

ORIENTATION THEORY OF CCD LINE-SCANNER IMAGES

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ABSTRACT

A new and general orientation theory is derived for three overlapped line-scanner images which are mathematically necessary for discussing the model construction and the transformation between the model and object spaces. In addition, possibilities of the self-calibration of a CCD line-camera are investigated. The orientation techniques presented are tested with simulated line-scanner images in order to clarify the difficulties when applying them to practical cases.

INTRODUCTION

The analysis of CCD line-scanner imagery such as SPOT imagery and DPS imagery is becoming of greater importance. However, little has been written that would provide a general approach to the orientation problem of such imagery, particularly to that of their overlapped images (Derenyi(1973), Okamoto(1981), Hofmann(1986), and Ebner and Mueller(1986)). Therefore this paper presents a general and rigorous orientation theory of line-scanner images, which is basically very important for the analysis of CCD line-scanner imagery. First, geometrical characteristics of line-scanner images are clarified and possibilities of the self-calibration of a line-camera are investigated. Then, the orientation techniques presented are tested with simulated line images so as to explore the difficulties when applying them to the practical analysis of CCD line-scanner imagery.

BASIC CONSIDERATION

Let a line-scanner image be photographed in a plane as is demonstrated in Figure-1. The collinearity condition relating an object point $P(Y,Z)$ and its measured image point $p_c(y_c)$ can be expressed in a following algebraic form

$$y_c = \frac{A_1 Y + A_2 Z + A_3}{A_4 Y + A_5 Z + 1} \quad (1)$$

in which $A_i (i=1, \dots, 5)$ are independent coefficients. Geometrically, these five independent orientation elements are consid-

ered to be a rotation parameter ω in the plane, two translation parameters Y_0 and Z_0 which represent the projection center of the line-camera, and the two interior orientation elements (the principal point coordinate y_H and the principal distance c). Thus, with five control points in the object plane (Y,Z) , these five elements can be uniquely determined.

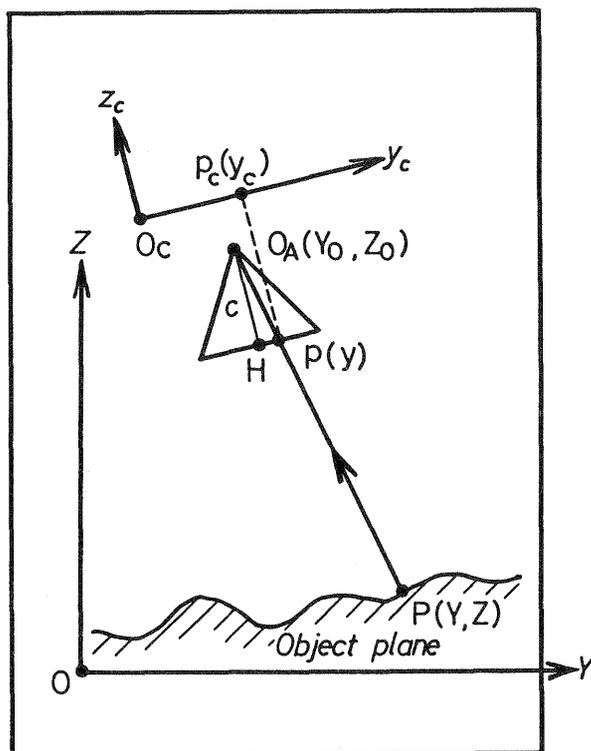


Figure-1: Geometry of a line-scanner image

In considering the model construction and the relationship between the model and object planes, we must employ three overlapped line images as is shown in Figure-2. The general collinearity equations are

