A STUDY ON THE IMPROVEMENT OF PHOTOGRAMMETRIC BLOCK
ADJUSTMENT PROCEDURES BY AUXILIARY DATA

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Presented paper to Commission III, Working Group I

Abstract

Direct determined camera position data during flight are highly effective to reduce ground control points in photogrammetric block adjustment. But it forces us to redefine the whole of photogrammetric procedures, planning and the influences of error sources in relation to the mathematical conditions. A pre-study is made, as an example, of the influences of the inner orientation elements of the metric camera.

The development of auxiliary instruments is fast and for photogrammetry very promising. In the Netherlands a test is already set up incorporating GPS data of camera position in large scale photogrammetry.

Introduction

In aerotriangulation optimum accuracy requires ground control points, around the perimeter of the block at intervals of two airbases and if precise elevations are to be determined, there must also be elevation control in the area, regular rows of height control in the sidelap of strips. These planimetric and height control points are necessary in order to determine the unknowns of the camera:

\[
\begin{align*}
\text{position} & : X^0, Y^0, Z^0 \\
\text{orientation} & : \omega, \phi, \kappa.
\end{align*}
\]

The utilization of navigation data to determine directly the position and orientation of the camera, have been proven to be very effective in aerotriangulation. The utilization of these directly measured camera orientation data as 'auxiliary data' dates back for several decennia. Especially extra height information, statoscope and APR-data, have been successfully utilized in practical block adjustment procedures; this is for the fact that the vertical position is a weak point if elevations are to be determined.

Modern flight navigation systems supply auxiliary data which in joint block adjustment allow considerable reduction of ground control points or even aerotriangulation with no ground control. With the Global Positioning System, GPS, one has the possibility to record the position data continuously during photo flight. Extensive simulation computations for small scale survey have shown that the influence of position data of the camera on the precision of photogrammetric blocks is considerable; see Fries (3). Even simulations for large scale photos are much promising, see Boswinkel (2). Especially the reduction of ground control will be of economic importance.

In this study not only attention is paid to the observation errors but in particular the influence of inner orientation elements of the camera is taken in account.
Large scale photographs are chosen in combination with GPS-data for the measuring of the position data of the camera during flight. Similar test-flights are done already in The Netherlands by the Survey Department of Rijkswaterstaat Delft in cooperation with KLM Aerocarto. It is to be expected that the propagation of errors and the influence of biases is quite different when the control is moved from ground level to flight level.

As opposed to the known simulations here the control points in the block are either on ground level or on flight level, but not a combination. Although such a separation is not the practical way in particular for large scale mapping it is chosen in order to get a better judgement of the various effects.

We will introduce the expressions Ground Control Point, GCP, respectively Flight Control Point, FCP; that is to say GCP-block is a block with only control points in the terrain and FCP-block is a block with only known coordinates of projection centres.

**Mathematical and stochastic model**

The mathematical model used is the algorithm as in bundle block adjustment. The introduction of measured camera position data does not give rise to any problem and it is done in the same way as for given ground control points. Each measured coordinate of a projection centre or of a terrain point provides an additional observation equation which is connected with the observation equations of the photo coordinates.

The transformation for each bundle is:

\[
\begin{bmatrix}
    x - x_h \\
    y - y_h \\
    -c
\end{bmatrix} = 1/ \lambda_i(R) \begin{bmatrix}
    X - X^0 \\
    Y - Y^0 \\
    Z - Z^0
\end{bmatrix}
\]

or

\[
(x - x_h) = -c \left[ \sum_{i=1}^{33} r_{ij} (X^0_i - X_h) + r_{ij} (Y^0_i - Y_h) + r_{ij} (Z^0_i - Z_h) \right]
\]

\[
(y - y_h) = -c \left[ \sum_{i=1}^{33} r_{ij} (X^0_i - X_h) + r_{ij} (Y^0_i - Y_h) + r_{ij} (Z^0_i - Z_h) \right]
\]

with

\[
R = \begin{bmatrix}
    r_{11} & r_{12} & r_{13} \\
    r_{21} & r_{22} & r_{23} \\
    r_{31} & r_{32} & r_{33}
\end{bmatrix}
\]

\[
r_{ij} = f(\omega, \phi, \kappa)
\]

In (1) and (2) is:

\[(x, y, -c) : \text{photo coordinates}\]

\[(x_h, y_h) : \text{coordinates of principal point}\]

\[(X^0, Y^0, Z^0) : \text{coordinates of projection centre 0; position parameters of the camera}\]

\[(\omega, \phi, \kappa) : \text{orientation parameters of the camera}\]

\[(X, Y, Z) : \text{terrain coordinates}\]
The equations (2) establish the relationship between the unknown and the observations:

unknowns: \((X, Y, Z), (X^0, Y^0, Z^0)\) and \((\omega, \phi, \kappa)\)

observations: \((x - x_h), (y - y_h)\) and control points.

