

# BUNDLE ADJUSTMENT OF SPOT IMAGERY

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## ABSTRACT

The high resolution and good base-height ratio of SPOT stereo imagery make it possible to evaluate SPOT imagery on analytical photogrammetric instruments accurately and efficiently. In order to provide analytical plotter, orthophotoprojector and digital image correlation system with orientation parameters, it is necessary to perform an aerotriangulation of SPOT images. The geometry and mathematical model of SPOT images are discussed. A bundle block adjustment program has been developed to handel CCD linear array imagery. Some results using a panchromatic SPOT stereo pair over the area of Marseille in the south of France are presented and compared with the results obtained by the University of HANNOVER.

## 1. INTRODUCTION

Nowadays the use of SPOT imagery to produce precise maps with small scale of 1:50,000 and 1:100,000 on analytical photogrammetric instruments is one of the most interesting topic in the field of photogrammetry and remote sensing. The first application results /Rivereau, 1987; Konecny et al, 1987; etc.../ encourage us to do some experiments in this direction.

Because the aerotriangulation of SPOT stereo images can provide the necessary orientation parameters for the plotting of SPOT imagery on analytical plotter, for the orthophoto production on orthophotoprinter as well as for the digital correlation procedure, a bundle adjustment of SPOT stereo images has been carried out in this paper. The mathematic model and the experimental results are given, the strong correlation among the orientation parameters and its effect on the plotting are discussed.

## 2. MATHEMATIC MODEL

### 2.1 Definition of coordinate systems

For a stereo pair of SPOT images (6000 x 6000 pixel, format 15x15 cm) only the side overlap is available, therefore the x-direction of image coordinate system is chosen to cross to the flight direction ( see Fig.1 )

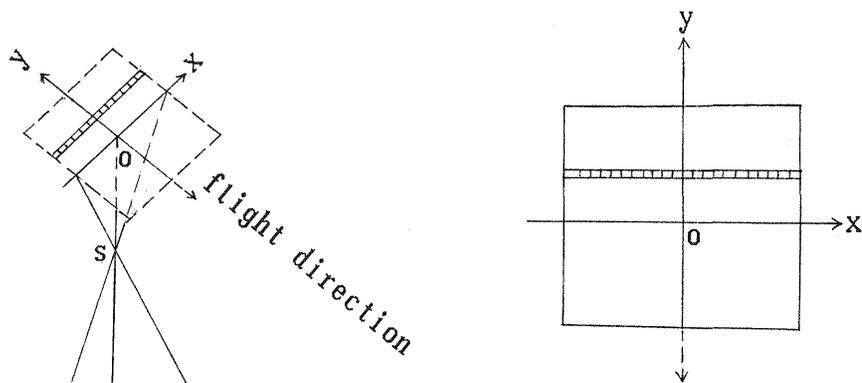


Fig.1 Image coordinate system

The concerned object coordinate system for space photogrammetry are :

- Gauss-Krueger coordinate system (Xt, Yt) and normal height;
- Geodetic coordinate system, i.e. geodetic longitude L, geodetic latitude B and geodetic height;
- Geocentric rectangular coordinate system (Xc, Yc, Zc) and
- Local tangent plane coordinate system, i.e. photogrammetric coordinate system (Xp, Yp, Zp).

The bundle adjustment of photogrammetric triangulation performs normally in local tangent plane coordinate system and the plotting for map making are expressed with Gauss-Krueger coordinates and normal height. So, before and after the bundle adjustment a transformation and its inverse transformation of coordinate are necessary,

$$( X_t, Y_t, h ) \leftrightarrow ( L, B, H ) \leftrightarrow ( X_c, Y_c, Z_c ) \leftrightarrow ( X_p, Y_p, Z_p )$$

The transformation formula can be found in geodetic literature ( also see Cheng, 1988 ). A direct coordinate transformation between Gauss-Krueger and photogrammetric coordinate system in consideration of the earth curvature correction is practically used.