$$Y_{c1} = \frac{A_{11}Y + A_{12}Z + A_{13}}{A_{14}Y + A_{15}Z + 1} \quad (2)$$

for the first line image,

$$Y_{c2} = \frac{A_{21}Y + A_{22}Z + A_{23}}{A_{24}Y + A_{25}Z + 1} \quad (3)$$

for the second line image, and

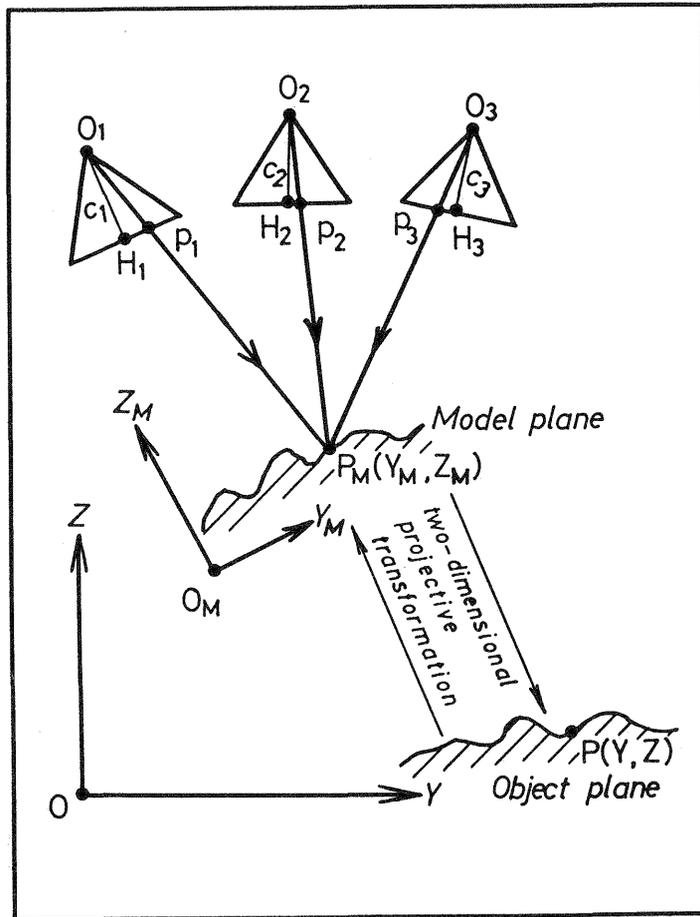


Figure-2: Orientation of three overlapped line images

$$y_{c3} = \frac{A_{31}Y + A_{32}Z + A_{33}}{A_{34}Y + A_{35}Z + 1} \quad (4)$$

for the third one, respectively. Equations 2, 3, and 4 can also be expressed in the linear form with respect to the object space coordinates (Y,Z) as

$$\begin{aligned} (y_{c1}A_{14} - A_{11})Y + (y_{c1}A_{15} - A_{12})Z + y_{c1} - A_{13} &= 0 \\ (y_{c2}A_{24} - A_{21})Y + (y_{c2}A_{25} - A_{22})Z + y_{c2} - A_{23} &= 0 \\ (y_{c3}A_{34} - A_{31})Y + (y_{c3}A_{35} - A_{32})Z + y_{c3} - A_{33} &= 0 \end{aligned} \quad (5)$$

The condition that Equation 5 is satisfied for an arbitrary object point P(Y,Z) is derived in the following determinant form

$$\begin{vmatrix} Y_{C1}A_{14} - A_{11} & Y_{C1}A_{15} - A_{12} & Y_{C1} - A_{13} \\ Y_{C2}A_{24} - A_{21} & Y_{C2}A_{25} - A_{22} & Y_{C2} - A_{23} \\ Y_{C3}A_{34} - A_{31} & Y_{C3}A_{35} - A_{32} & Y_{C3} - A_{33} \end{vmatrix} = 0 \quad (6)$$

Under the condition of Equation 6 we can form a two-dimensional space (Y_M, Z_M) with the three overlapped line images, which can be transformed into the object plane by the two-dimensional projective transformation having eight independent coefficients, i.e.,

$$\begin{aligned} Y_M &= \frac{B_1Y + B_2Z + B_3}{B_7Y + B_8Z + 1} \\ Z_M &= \frac{B_4Y + B_5Z + B_6}{B_7Y + B_8Z + 1} \end{aligned} \quad (7)$$

From the fact that a line image has five independent orientation elements, we can accordingly find the following facts that:

- 1) Seven orientation parameters can be determined from the model construction condition (Equation 6), and
- 2) All the fifteen orientation parameters of the three line-images can be uniquely provided, if we set up the collinearity equations for seven object points including four control points.