The non-linear equations (2) are developed into Taylor series for linearization. This involves determinations of approximate values for all unknown. After linearization the following type of observation equations is applied for the photo coordinates with \(x_h = y_h = 0:\)

\[
\begin{align*}
\begin{bmatrix}
\Delta x + \varepsilon \Delta x \\
\Delta y + \varepsilon \Delta y
\end{bmatrix}
&= (A)_i^j \begin{bmatrix}
\Delta \omega \\
\Delta \phi \\
\Delta \kappa
\end{bmatrix}^j + (B)_i^j \begin{bmatrix}
\Delta X^0 \\
\Delta Y^0 \\
\Delta Z^0
\end{bmatrix}^j - \begin{bmatrix}
\Delta X \\
\Delta Y \\
\Delta Z
\end{bmatrix}_i
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix}
\Delta X \\
\Delta Y \\
\Delta Z
\end{bmatrix}_i
&= (E) \begin{bmatrix}
\Delta X \\
\Delta Y \\
\Delta Z
\end{bmatrix}_i
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix}
\Delta X^0 + \varepsilon \Delta X^0 \\
\Delta Y^0 + \varepsilon \Delta Y^0 \\
\Delta Z^0 + \varepsilon \Delta Z^0
\end{bmatrix}^j
&= (E) \begin{bmatrix}
\Delta X^0 \\
\Delta Y^0 \\
\Delta Z^0
\end{bmatrix}^j
\end{align*}
\]

In (3a) the observation equation for the bundle \(j:\)

\[
\begin{align*}
\begin{bmatrix}
\Delta x_i^j \\
\Delta y_i^j
\end{bmatrix}
&: \text{the differentials of the observations, } x, y \text{ the photo coordinates of point } i \text{ in photo } j. \\
(A)_i^j \text{ and } (B)_i^j
&: \text{design matrices of point } i \text{ in photo } j. \\
(\Delta \omega, \Delta \phi, \Delta \kappa)_i^j
&: \text{the 3 unknown orientation parameters of bundle } j. \\
(\Delta X^0, \Delta Y^0, \Delta Z^0)_i^j
&: \text{the 3 unknown position parameters of bundle } j. \\
(\Delta X, \Delta Y, \Delta Z)
&: \text{the unknown terrain coordinates of point } i. \\
(\varepsilon, \Delta x, \text{etc.})
&: \text{least squares residuals.}
\end{align*}
\]

The observation equations (3b) refer to point \(i\) in the terrain, GCP, and (3c) refer to the coordinates of projection centre of bundle \(j, FCP.\)

\[
(\varepsilon)
: \text{unit matrix of } 3 \times 3.
\]

The observation equations (3b) and (3c) tie together with the observation equations of the photogrammetric block (3a) via the common unknowns:

\[
\begin{align*}
(\Delta x, \Delta y, \Delta z)_i
&: \text{coordinates of terrain point } i \\
(\Delta X^0, \Delta Y^0, \Delta Z^0)_i^j
&: \text{position coordinates of camera } j.
\end{align*}
\]

In the mathematical model the unknown systematic errors, such as date, drift, etc. are left out of consideration.
The stochastic model. The observations, photo coordinates, GCP and FCP, should be given appropriate standard deviations and correlation numbers, a full variance-covariance matrix. In this study the matrix is simplified.

According the algorithm of least squares adjustment the adjusted unknowns and their stochastic behavior can be computed.

**Simulation studies**

The simulations are based on a small block and are greatly simplified. The block is composed of two strips with two photographs each. The terrain is flat, ideal overlap and the points are regular distributed. For the connection of the bundles in the strip the well known 6 points are chosen and in the sidelap 2 connection points. The flying height is constant and the photographs are vertical.

