COMBINED POINT DETERMINATION USING DIGITAL TERRAIN MODELS AS CONTROL INFORMATION

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Abstract

The paper discusses aspects of aerial triangulation using Digital Terrain Models (DTMs) as additional or exclusive control information. The mathematical model for the integration of this information into the block adjustment is described. Then the conditions for the absolute orientation of a block by the exclusive use of a DTM are discussed. The accuracy of combined point determination using DTM information is investigated by simulations. Finally, the results of a practical test are presented and possible future applications are mentioned.

1. Introduction

The determination of control points or control information for orientation and accuracy improvement of a photogrammetric block in aerial triangulation is usually quite expensive and time-consuming. At many private and public organizations, however, local, regional or country-wide Digital Terrain Models (DTMs) are generated and available in data bases. Therefore, it is reasonable to consider this information about the shape of the terrain in a combined block adjustment.
The method was first proposed in connection with simulation studies for digital three line imagery (Ebner and Müller, 1986). Investigations concerning the orientation of photogrammetric models by means of DTM were performed (Rosenholm and Torlegard, 1987). A practical example was presented by (Müller and Strunz, 1987).
In the following chapter, the mathematical model for the integration of DTM information into the block adjustment will be described. Then the conditions for the absolute orientation of a block, i.e. the definition of the datum by the exclusive use of a DTM are discussed. The accuracy of combined point determination using a DTM as exclusive or additional control information is investigated by simulations. Finally, the results of a practical test are presented and possible future applications are mentioned.

2. Mathematical Model

In point determination by photogrammetric means, non-photogrammetric information is on the one hand required for the definition of the datum and on the other hand for the improvement of the accuracy and reliability of the results. Normally, coordinates of control points are used for this purpose. Moreover, general control information, e.g. geodetic observations, data from navigation or positioning systems or any geometric information about the object, can be used in a combined adjustment with all available data (Ebner, 1984; Ackermann, 1986). In this paper the use of a given DTM as control information in aerial triangulation will be discussed.

A Digital Terrain Model is a mathematical description of the terrain surface. If the DTM is given in the coordinate system, in which the coordinates of the points are requested, for each object point P_k an observation equation can be formulated:

$$\hat{v}_{z_k} = \hat{z}_k - z_k(\hat{x}_k, \hat{y}_k) \qquad (1)$$

with:

$\hat{x}_k, \hat{y}_k, \hat{z}_k$: unknown coordinates of object point P_k

$z_k(\hat{x}_k, \hat{y}_k)$: observation z_k derived from the DTM

$\hat{v}_{z_k}$: residual of z_k

The observation z_k is given by the planimetric position of the point P_k and the parameters for the description of the DTM. In case of a raster DTM these are the coordinates of raster points and the algorithm for the interpolation of the height of an arbitrary point from the raster heights. If a bilinear interpolation is used, this height is calculated from the surrounding 4 raster points. Assuming constant raster width $z_k(\hat{x}_k, \hat{y}_k)$ is given by:

$$z_k(\hat{x}_k, \hat{y}_k) = \left(1 - \frac{\hat{x}_k - x_i}{d}\right)\left(1 - \frac{\hat{y}_k - y_j}{d}\right) \cdot z_{i,j} + \frac{\hat{x}_k - x_i}{d} \cdot \left(1 - \frac{\hat{y}_k - y_j}{d}\right) \cdot z_{i+1,j} +$$
$$+ \left(1 - \frac{\hat{x}_k - x_i}{d}\right) \cdot \frac{\hat{y}_k - y_j}{d} \cdot z_{i,j+1} + \frac{\hat{x}_k - x_i}{d} \cdot \frac{\hat{y}_k - y_j}{d} \cdot z_{i+1,j+1}$$

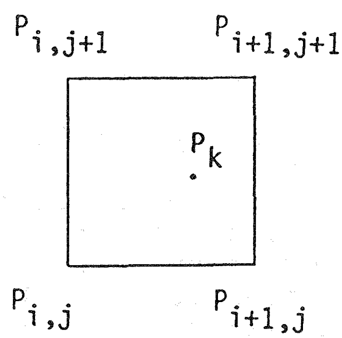
with:

grid points $P_{i,j}\ (x_i, y_j)$

$P_{i+1,j}(x_{i+1}, y_j)$

$P_{i,j+1}(x_i, y_{j+1})$

$P_{i+1,j+1}(x_{i+1}, y_{j+1})$



grid width $d = x_{i+1} - x_i = y_{j+1} - y_j$

The linearization of (1) using the initial values $\overset{o}{x}_k$, $\overset{o}{y}_k$, $\overset{o}{z}_k$ yields:

$$\hat{v}_{z_k} = \Delta\hat{z}_k - \frac{\partial z_k}{\partial \overset{o}{x}_k} \cdot \Delta\hat{x}_k - \frac{\partial z_k}{\partial \overset{o}{y}_k} \cdot \Delta\hat{y}_k - [z_k(\overset{o}{x}_k,\overset{o}{y}_k) - \overset{o}{z}_k] \qquad (2)$$

with:

$$\frac{\partial z_k}{\partial \overset{o}{x}_k} = (-\frac{1}{d} + \frac{\overset{o}{y}_k-y_j}{d^2}) \cdot z_{i,j} + (\frac{1}{d} - \frac{\overset{o}{y}_k-y_j}{d^2}) \cdot z_{i+1,j} +$$

$$+ (-\frac{\overset{o}{y}_k-y_j}{d^2}) \cdot z_{i,j+1} + (\frac{\overset{o}{y}_k-y_j}{d^2}) \cdot z_{i+1,j+1} \quad ;$$

$$\frac{\partial z_k}{\partial \overset{o}{y}_k} = (-\frac{1}{d} + \frac{\overset{o}{x}_k-x_i}{d^2}) \cdot z_{i,j} + (-\frac{\overset{o}{x}_k-x_i}{d^2}) \cdot z_{i+1,j} +$$

$$+ (\frac{1}{d} - \frac{\overset{o}{x}_k-x_i}{d^2}) \cdot z_{i,j+1} + (\frac{\overset{o}{x}_k-x_i}{d^2}) \cdot z_{i+1,j+1} \quad ;$$

A remark shall be made concerning the observation z_k. The value $z_k(\overset{o}{x}_k,\overset{o}{y}_k)$ is derived from approximate values $\overset{o}{x}_k$ and $\overset{o}{y}_k$, which change in each iteration of the adjustment. Therefore, the observation z_k also varies from iteration to iteration.
The stochastic properties of the DTM information can be described by its covariance matrix. In the following, the DTM observations are simply assumed to be uncorrelated and equally accurate.

3. Absolute Orientation of a Block Using a DTM

In this chapter the absolute orientation of a photogrammetric block by the exclusive use of a DTM will be discussed. The basic results are also valid for the absolute orientation of a single model.
From photogrammetric image coordinates the position of points can be derived according to the collinearity equations. The rank deficiency of the normal equation system with regard to three translations, three rotations and the scale factor cannot be compensated for by photogrammetric data. If the position of the points shall be expressed by coordinates, these 7 parameters have to be fixed. We denote the object coordinate system by (x,y,z) and describe the position of points determined by photogrammetry only in an arbitrary coordinate system (x',y',z').
Let us first consider the "normal" case, where 7 coordinates of identical points in both systems are necessary to determine the transformation parameters. For these coordinates 7 transformation equations can be formulated. If the arbitrary coordinate system (x',y',z') is chosen in a way that only differential translations, rotations and a differential deviation of the scale factor from 1 are necessary for the transformation from (x',y',z') to (x,y,z), these equations read:

$$
\begin{aligned}
\hat{x}_k &= \hat{X}0 + z'_k \cdot \hat{\phi} - y'_k \cdot \hat{\kappa} + x'_k \cdot \hat{m} + x'_k \\
\hat{y}_k &= \hat{Y}0 - z'_k \cdot \hat{\omega} + x'_k \cdot \hat{\kappa} + y'_k \cdot \hat{m} + y'_k \\
\hat{z}_k &= \hat{Z}0 - x'_k \cdot \hat{\phi} + y'_k \cdot \hat{\omega} + z'_k \cdot \hat{m} + z'_k
\end{aligned} \tag{3}
$$

with:

$\hat{X}0, \hat{Y}0, \hat{Z}0$: translation parameters

$\hat{\phi}, \hat{\omega}, \hat{\kappa}$: rotation parameters

$(1+\hat{m})$: scale factor

The coordinates x'_k, y'_k, z'_k are regarded as error-free values in order to exclude effects due to project parameters, which are specific for a particular block, e.g. image scale, overlap, measurement accuracy.