POTENTIAL THEORY FOR OVERLAPPED LINE-SCANNER IMAGES

A potential theory for conventional two-dimensional photographs was given by Okamoto(1986), which can clarify characteristics of the orientation problem in special cases where we have constraints among the orientation parameters. This theory can also be derived for the orientation analysis of overlapped line-scanner images. The geometry of a line image can be determined by the five orientation elements (ω, Y_O, Z_O, Y_H, c). These five parameters have central-perspective properties, because the relationship relating an object plane and its line image can be geometrically expressed in one central-perspective projection. Furthermore, the first three exterior orientation elements (ω, Y_O, Z_O) are related to a coordinate transformation (a congruence transformation). A model constructed with more than two overlapped line images can be transformed into the object by the projective transformation. Geometrical properties of this projective transformation can be explored by consider-

ing the geometry of a conventional photograph taken of a flat terrain. As is well-known, the relationship between an object plane and its conventional picture can be determined with the six exterior orientation elements (the three rotation elements $(\omega, \varphi, \kappa)$, the three translation elements (X_o, Y_o, Z_o) which represent the projection center of the photograph), and the two interior orientation parameters (x_H, y_H) . This transformation is central-perspective. However, it can be geometrically divided into three transformations in a three-dimensional space: a two-dimensional similarity transformation having four independent elements, a central-perspective transformation with two independent parameters, and an affine transformation having two independent elements. Thus, expressing the eight parameters describing the relationship between the object and the model constructed with overlapped line images as $(\hat{\Omega}, \hat{X}_{og}, \hat{Y}_{og}, \hat{m}, \hat{\lambda}, \hat{\mu}, \hat{a}_1, \hat{a}_2)$, these parameters are considered to have the following geometrical properties:

- 1) The first four parameters $(\hat{\Omega}, \hat{Y}_{og}, \hat{Z}_{og}, \hat{m})$ are related to a two-dimensional similarity transformation.
- 2) The fifth and sixth elements $(\hat{\lambda}, \hat{\mu})$ have central-perspective properties, and
- 3) The last two elements (\hat{a}_1, \hat{a}_2) have both central-perspective and affinitive properties.

Relating orientation parameters $(\omega_i, Y_{oi}, Z_{oi}, y_{Hi}, c_i (i=1, \dots, n))$ of n overlapped line images with the above eight parameters, we can accordingly find following relationships:

$$\begin{aligned}
 1) \quad \hat{S}_l &= \hat{S}_l(\omega_i, Y_{oi}, Z_{oi} (i=1, \dots, n)) \quad (l=1, \dots, 4) \\
 \hat{S}_l &: \hat{\Omega}, \hat{Y}_{og}, \hat{Z}_{og}, \hat{m}, \\
 2) \quad \hat{C}_m &= \hat{C}_m(\omega_i, Y_{oi}, Z_{oi}, y_{Hi}, c_i (i=1, \dots, n)) \quad (m=1, 2) \quad (8) \\
 \hat{C}_m &: \hat{\lambda}, \hat{\mu} \\
 3) \quad \hat{A}_p &= \hat{A}_p(\omega_i, Y_{oi}, Z_{oi}, y_{Hi}, c_i (i=1, \dots, n)) \quad (p=1, 2) \\
 \hat{A}_p &: \hat{a}_1, \hat{a}_2
 \end{aligned}$$

From Equation 8 we can find characteristics of the orientation problem of overlapped line images in various cases where we have constraints among the orientation parameters, e.g.,

- 1) The principal distance c and the principal point coordinate y_H of a line image are related to four parameters of the constructed model, which means that the model becomes similar to the object when the interior orientation parameters of two line images are known. However, it will be noted that more than two line images are required to form the model.
- 2) If the interior orientation is unchanged between three overlapped line images, we have four constraints among the interior orientation parameters. Using these constraints

we can form a model similar to the object.