**Flight parameters:**

- Photo scale: 1 : 6000
- Principal distance: \( c = 15 \text{ cm} \)
- Photobase: \( b = 9 \text{ cm} \)
- Sidelap: 8.7%

The position of the points is given in the figures 1 and 2. The distance between the points in X-direction is 540m and in Y-direction 630m; so the whole block is:

- **Size**: 540m by 2520m
- **Flying height**: 900m

Further simplifications are:

- Observed photo coordinates are random and not correlated: \( \sigma_x = \sigma_y \).
- No film deformation, etc.
- For the standard deviation of the ground control points is applied: \( \sigma_x = \sigma_y = \sigma_z \) and no correlation.

We distinguish in the block for the absolute orientation either ground control points or measured coordinates of projection centra, that is to say either GCP or FCP.

The position of the control points is chosen as follows:

**GCP-block:** \( X_i, Y_i, Z_i \) \( i = 11 \text{ and } 52 \)

\( Z_i \) \( i = 31 \text{ and } 32. \)

**FCP-block:** \( X_j, Y_j, Z_j \) \( j = 11, 12, 21 \text{ and } 22. \)

Figures 1 and 2 give a review with:

- The two strips with two photographs each, photos 11 and 12 respectively 21 and 22.
- The position and numbering of the points.

For the GCP-block a minimum of ground control points is introduced, 8 coordinates, in order not to improve the results by additional terrain measurements.

For the FCP-block the 3 coordinates of the 4 projection centres are measured during flight in the XYZ-system. This assumption is of course far from reality. This holds among others:
- neither drift nor timing or other parameters are introduced for the GPS-measurements;
- in addition unknown drift or other parameters will make the problem as defined here singular.

Remarks:
a. To realize a connection between the two systems and to eliminate or minimize the influence of a larger part of those parameters a second GPS-receiver can be used on the ground in the same area during flight mission. The GPS-data of the receiver in the plane can be corrected with those of the second one which has a fixed position and receives data of the same satellites; see (5).
b. The control points for absolute orientation in the FCP-block are 4 full control points. If also a minimum 8 coordinates, had been chosen as in the GCP-block, the conditions are too weak.

These two blocks are used to simulate and compare
1. the observation errors of photo coordinates and coordinates of control points as the inner orientation elements are errorless;
2. the influence of bias in inner orientation elements on the coordinates of terrain points and projection centres.
1. Observation errors

The observations are the photocordinates and the control points; the control points are respectively GCP and FCP. The variance-covariance matrix of the observations is simplified by only taken into account the diagonal elements of the matrix:

GCP-block: \( \sigma_x = \sigma_y = 10 \mu \)
\[
\begin{align*}
\sigma_x^2 &= \sigma_y^2 = \sigma_z^2 = 6 \text{ cm} \\
\end{align*}
\]

(4a)

FCP-block: \( \sigma_x = \sigma_y = 10 \mu \)
\[
\begin{align*}
\sigma_x^2 &= \sigma_y^2 = \sigma_z^2 = 6 \text{ cm} \\
\end{align*}
\]

(4b)

The stochastic behaviour of the adjusted unknowns, the coordinates of terrain points and projection centres, is described by a full covariance matrix. Only the diagonal elements are computed by the stochastic model of the BINGO bundle block adjustment program and tabulated in table 1. Detailed interpretations need the complete matrix but in this case the diagonal elements are sufficient.

Although there is only a minimum number of control points in the GCP-block the results for terrain coordinates are better than for the FCP-block. As to be expected the opposite holds for the coordinates of the projection centres.

<table>
<thead>
<tr>
<th>GCP-block</th>
<th>FCP-block</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_x )</td>
<td>( \sigma_y )</td>
</tr>
<tr>
<td>11</td>
<td>17</td>
</tr>
<tr>
<td>21</td>
<td>32</td>
</tr>
<tr>
<td>31</td>
<td>20</td>
</tr>
<tr>
<td>41</td>
<td>17</td>
</tr>
<tr>
<td>51</td>
<td>17</td>
</tr>
<tr>
<td>12</td>
<td>20</td>
</tr>
<tr>
<td>22</td>
<td>32</td>
</tr>
<tr>
<td>32</td>
<td>17</td>
</tr>
<tr>
<td>42</td>
<td>6</td>
</tr>
<tr>
<td>52</td>
<td>37</td>
</tr>
<tr>
<td>0(11)</td>
<td>39</td>
</tr>
<tr>
<td>0(21)</td>
<td>39</td>
</tr>
<tr>
<td>0(12)</td>
<td>37</td>
</tr>
<tr>
<td>0(22)</td>
<td>37</td>
</tr>
</tbody>
</table>