In the following it will be investigated, whether a DTM can be used for the determination of the 7 transformation parameters. Instead of given coordinates in the system (x,y,z) the condition has to be fullfilled that the points lie on the surface defined by the DTM. In the non-redundant case 7 observation equations (2) can be formulated; the results, however, can easily be applied to redundant cases.

With the definitions

$$\overset{o}{x}_k = x'_k \, , \quad \overset{o}{y}_k = y'_k \, , \quad \overset{o}{z}_k = z'_k$$

equations (3) can be changed into:

$$
\begin{aligned}
\Delta\hat{x}_k &= \hat{x}_k - \overset{o}{x}_k = \hat{x}_k - x'_k = \hat{X}0 + z'_k \cdot \hat{\phi} - y'_k \cdot \hat{\kappa} + x'_k \cdot \hat{m} \\
\Delta\hat{y}_k &= \hat{y}_k - \overset{o}{y}_k = \hat{y}_k - y'_k = \hat{Y}0 - z'_k \cdot \hat{\omega} + x'_k \cdot \hat{\kappa} + y'_k \cdot \hat{m} \\
\Delta\hat{z}_k &= \hat{z}_k - \overset{o}{z}_k = \hat{z}_k - z'_k = \hat{Z}0 - x'_k \cdot \hat{\phi} + y'_k \cdot \hat{\omega} + z'_k \cdot \hat{m}
\end{aligned} \tag{4}
$$

Replacing $\Delta\hat{x}_k$, $\Delta\hat{y}_k$, $\Delta\hat{z}_k$ in (2) using equations (4) we obtain linearized DTM observation equations, containing the unknown transformation parameters:

$$
\begin{aligned}
\hat{v}_{z_k} = & -\frac{\partial z_k}{\partial x'_k} \cdot \hat{X}0 - \frac{\partial z_k}{\partial y'_k} \cdot \hat{Y}0 + \hat{Z}0 + \left(- x'_k - \frac{\partial z_k}{\partial x'_k} \cdot z'_k\right) \cdot \hat{\phi} \\
& + \left(y'_k + \frac{\partial z_k}{\partial y'_k} \cdot z'_k\right) \cdot \hat{\omega} + \left(\frac{\partial z_k}{\partial x'_k} \cdot y'_k - \frac{\partial z_k}{\partial y'_k} \cdot x'_k\right) \cdot \hat{\kappa} \\
& + \left(z'_k - \frac{\partial z_k}{\partial x'_k} \cdot x'_k - \frac{\partial z_k}{\partial y'_k} \cdot y'_k\right) \cdot \hat{m} - [z_k(x'_k, y'_k) - z'_k]
\end{aligned} \tag{5}
$$

or in matrix notation:

$$\hat{v} = A \cdot \hat{t} - h$$

with:

$$\hat{v}^T := [\hat{v}_{z_1}, \ldots, \hat{v}_{z_7}]$$

$$\hat{t}^T := [\hat{X}0, \ldots, \hat{m}]$$

$$h^T := [(z_1(x_1', y_1') - z_1'), \ldots, (z_7(x_7', y_7') - z_7')]$$

$$A := \text{observation equation matrix}$$

The accuracy of the unknown transformation parameters is obtained as:

$$K_{\hat{t}\hat{t}} := N^{-1} = (A^T K_{hh}^{-1} A)^{-1} \tag{6}$$

with:

$K_{\hat{t}\hat{t}}$: covariance matrix of the transformation parameters

K_{hh} : covariance matrix of the DTM observations

A detailed analysis of the accuracy of the transformation parameters requires the inversion of N. Some main conclusions, however, can already be drawn from the diagonal elements of N. Assuming $K_{hh} = \sigma_h^2 * I$ (I: unity matrix), the diagonal elements of N read:

$$n(X0,X0) = \frac{1}{\sigma_h^2} \cdot \sum_{k=1}^{7} \left(\frac{\partial z_k}{\partial x_k'}\right)^2$$

$$n(Y0,Y0) = \frac{1}{\sigma_h^2} \cdot \sum_{k=1}^{7} \left(\frac{\partial z_k}{\partial y_k'}\right)^2$$

$$n(Z0,Z0) = \frac{1}{\sigma_h^2} \cdot 7$$

$$n(\phi,\phi) = \frac{1}{\sigma_h^2} \cdot \sum_{k=1}^{7} \left(-x_k' - \frac{\partial z_k}{\partial x_k'} \cdot z_k'\right)^2 \tag{7}$$