- 3) When four relative relationships among the exterior orientation parameters of overlapped line images are given, a model similar to the object can also be formed from the model construction condition. It should, however, be noted that seven independent orientation parameters can also be provided during the phase of the relative orientation.

The second property may be very important for the self calibration of a line camera without object space information.

DISCUSSION ON THE ANALYSIS OF CCD LINE SCANNER IMAGERY

CCD line-scanner imagery is consecutively taken with a central projection in the across-track direction and with an orthogonal projection in the along-track direction. Overlapped line-scanner imagery is photographed along different flight paths (SPOT imagery) or along the same flight path (DPS imagery). Extracting one line image from the CCD line-scanner imagery of the first flight path and the corresponding line images from those of the other flight paths, the orientation theory presented in the previous sections can, in principle, be applied to the analysis of the overlapped line images. However, due to the fact that ground control is necessary for every set of overlapped line images, the orientation calculation is usually carried out three-dimensionally, which means that orientation parameters of each set of overlapped line images are provided together with parameters describing the corresponding object plane with respect to the ground coordinate system. Further, the changes of exterior orientation parameters of imaging devices along the flight paths are assumed to be modeled with some functional form in order to connect adjacent models without object space information. We have another problem in the analysis of actual CCD line-scanner imagery, that all points selected on the first line image are recorded at the same instant, while the recording of the corresponding points on the other overlapped images are spread over a certain time period. However, this phenomenon is not harmful but helpful for connecting adjacent models accurately if the changes of the exterior orientation elements of the imaging devices is modeled appropriately (See Hofmann(1986)). An efficient algorithm for such orientation calculation is shown by Hofmann(1986), Ebner and Mueller(1986). From theoretical results obtained in the previous sections, we may note that:

The model construction condition (Equation 6) for three overlapped line images has an ability to provide seven independent orientation parameters among their 15 unknown elements in the general case and five independent orientation elements of the nine unknown ones in the case of employing a metric line camera with the known interior orientation. Thus, at least five orientation points might be selected for each set of three overlapped images so as to use this ability effectively.

TEST WITH SIMULATED LINE-SCANNER IMAGES

The orientation theories derived in the foregoing sections will be checked with simulated line-scanner images. In the construction of the simulated models three convergent line-scanner images are considered to be employed and the image coordinates of 21 object points are calculated by means of the collinearity equations under the following conditions (See Figure-3):

flying height:	$H = 1500\text{m}$
focal length of the used line camera:	$c = 15\text{cm}$
convergent angles:	$\pm 20\text{deg.}$
maximum height difference between the 21 object points:	ca.100m