Table 1: Standard deviation in centimeters of the coordinate of terrain points and projection centres.
Table 2: The deviation in the coordinates in centimeters caused by biases of the inner orientation elements.

<table>
<thead>
<tr>
<th></th>
<th>GCP-block</th>
<th></th>
<th>FCP-block</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\Delta X)</td>
<td>(\Delta Y)</td>
<td>(\Delta Z)</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>-2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>-3</td>
<td>-20</td>
</tr>
<tr>
<td>4</td>
<td>+9</td>
<td>-4</td>
<td>-11</td>
</tr>
<tr>
<td>5</td>
<td>+17</td>
<td>-4</td>
<td>-4</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>-9</td>
<td>+4</td>
<td>-11</td>
</tr>
<tr>
<td>8</td>
<td>+21</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>-12</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>+12</td>
<td>12</td>
<td>+12</td>
</tr>
<tr>
<td>11</td>
<td>-12</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>+12</td>
<td>12</td>
<td>+12</td>
</tr>
<tr>
<td>13</td>
<td>-12</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>+12</td>
<td>12</td>
<td>+12</td>
</tr>
<tr>
<td>15</td>
<td>-12</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>16</td>
<td>+12</td>
<td>12</td>
<td>+12</td>
</tr>
</tbody>
</table>

2. The inner orientation elements

The coordinates of the projection centre in the photo system are introduced with a bias of \(20^\circ\).

The sign of the bias \(\Delta x_h\) and \(\Delta y_h\), as defined in (1), is different for the two strips, for they are flown in opposite direction; so the camera is rotated 200 grades.

\[
\text{photo 11 and 12: } \Delta x_h = \Delta y_h = \Delta c = 20^\circ
\]

\[
\text{photo 21 and 22: } \Delta x_h = \Delta y_h = \Delta c = 20^\circ.
\]

A bias of the inner orientation elements will produce deviations in the coordinates, \(\Delta X\), \(\Delta Y\) and \(\Delta Z\). The computation is done with the functional model of the BINGO-program and the results are given in table 2. The position of the control points for both blocks is the same as in figure 1 and 2.

Evaluation

Observation errors. In table 1 the standard deviations are given for terrain point and projection centre. These figures show, as to be expected, that in the GCP-block the highest values for the standard deviations of the projection centre coordinates, up to 48 cm. In the FCP-block the highest values are for the standard deviations of the terrain point coordinates, by coincidence also up to 48 cm.
In the FCP-block all projection point coordinates have the same standard deviations and for the coordinates of terrain points they range from 18 to 48 cm. In the GCP-block the range for terrain points is large 6 to 37 cm.

The magnitude of these numbers is of course mainly determined by the standard observations of the observations as given in (4) and the block parameters. But for both blocks they are the same.

The inner orientation elements. The behaviour of the biases of the inner orientation elements is quite different, see table 2. The deviations for the terrain coordinates in the GCP-block is less than 1 cm and in the projection centre coordinates 12 cm. remark. Photoscale 1:6000; the biases of $20 \mu$ give $6000 \times 12 \mu = 12$ cm; the positive and negative values correspond with those of given in (5).

In the FCP-block the influence of the inner orientation elements is quite different. Projection centre coordinates change hardly any. The positive $\Delta c = 20 \mu$ gives a scale reduction in the terrain point coordinates and the position of the principal point introduces deviations of more than 12 cm, they vary from 0 to 17 cm.

These values for a small block can only in a general way be extrapolated to larger blocks. But this study shows:
- the influence of the inner orientation elements in a block with only FCP is remarkable large.
- the behaviour of observation errors makes all the difference in comparison with the biases of inner orientation elements.

Acknowledgement

The author thanks ir. D. Boswinkel for careful giving assistance in the computation and for his advice and suggestions on the subject.

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