$$n(\omega,\omega) = \frac{1}{\sigma_h^2} \cdot \sum_{k=1}^{7} \left(y_k' + \frac{\partial z_k}{\partial y_k'} \cdot z_k'\right)^2$$

$$n(\kappa,\kappa) = \frac{1}{\sigma_h^2} \cdot \sum_{k=1}^{7} \left(\frac{\partial z_k}{\partial x_k'} \cdot y_k' - \frac{\partial z_k}{\partial y_k'} \cdot x_k'\right)^2$$

$$n(m,m) = \frac{1}{\sigma_h^2} \cdot \sum_{k=1}^{7} \left(z'_k - \frac{\partial z_k}{\partial x'_k} \cdot x'_k - \frac{\partial z_k}{\partial y'_k} \cdot y'_k \right)^2$$

In principle, the translation ZO can always be determined by the use of DTM information, independent of terrain shape and point location. Also the orientations ϕ and ω can be determined in general. The reason is, that the DTM primarily gives height information, i.e. every point can be regarded as control point in z. The translation parameters XO, YO, the scale factor (1+m) and the rotation κ are only determinable, if the respective diagonal elements of N are not equal to zero. The translations XO and YO can only be determined, if there is local slope at the points. The scale factor (1+m) and the rotation κ can only be computed, if the local slope at the considered points fulfills specific conditions.
The accuracy, by which the 7 parameters can be determined, results essentially from the values of the diagonal elements of N. It depends primarily on the accuracy of the DTM (σ_h), the location of the points (x'_k, y'_k, z'_k) and the local slope characteristics of the terrain ($\partial z_k / \partial x'_k$, $\partial z_k / \partial y'_k$).

4. Accuracy of Combined Point Determination Using DTM Information

The accuracy of aerial triangulation using a DTM as exclusive or additional control information was investigated by simulations based on a given DTM.

The parameters of the simulated block are:

- block size: 8.28 km * 11.04 km
- image scale: 1 : 10 000
- focal length: 153.0 mm
- forward / side overlap: 60% / 20%
- number of photographs: 60 (6 strips, 10 photographs each)
- distance between object points in x and y: 600m,300m,150m
- minimum number of points per image: 9, 36, 144
- mean terrain slope: 13.0%, 6.5%, 1.3%
- standard deviation of image coordinates: 10 μm
- standard deviation of control point coordinates x,y,z: 0.10 m
- standard deviation of DTM observations: 20m,10m,5m,2m,1m,0m

The bundle block adjustments were performed using the program system for combined point determination of the Chair of Photogrammetry of the Technical University of Munich, which allows for the introduction of various types of non-photogrammetric information (Müller and Strunz, 1987).

In the simulation, the number of points per image, the mean terrain slope and the accuracy of the DTM were varied. The different terrain slopes were generated by changing the z coordinates of all object points accordingly.

Figure 1 shows perspective views of the DTMs with the different mean slopes used in the investigations. The exaggeration factor in z direction is 3 in each case.