## 2.2 Collinearity equation of SPOT imagery

The SPOT satellite has two CCD linear array sensors, mounted in the focal plane of the optics. While the platform moves, the terrain is continuously projected onto the sensor. The image swath may be through a mirror offset from the vertical sideways by  $\pm 5^\circ, \pm 10^\circ, \pm 15^\circ, \pm 20^\circ, \pm 25^\circ, \text{ and } \pm 27^\circ$  for each sensor. This also permits stereo imagery from several orbits.

The exterior orientation of each single line is given by six orientation parameter as in the case of aerial photography. For a discrete point in the  $i$ -line of an image its collinearity equation has the following form /Wang, 1986/:

$$\begin{bmatrix} x_i \\ 0 \\ -f \end{bmatrix} = \lambda \cdot R_i^T \begin{bmatrix} X_i - X_{si} \\ Y_i - Y_{si} \\ Z_i - Z_{si} \end{bmatrix} \quad (1)$$

where,  $x_i$  - image coordinate of the point in the  $i$ -line;

$X_i, Y_i, Z_i$  - ground point coordinates;

$X_{si}, Y_{si}, Z_{si}, \varphi_i, \omega_i, \kappa_i$  - exterior orientation parameters

for the  $i$ -line of the image ; and

$$R_i = R_{\varphi_i} \cdot R_{\omega_i} \cdot R_{\kappa_i} \cdot R_{\Phi_0} = \bar{R}_i \cdot R_{\Phi_0} = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

in which  $\varphi_i, \omega_i, \kappa_i$  are the rotation angles of the sensor at the orbit, while  $\Phi_0$  is the mirror offset angle.

According to eq. (1) we can write

$$\begin{aligned} x_i &= -f \frac{a_1(X_i - X_{si}) + b_1(Y_i - Y_{si}) + c_1(Z_i - Z_{si})}{a_3(X_i - X_{si}) + b_3(Y_i - Y_{si}) + c_3(Z_i - Z_{si})} \\ 0 &= -f \frac{a_2(X_i - X_{si}) + b_2(Y_i - Y_{si}) + c_2(Z_i - Z_{si})}{a_3(X_i - X_{si}) + b_3(Y_i - Y_{si}) + c_3(Z_i - Z_{si})} \end{aligned} \quad (2)$$

Differing from the conventional central projection for aerial photography all six orientation parameters of each line for SPOT image are not constant but a function of time. By the assumption of a linear non-accelerated movement of the sensor during the period of one image we can make the following approximation,

$$\begin{aligned} \varphi_i &= \varphi_0 + k_1 \cdot y_i \\ \omega_i &= \omega_0 + k_2 \cdot y_i \\ \kappa_i &= \kappa_0 + k_3 \cdot y_i \\ X_{si} &= X_{s0} + k_4 \cdot y_i \\ Y_{si} &= Y_{s0} + k_5 \cdot y_i \\ Z_{si} &= Z_{s0} + k_6 \cdot y_i \end{aligned} \quad (3)$$

in which

$\varphi_i \dots Z_{si}$  - the exterior orientation element for the  $i$ -line of the image;

$\varphi_0 \dots Z_{s0}$  - the exterior orientation element for  $y=0$ ;

$k_1 \dots k_6$  - linear corrections for six orientation parameters and

$y_i$  - the  $y$  coordinate for the point in the  $i$ -line

### 2.3 Linearized observation equation of SPOT image

There are three sorts of observation equation for SPOT bundle adjustment .

