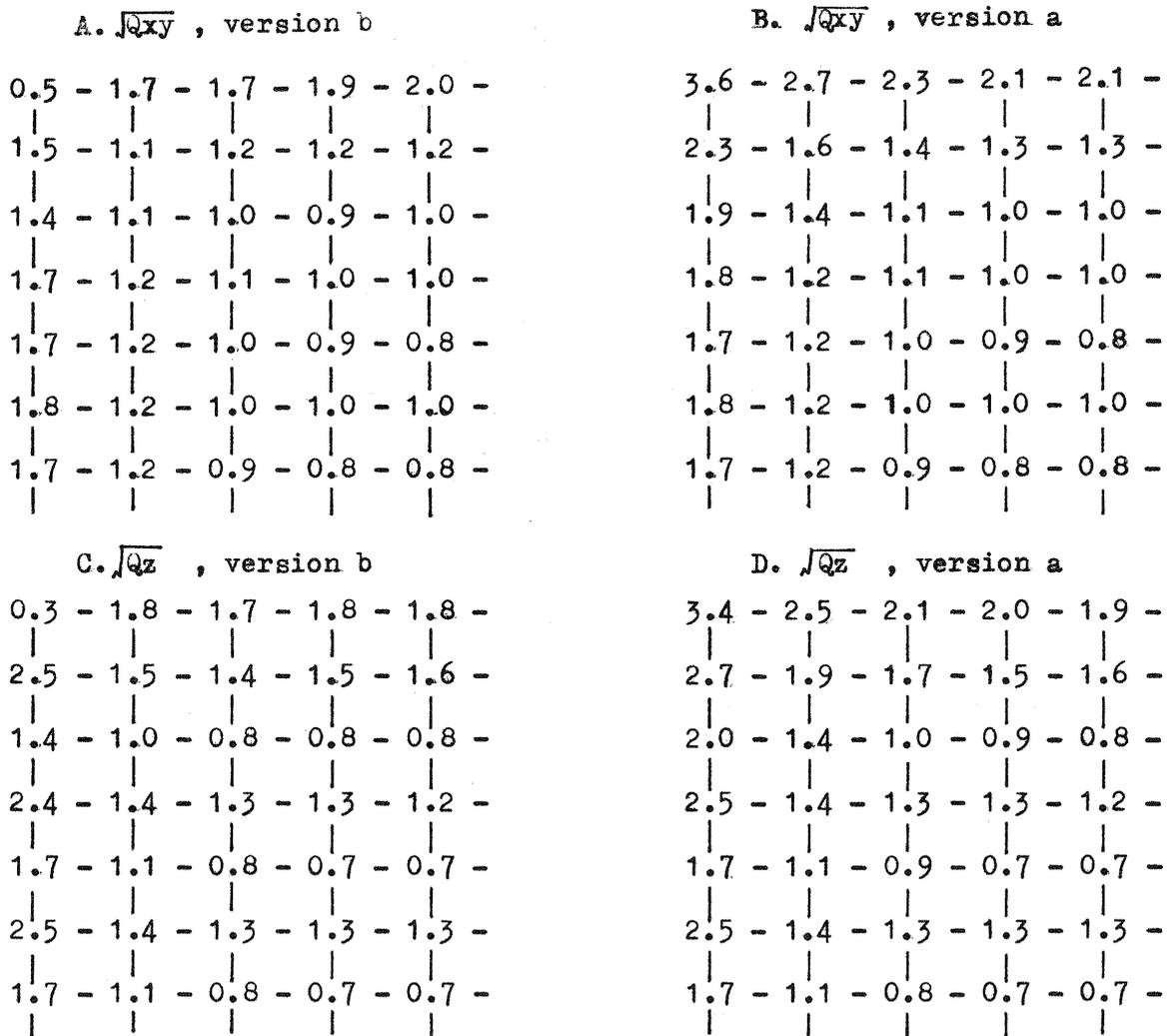


Fig.3 Weight coefficients of adjusted object points
(upper left corner of the block, Version C-1)

---For aerotriangulation with navigation data, precision is the worst both at corners and edges of the blocks, especially of those blocks without ground control. Therefore the strategy of extra strips and photos in the direction of side overlap and forward overlap around the block is also recommended.