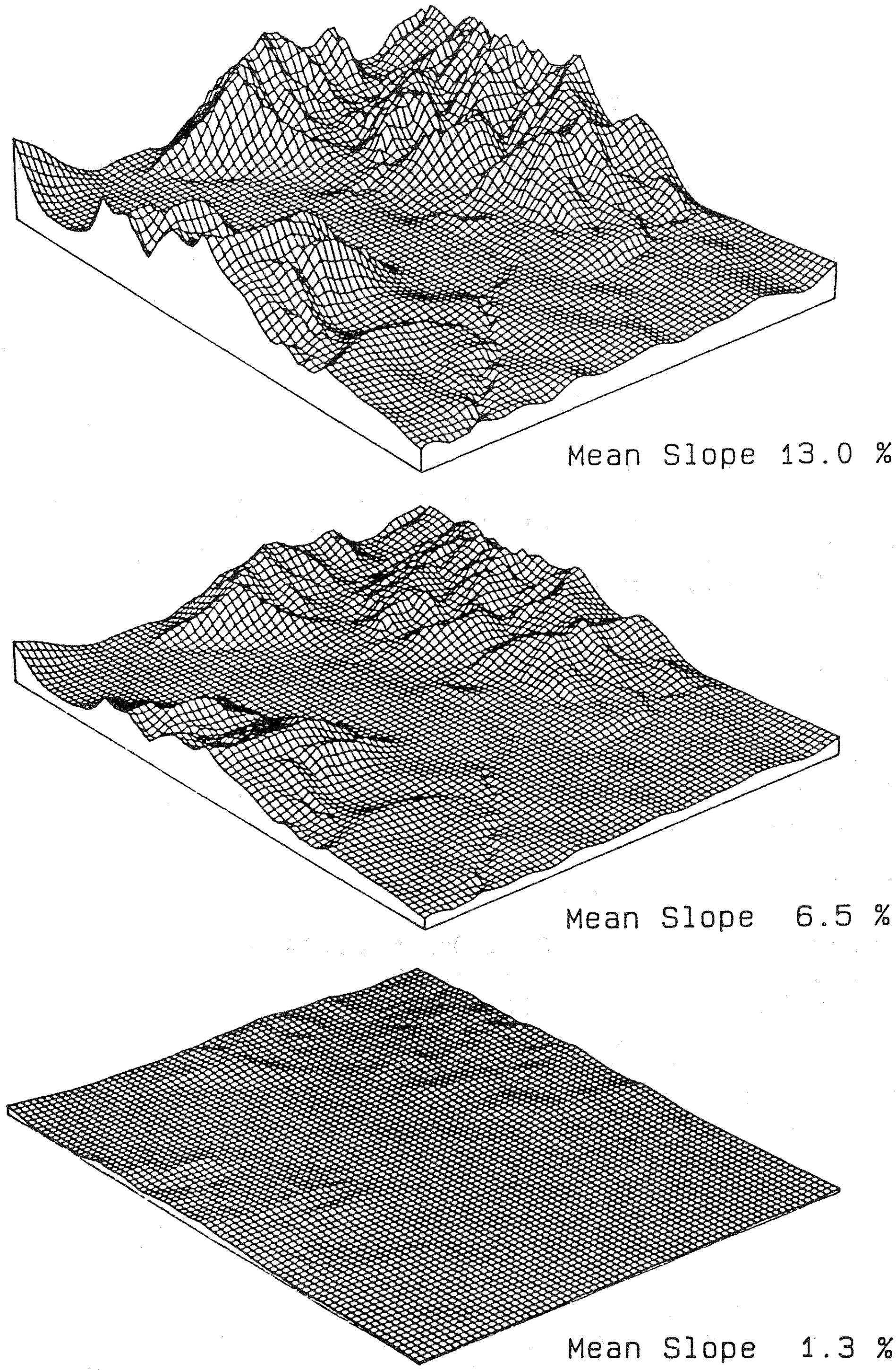


Figure 1: perspective views of the DTMs

In the block adjustments the standard deviations $\sigma_{\hat{x}k}$, $\sigma_{\hat{y}k}$, $\sigma_{\hat{z}k}$ of the object points were calculated. The r.m.s values $\mu_{\hat{x}\hat{y}}$ of all $\sigma_{\hat{x}k}$ and $\sigma_{\hat{y}k}$ and $\mu_{\hat{z}}$ of all $\sigma_{\hat{z}k}$ were used for the description of the theoretical accuracy.

Conventional aerial triangulations with control points were simulated for comparison reasons. The results of these simulations are given in table 1.

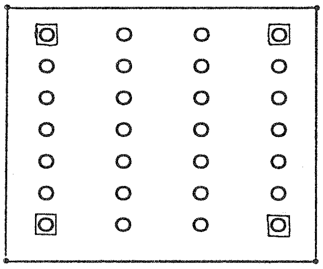
Control points		Points per image (min)	$\mu_{\hat{xy}}$ (m)	$\mu_{\hat{z}}$ (m)
4 xy 28 z		9 36 144	0.32 0.17 0.13	0.24 0.21 0.19

Table 1: results of aerial triangulations with control points for a mean terrain slope of 13%.

Various simulations were performed using a DTM as the only control information. The effect of the number of points per image, the mean slope of the terrain and the accuracy level of the DTM on the accuracy of the oject points was investigated. Only some representative results of the simulated block adjustments shall be presented in tables 2 - 4.

Points per image (min)	$\mu_{\hat{xy}}$ (m)	$\mu_{\hat{z}}$ (m)
9	5.11	1.52
36	2.50	0.64
144	1.25	0.35

(a)

Points per image (min)	$\mu_{\hat{xy}}$ (m)	$\mu_{\hat{z}}$ (m)
9	1.16	0.47
36	0.54	0.26
144	0.28	0.20

(b)

Table 2: effect of the number of points per image for a mean terrain slope of 13% and an accuracy of the DTM of 10 m (a) and 2 m (b).