The first one is the observation equation for image coordinates. Linearizing eq(2) and considering to eq(3) we get

$$\begin{aligned} V_{xi} &= a_{11}\Delta\varphi_0 + a_{12}\Delta\omega_0 + a_{13}\Delta\kappa_0 + a_{14}\Delta X_{so} + a_{15}\Delta Y_{so} + a_{16}\Delta Z_{so} \\ &\quad + a_{11}y_{\Delta k1} + a_{12}y_{\Delta k2} + a_{13}y_{\Delta k3} + a_{14}y_{\Delta k4} + a_{15}y_{\Delta k5} + a_{16}y_{\Delta k6} \\ &\quad - a_{14}\Delta X_i - a_{15}\Delta Y_i - a_{16}\Delta Z_i - L_{xi} \\ V_{yi} &= a_{21}\Delta\varphi_0 + a_{22}\Delta\omega_0 + a_{23}\Delta\kappa_0 + a_{24}\Delta X_{so} + a_{25}\Delta Y_{so} + a_{26}\Delta Z_{so} \\ &\quad + a_{21}y_{\Delta k1} + a_{22}y_{\Delta k2} + a_{23}y_{\Delta k3} + a_{24}y_{\Delta k4} + a_{25}y_{\Delta k5} + a_{26}y_{\Delta k6} \\ &\quad - a_{24}\Delta X_i - a_{25}\Delta Y_i - a_{26}\Delta Z_i - L_{yi} \end{aligned} \quad (4)$$

in which

$$\begin{aligned} L_{xi} &= x + f \cdot \frac{a_1(X_i - X_{si}) + b_1(Y_i - Y_{si}) + c_1(Z_i - Z_{si})}{a_3(X_i - X_{si}) + b_3(Y_i - Y_{si}) + c_3(Z_i - Z_{si})} \\ L_{yi} &= f \cdot \frac{a_2(X_i - X_{si}) + b_2(Y_i - Y_{si}) + c_2(Z_i - Z_{si})}{a_3(X_i - X_{si}) + b_3(Y_i - Y_{si}) + c_3(Z_i - Z_{si})} \end{aligned} \quad (5)$$

and the coefficients are the corresponding partial differentials.

The ground control points are usually identified by the existing map , their coordinates should be better considered as weighted observations,

$$\begin{bmatrix} V_{xi} \\ V_{yi} \\ V_{zi} \end{bmatrix} = \begin{bmatrix} \Delta X_i \\ \Delta Y_i \\ \Delta Z_i \end{bmatrix} - \begin{bmatrix} L_{xi} \\ L_{yi} \\ L_{zi} \end{bmatrix}, \quad P_{\tau i} \quad (6)$$

where the weight depends on the map scale and the identifiability of control point .

The unknowns of orientation parameters of SPOT linear array image are also treated as weighted observations, especially in the case without satellite orbital data , in order to avoid high correlation between them /li, 1985/.

$$\begin{bmatrix} V_{\varphi_0} \\ V_{\omega_0} \\ V_{\kappa_0} \\ V_{x_{so}} \\ V_{y_{so}} \\ V_{z_{so}} \\ V_{k1} \\ V_{k2} \\ V_{k3} \\ V_{k4} \\ V_{k5} \\ V_{k6} \end{bmatrix} = \begin{bmatrix} \Delta \varphi_0 \\ \Delta \omega_0 \\ \Delta \kappa_0 \\ \Delta X_{so} \\ \Delta Y_{so} \\ \Delta Z_{so} \\ \Delta k_1 \\ \Delta k_2 \\ \Delta k_3 \\ \Delta k_4 \\ \Delta k_5 \\ \Delta k_6 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{Psi} \quad (7)$$

According to above three observation equations, we can summarize them in matrix form,

$$\begin{aligned}
 V_p &= A_{11} X_1 + A_{12} X_2 + A_{13} X_3 - L_p & , P_p \\
 V_x &= & A_{22} X_2 & -0 & , P_T \\
 V_s &= A_{31} x_1 & & -0 & , P_s
 \end{aligned} \tag{8}$$

where  $X_1$  - 24 orientation parameters for a pair of SPOT images  
 $X_2$  - corrections of control point coordinates  
 $X_3$  - unknowns of ground point coordinates

Some additional parameters can<sup>be</sup> introduced in order to compensate the possible systematic errors, such as the non-uniform movement and earth rotation.