---In inner of the block, precision of object point field is quite homogeneous both in planimetry and in elevation. The precision of points within one strip are slightly worse than the points located at two neighbour strips. This could be avoided by flight with 60% side overlap.

---Comparing with the conventional adjustment without navigation data, the elevation precision for block adjustment with navigation data is obviously more homogeneous, while the planimetric precision has the same homogeneity. This means, in aerotriangulation with navigation data, there is no need of elevation control in inner of block at all.

4. RELIABILITY ANALYSIS

For the analysis of reliability, the local redundancy

$$r_i = (Q_{vv} \times P)_{ii} \quad (6)$$

is computed. Generally, only one single blunder is assumed. The internal reliability and external reliability are expressed respectively with the following two formulas (Li, 1986)

$$\sigma_{l_i} = \sigma_{l_0} \cdot \frac{\delta_0}{\sqrt{r_i}} = \sigma_{l_0} \cdot \delta'_{0,i} \quad (7)$$

and

$$\bar{\delta}_{0,i} = \delta_0 \cdot \sqrt{\frac{1-r_i}{r_i}} = \sqrt{1-r_i} \cdot \delta'_{0,i} \quad (8)$$

With the chosen significant level $\alpha = 0.1\%$ and the test power $\beta = 80\%$, the non-centrality parameter δ_0 is then equal to 4.13.

The average local redundancies of various observations are shown in Tab.3. Following remarks can be obtained with the analysis of Tab.3.

Tab.3 The average local redundancies of various observations

version	E-1		C-1		F-2		A-3		A-5		C-4		c	d
	a	b	a	b	a	b	a	b	a	b	a	b		
σ_s	0.1 m		0.1 m		1 m		3 m		10 m		10 m		∞	
σ_α	2."3		∞		22."9		68."8		68."8		∞		∞	
\bar{r}_x	0.49	0.49	0.39	0.40	0.31	0.31	0.28	0.28	0.28	0.28	0.27	0.27	0.29	0.29
\bar{r}_y	0.59	0.60	0.51	0.51	0.39	0.39	0.35	0.35	0.34	0.34	0.33	0.33	0.36	0.36
\bar{r}_s	0.22	0.22	0.09	0.09	0.79	0.79	0.92	0.93	0.98	0.98	0.96	0.97	/	/
\bar{r}_α	0.34	0.34	/	/	0.87	0.88	0.96	0.97	0.94	0.95	/	/	/	/
\bar{r}_X	/	0.15	/	0.04	/	0.02	/	.004	/	.002	/	.001	.017	.015
\bar{r}_Y	/	0.14	/	0.03	/	0.02	/	.004	/	.001	/	.001	.017	.015
\bar{r}_Z	/	0.08	/	0.03	/	0/01	/	.003	/	.002	/	.001	.006	.005

1) In aerotriangulation with navigation data, the average reliability of both image points and navigation data would not be affected practically, no matter whether or not with ground control points at corners of block (see column a and b in Tab.3). From the reliability point of view, the ground control points would not necessarily be introduced in the aerotriangulation with navigation data. They can be used to monitor the quality of adjustment and to discover or delete blunders among themselves.

2) In aerotriangulation with navigation data, the average reliability of image points is generally not less than that in conventional adjustment without navigation data. With the more precise navigation data, the image points would become more reliable (see Fig. 4). If the navigation data have high precision, the reliability of image points can be improved quite efficiently, whereas it would not be affected by control points in conventional adjustment without navigation data.

AEROTRIANGULATION WITH NAVIGATION
DATA AND NO GROUND CONTROL
(version E-1)

CONVENTIONAL BUNDLE
BLOCK ADJUSTMENT
(version d)