Mean terrain slope (%)	$\mu_{\hat{xy}}$ (m)	$\mu_{\hat{z}}$ (m)
1.3	26.62	0.95
6.5	5.31	0.91
13.0	2.68	0.89

(a)

Mean terrain slope (%)	$\mu_{\hat{xy}}$ (m)	$\mu_{\hat{z}}$ (m)
1.3	5.62	0.31
6.5	1.18	0.31
13.0	0.67	0.31

(b)

Table 3: effect of the mean terrain slope for a minimum of 9 points per image and an accuracy of the DTM of 5 m (a) and 1 m (b).

Accuracy of the DTM (m)	$\mu_{\hat{xy}}$ (m)	$\mu_{\hat{z}}$ (m)
20	10.44	2.72
10	5.11	1.52
5	2.68	0.89
2	1.16	0.47
1	0.67	0.31
0	0.19	0.04

(a)

Accuracy of the DTM (m)	$\mu_{\hat{xy}}$ (m)	$\mu_{\hat{z}}$ (m)
20	4.97	1.16
10	2.50	0.64
5	1.26	0.40
2	0.54	0.26
1	0.31	0.21
0	0.09	0.03

(b)

Table 4: effect of the accuracy of the DTM for a mean terrain slope of 13 % and a minimum of 9 (a) and 36 (b) points per image.

In general the following conclusions can be drawn from the results of the simulation study.
In planimetry the accuracy $\mu_{\hat{xy}}$ of the object points is practically inversely proportional to the mean terrain slope and the square root of the number of points per image. Moreover, $\mu_{\hat{xy}}$ depends linearly on the accuracy of the DTM. The height accuracy $\mu_{\hat{z}}$ is roughly speaking inversely proportional to the square root of the number of points per image and depends almost linearly on the accuracy of the DTM. However, the height accuracy does not depend on the terrain slope.

A number of simulations was performed using both, control points and DTM observations, as control information. The results of the simulations can be summarized as follows.
The xy accuracy obtained by control points only (see table 1) can not or only slightly be improved by DTM information under the accuracy assumptions used in these investigations. Only DTM information of high accuracy can improve the xy accuracy obtained by 4 control points. The effect of the additionally introduced DTM information is more significant, if the control points are less accurate than assumed here. Because of practical and accuracy reasons the use of xy control points is recommended. Only if the accuracy demands are low, the exclusive use of a DTM might be a practical alternative to control points.
Considering the z accuracy the results are promising. Even at low accuracy levels of the DTM, the introduction of this information gives good results. A certain accuracy in z can be obtained by a dense distribution of z control points over the block or by DTM information with normally less precision than the control point coordinates. The influence of the point density or number of points per image is high, because the higher the number of points, the more DTM information is used. For the replacement or the reduction of the number of z control points the use of a given DTM is recommended.

5. Conclusions

In the paper the use of a DTM for the orientation of photogrammetric blocks has been discussed and the accuracy of the block adjustment using DTM information has been investigated. The results indicate, that the method gives sufficient accuracy in height for many applications. This is valid even for a DTM with low quality. In planimetry the DTM information is normally less effective. The effect depends on the local slope characteristics of the terrain. This corresponds to the results of a practical test with real data published in (Müller and Strunz, 1987). The method has been applied to a block in the Ötztal Alps, Austria. The image scale was 1 : 20 000, approximately 20 points per image were measured and a DTM was given with an accuracy of 2 m. The accuracy of point determination with the use of a DTM as the exclusive control information was determined empirically by comparing the results with independent control point coordinates. The differences indicated an accuracy of the block adjustment in planimetry and height of 1.5 m and 0.8 m, respectively.

Applications of the method are conceivable in all projects, where a DTM is available. For instance in topographic mapping, the cost for the establishment and signalization of classical control points, especially z control, can be reduced. Mapping from satellite imagery, when the accuracy requirements are not so high, or photogrammetric map revision could be possible fields of application for the method. Also for the production of orthophoto maps, where a DTM is necessary anyway, the use of the DTM information for the aerial triangulation seems reasonable. The principle of the method, i.e. the inclusion of geometric information about the object, can also be applied to close-range applications, because often digital models for the surface of objects are available. In industrial applications, e.g. for shape control, the method could in principle also be used. In these cases aberrations from the theoretical shape can be detected quite easily.

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