### 3. EXPERIMENT AND ANALYSIS

A Fortran 77 program for bundle adjustment of SPOT imagery has been developed on the SIEMENS 7570-c computer .

The data snooping is used for detecting gross errors.

This program was tested using a copy of panchromatic stereo pair in the area of Marseille . The basic data of this stereo are described in Tab.1

The results of the bundle adjustment are shown in table 2

Tab.1 Basic data of Marseille Model

Instrument	HRV 1	HRV 1
Mean orbit altitude	832 km	832 km
Focal length	2087.4 mm	2087.4 mm
Photo scale	1,400,000	1,400,000
Format	15x15 cm	15x15 cm
Area	60x60 km	60x60 km
View angle	25°02' left	26°11' right
Date	11-5-86	18-5-86
Basa-height ratio	1:1.05	

Tab.2 Results of bundle adjustment

	Number of independ. check points	Number of control points	RMSE of control points (M)			RMSE of check points (M)			$\sigma_0$ ( $\mu\text{M}$ )
			X	Y	Z	X	Y	Z	
WTUSM	34	37	8.9	7.4	1.5	9.8	11.7	9.9	34
	44	27	9.9	8.1	1.5	9.8	10.9	8.5	35
	59	12	4.6	7.4	0.7	19.2	12.1	8.6	25
	0	66	6.4	6.7	1.6	/	/	/	31
Hannover Univer.	68	18	$\mu_{xy}$		8.5	10.9	13.7	6.5	8.4
	52	34	5.2	7.1					

In Tab.2 the results are compared with the results obtained by Hannover University / Konecny et al,1987 /.

Considering that over 50% of the model is covered by water , the ground control has been taken from existing 1,25,000 maps , the photo copies are not directly produced from CCT and no priori-orbital data are available, the obtained results are reasonable and acceptable

Because SPOT image is taken by a CCD-line sensor with a super lang focal length on a platform of a high altitude satellite, there are unavoidably high correlation between the exterior orientation elements.In order to know if these differences have any effect on the SPOT plotting, two groups of image coordnates of the high and low DEM Grid, which are available for SPOT plotting on analytical plotter / Konecny et al.,1987 /, are calculated using different exterior orientation obtained from two different control point distributions. A comparison between these two groups of image coordinates is given in Tabel 3 , from which it can be found that the high correlation between exterior oritation has almost no effect on the SPOT plotting.

SUMMARY

The presented approach of the bundle adjustment of SPOT images and the limited experiment has shown that the SPOT images are available for topographic mapping with a scale of 1,50,000 - 1,100,000 . The experiments for plotting on analytical plotter C 100 and for producing orthophoto map on orthophotoprinter OR-1 are being carried out at Wuhan Technical University of Surveying and Mapping.

Tab. 3 Differences of Image Coordinate for DEM-Grid  
 calculated from different exterior orientation  
 (obtained from the x-and y-coordinate differences at 450 points)

	left image		right image	
	hight Z=600 m	low Z=0 m	hight Z=600 m	low Z=0 m
$\hat{\sigma}_d$	+10.0 $\mu\text{m}$ (0.29 $\hat{\sigma}_0$ )	+6.3 $\mu\text{m}$ (0.18 $\hat{\sigma}_0$ )	+7.6 $\mu\text{m}$ (0.22 $\hat{\sigma}_0$ )	+4.3 $\mu\text{m}$ (0.12 $\hat{\sigma}_0$ )
$ \Delta x _{\text{max}}$	13 $\mu\text{m}$		14 $\mu\text{m}$	
$ \Delta y _{\text{max}}$	20 $\mu\text{m}$		12 $\mu\text{m}$	

$$\hat{\sigma}_d = \sqrt{\frac{\sum (\Delta x)^2 + \sum (\Delta y)^2}{n_x + n_y}}$$

#### REFERENCES

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