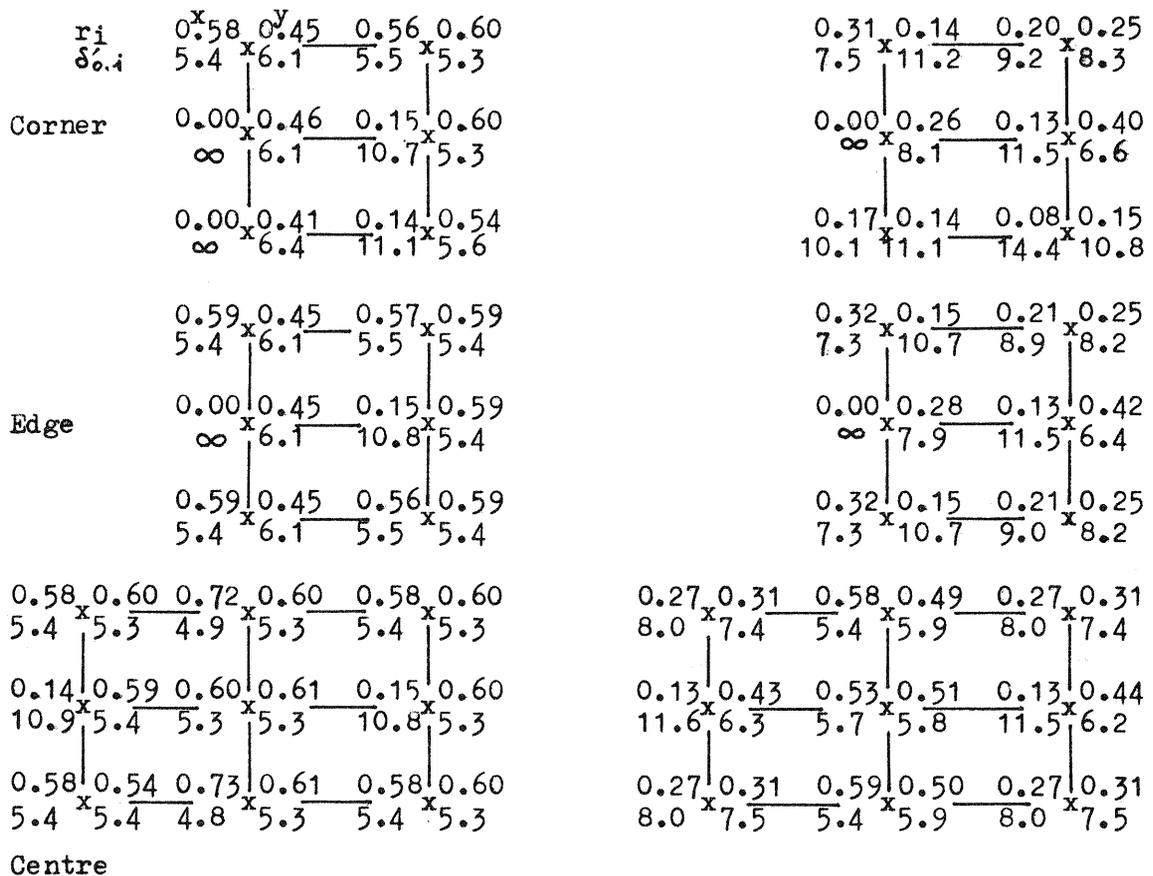


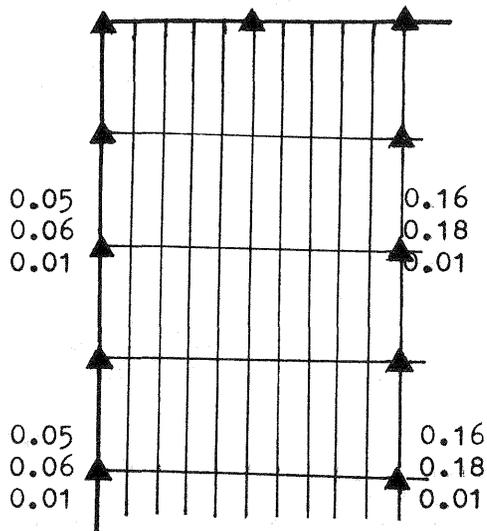
Fig.4 Local redundancy and reliability factor of image points

3) The more precise the navigation data are, the higher weights they get, and then the less the corresponding local redundancies become. But, in a comparison with the poor reliability of ground control points ($\bar{r}=0.02\text{---}0.20$) in conventional adjustment, the reliability of navigation data themselves ($\bar{r}=0.2\text{---}0.9$) is absolutely much better and very homogeneous all over the block. While it is well-known, that the reliability of ground control points in conventional adjustment without navigation data is dependent on their locations in the block (see Fig.5). Therefore the poor reliability of ground control points in conventional adjustment can be thoroughly avoided and overcome by the replacement of ground control points with navigation data in aerotriangulation.