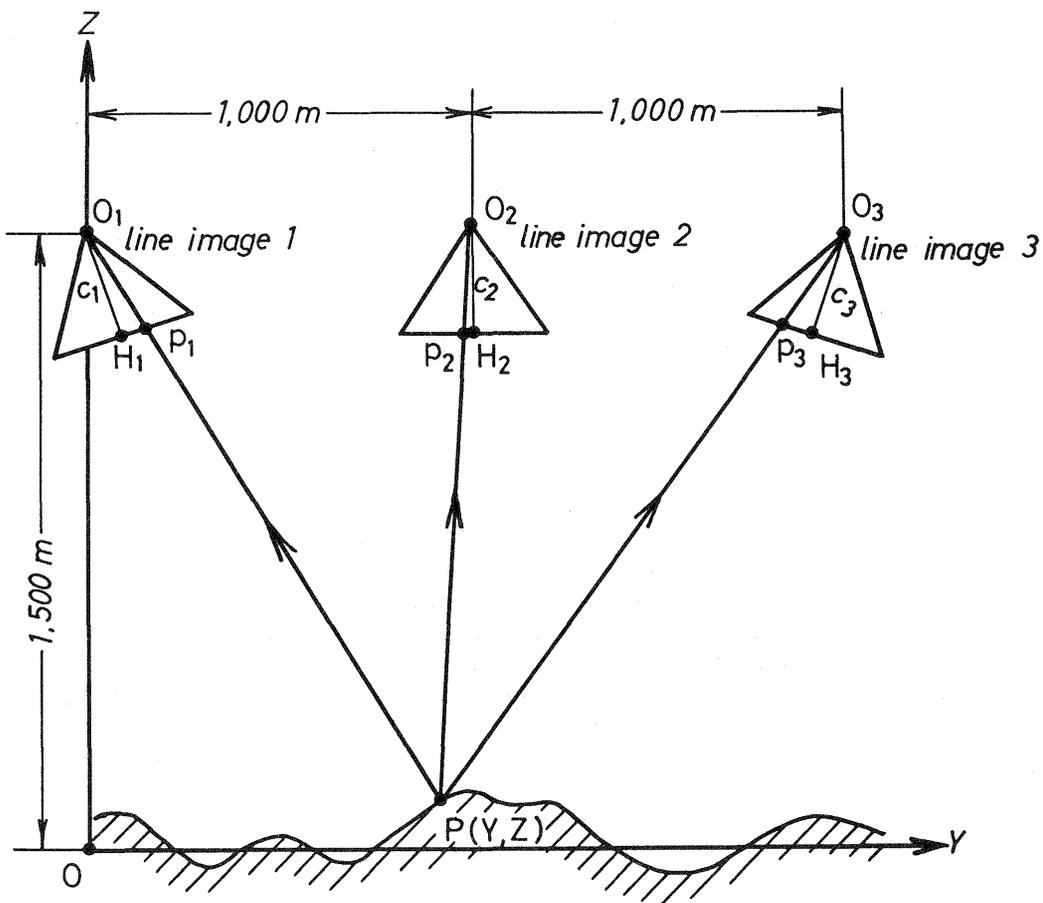


Figure-3: Convergent three line images

Then, the perturbed photo coordinates are provided in which the perturbation consists of random normal deviates having a standard deviation of 5 micrometers.

Using these three simulated line images following four cases are analyzed:

- 1) the general case (1) where a line image has five independent orientation unknowns $A_i (i=1, \dots, 5)$; four control

- points are used for the analysis.
- 2) the general case (2) where the geometry of a line image can be determined by five independent geometric orientation unknowns (ω, Y_O, Z_O, Y_H, c); the number of used control points is also four.
 - 3) a special case (1) where three line images are taken with a metric line camera; only two control points are employed.
 - 4) a special case (2) where the interior orientation is unchanged between the three line images; the analysis is performed with only two control points mathematically required.

The obtained results regarding the standard error of unit weight, the average internal error of the 21 object points, and the average external error are shown in Table-1.

Table-1: the obtained results

	$\hat{\sigma}_0$	average internal error at the ground scale	average external error at the ground scale
general case (1)	2.2 μ m	3.2cm	10.5cm
general case (2)	3.8 μ m	4.7cm	7.1cm
special case (1)	2.4 μ m	4.1cm	13.3cm
special case (2)	2.1 μ m	3.6cm	6.8cm

We can find in Table-1 the following characteristics of the orientation problem of overlapped line-scanner images:

- 1) The average internal error in each case is almost identical to that to be expected when the image coordinates are contaminated by random errors having a standard deviation of 5 micrometers. This means that the derived orientation theories are mathematically sound.
- 2) The external error in each case is somewhat larger than the internal one. This is mainly due to the fact that only control points mathematically necessary have been employed in each analysis. With redundant controls the external precision may be improved.

CONCLUDING REMARKS

In this paper a general theory for the analysis of CCD line-scanner imagery has been derived. From this theory it has been revealed that triplet line images are mathematically required for performing the orientation of line imagery in the same way as that for conventional two-dimensional photographs. Next, a potential theory for overlapped line images has been constructed, which can clarify the geometrical characteristics of various special cases where we have constraints among the orientation parameters. Based on this theory some self-calibration methods of a line camera can be developed.

The orientation theories derived here have been checked with simulated line-scanner images. The obtained results indicate that the orientation of CCD line-scanner images can be carried out with the same accuracy as that of conventional photographs. In this research continuous CCD line-scanner imagery has not been analyzed with simulated models. Such orientation calculation is now ready to be performed and the results will be reported in a later paper.

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