X 0.145
Y 0.138
Z 0.081

X_S	0.16	0.21	0.19	0.19
Y_S	0.18	0.22	0.22	0.22
Z_S	0.12	0.21	0.20	0.20
φ	0.23	0.35	0.31	0.31
ω	0.32	0.39	0.38	0.38
κ	0.16	0.27	0.24	0.24
X_S	0.17	0.22	0.23	0.23
Y_S	0.16	0.22	0.23	0.24
Z_S	0.12	0.21	0.23	0.23
φ	0.26	0.36	0.38	0.38
ω	0.29	0.37	0.39	0.40
κ	0.17	0.28	0.30	0.31
X_S	0.17	0.22	0.23	0.23
Y_S	0.17	0.22	0.23	0.24
Z_S	0.13	0.21	0.23	0.24
φ	0.26	0.36	0.38	0.38
ω	0.28	0.37	0.40	0.40
κ	0.17	0.28	0.30	0.31

X 0.02 0.03 0.04
Y 0.01 0.02 0.03
Z 0.00 0.00 0.01



a) local redundancy of navigation data in block adjustment with navigation data. (Version E-1)

b) local redundancy of control points in block adjustment with ground control. (ground control Version d)

Fig.5 Reliability comparison of block adjustment with and without navigation data.

4) Blunders in navigation data, which can not be found, do not significantly affect adjusted object points (see. Tab.4). The external reliability of navigation data is almost always much better than that of ground control points at edges of block in conventional adjustment, especially in elevation. The reliability of Z-coordinates of ground control points in conventional adjustment is always lower than that of navigation data.

Tab.4 External reliability of observations.

version	E-1		C-1		F-2		A-3		A-5		C-4		c	d
	a	b	a	b	a	b	a	b	a	b	a	b		
σ_s	0.1 m		0.1 m		1 m		3 m		10 m		10 m		∞	
σ_α	2."3		∞		22."9		68."8		68."8		∞		∞	
$\bar{\delta}_x$	4.2	4.2	5.1	5.1	6.1	6.2	6.6	6.6	6.7	6.6	6.8	6.7	6.3	6.4
$\bar{\delta}_y$	3.4	3.3	4.1	4.1	5.1	5.2	5.7	5.7	5.8	5.8	5.9	5.9	5.5	5.6
$\bar{\delta}_{x_s y_s z_s}$	7.8	7.8	13.0	13.0	2.2	2.1	1.2	1.2	0.6	0.6	0.8	0.8	/	/
$\bar{\delta}_{\varphi \omega \kappa}$	5.8	5.8	/	/	1.6	1.5	0.8	0.8	1.0	0.8	/	/	/	/
$\bar{\delta}_{XY}$	/	10.2	/	23.2	/	32.5	/	64.3	/	106.0	/	123.6	30.6	33.4
$\bar{\delta}_Z$	/	13.9	/	24.7	/	37.0	/	70.5	/	106.5	/	152.8	51.1	60.5

5. SUMMARY AND CONCLUSIONS

From the above simulated calculation and quality analysis of bundle block adjustment with navigation data, following conclusions are summarized.

- 1) The aerotriangulation with navigation data is applicable to the mapping with the scale from 100,000 to 10,000, if navigation systems can supply camera stations with the precision around 1m, in which systematical errors are already deleted.

2) In aerotriangulation with navigation data, the camera stations are usually dominant. The elevation precision of adjusted object points can be improved significantly with the aid of orientation angles, if the camera stations are not precise enough.

3) Comparing with the conventional adjustment without navigation data, the elevation precision for block adjustment with navigation data is obviously more homogeneous, while the planimetric precision has the same homogeneity. This means, in aerotriangulation with navigation data, there is no need of elevation control in inner of block at all.

4) The reliability of bundle block adjustment with navigation data is quite good. Comparing with the reliability of image points in conventional adjustment, the reliability of image points in aerotriangulation with navigation data is at the same level or even better. The navigation data are always much more reliable than ground control points in conventional adjustment with ground control. This shows the practical potential of aerotriangulation with navigation data to replace the conventional one with ground control.

The availability of aerotriangulation with navigation data is confirmed both in precision and in reliability through the analysis of its quality. But the systematical errors in navigation data are unavoidable. Therefore the characters of systematical errors in navigation data, especially their separability with the systematical errors in image points, should be examined and researched further.

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