

OPTIMAL ROBUST SCALING SCHEME FOR A
ROBUSTIFIED BUNDLE BLOCK ADJUSTMENT

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ABSTRACT

Robust estimation methods are now making in-roads into the adjustment of geodetic and photogrammetric data, especially for the simultaneous detection and elimination of blunders in such data. The procedure requires the use of the standard deviation of the residuals in order to make the resulting estimators scale invariant. For situations where the covariance matrix of the residuals cannot be computed economically, this paper investigates the best robust scale estimator that could be used to scale the residuals in a robustified bundle adjustment.

1. INTRODUCTION

Robust estimation methods have promising applications in the adjustment of geodetic and photogrammetric data, especially for the detection and elimination of blunders in such data. Although statisticians have been applying robust statistics for more than three decades now, the surveying community is gradually embracing the advantages of robust estimation methods for its data processing activities. The Danish Geodetic Institute is probably the first to apply this new concept to geodetic data (cf. Krarup et al., 1980). Thereafter, research efforts have been directed towards its application in blunder detection in independent model triangulation (Klein and Forstner, 1984), single photo resection (Veress and Youcai, 1987; Chong, 1987), relative and absolute orientation of stereo models (Kubik et al., 1986; 1987), accuracy problems in GPS navigation data (Mertikas, 1987) and deformation surveys (Caspary and Borutta, 1987). Using the concept of Huber's distributional robustness (Huber, 1977), Faig and Owolabi (1988) demonstrated the advantages of robust estimation methods over data snooping technique in a simultaneous blunder removal and estimation process for a bundle block adjustment.

In robust regression with M-estimation methods, a specified objective function of the residuals is optimized, or equivalently a set of equations derived from the partial derivatives of the objective function is solved for the unknowns. In order to make the resulting estimators scale invariant, the residuals are scaled with their respective standard deviations. Assuming a parametric-type model, the standard deviations of the residuals are obtained as the square root of the diagonal elements of the covariance matrix:

$$C = \sigma_v^2 Q = \sigma_v^2 (P - AN^{-1}A^T), \text{ where } P \text{ is the weight matrix of the observations, } \sigma_v^2 = V PV/df, N = A^T P A, A \text{ is the}$$

design matrix and df is the degrees of freedom. Reasonable estimates of these values could be elusive if the data set is contaminated with outlying observations. It then becomes imperative to replace σ_v by some approximate values. However, various robust scale estimators are available for this purpose. The objective of this paper is to investigate the most efficient robust scaling scheme for a bundle block adjustment which has capability for using robust estimation methods in carrying out in one step, the simultaneous detection and elimination of blunders in the photogrammetric data processing for the estimation of the desired parameters.

2. ROBUST ESTIMATION METHODS

The poor performance of least squares estimators in the presence of outliers or of minor deviations from the assumptions of the error distribution, led statisticians to search for an alternative method to least squares as early as the 1950s. The historical development of robust estimation methods abounds in statistical literature (see, for example, Tukey, 1960; Huber, 1972). Studies were initially concentrated on the location case, culminating in the famous Princeton Robustness Study (Andrews et al., 1972). The satisfactory result obtained for robust estimation of location parameters encouraged the natural generalization of the technique to the more complicated regression case and to other more structured data such as surveying data.

Thus, robust estimation procedures can conceptually be grouped into two major parts: (i) robust estimation of location parameters and (ii) robust regression. The first part has direct applications for repeated single variable measurements and can readily be used at the input stage of a bundle adjustment software to eliminate simple blunders such

as those due to misidentification of points. On the other hand, robust regression has direct application at the adjustment stage and is the method considered in this paper.

Huber (1964) classifies robust estimation methods into three categories: (i) *M*-estimation methods, which are related to the maximum likelihood estimation method, (ii) *L*-estimation methods, which are linear combinations of the ordered statistics and (iii) *R*-estimation methods which are based on ranks or scores of the observed data. Extensive studies of these methods in the location problem have shown that the *M*-estimation is easier, more flexible and has better statistical properties than *L*- and *R*-estimation methods. Moreover, only the *M*-estimation method has a clear and flexible generalization to the regression case. Hence, it is the only method considered in this study.

2.1 ROBUST M-ESTIMATION METHODS

The classical least squares method minimizes the weighted sum of squares of the residuals given by:

$$\rho(v) = v^T P v = \min \quad (1)$$

where, *P* is the weight matrix of the observations. The ρ function in equation (1) can be made more general by replacing the weighted sum of squares of the residuals by a less rapidly increasing function (Huber, 1977). The objective then boils down to minimizing

$$\sum_i \rho(v_i / \sigma_i) \quad \text{or equivalently solving the system of equations}$$

$$\sum_i \psi(v_i / \sigma_i) \frac{\partial f}{\partial x_j} = 0, \quad j = 1, 2, \dots, u \quad (4)$$

in which the previous objective function $\rho(v) = v^T P v$ for the least squares method is a subclass. To make the estimator scale invariant (see Huber, 1973; Hogg, 1979), ψ is set equal to ρ' in equation (4) and the expression is divided by σ_i , the standard deviation of *v*.

The ψ functions and their associated tuning constants further divide the *M*-estimators into subclasses. Thus, there are Huber's *M*-estimator, Hampel's *M*-estimator, Andrews' *M*-estimator, etc. Also Huber's *M*-estimator with a tuning constant equal to infinity gives the usual least squares estimator. A collection of presently available ψ functions is given in Faig and Owolabi (1988).

Quite apart from the possibility of nonlinearity of the functional model *f*, most ψ functions are available in

nonlinear form, requiring that the solution of equation (4) be iterative in nature. Of the three approaches available to solve equation (4) (Holland and Welsch, 1977), the iteratively reweighted least squares method is the most favoured and widely used, because of its flexibility. Furthermore, it only requires computing a weight function as a function of the scaled residuals, that is, $w(v/\hat{\sigma}_v) = \psi(v/\hat{\sigma}_v)/(v/\hat{\sigma}_v)$ and then using an existing weighted least squares algorithm.

2.2 SCALE ESTIMATORS FOR ROBUST REGRESSION

Scale invariant properties for robust estimation of location parameters have been treated in Andrews et al. (1972) and Hogg (1979). Their generalization to robust regression is considered in this paper.

In large data sets in which we are not certain that the data being processed is free of blunders, robust estimation technique is one of the alternatives available for data processing. In that case, the standard deviation of the residuals obtained from the appropriate estimated covariance matrix cannot be used for scaling the residuals because it is influenced too much by outliers. The immediate offending component is the a-posteriori variance of unit weight computed from the residuals which are themselves prone to inflation from outlying data.

Consequently, the following alternatives are suggested and implemented in a robustified bundle adjustment software (ROBUD).

$$(1) \hat{\sigma}_v = \text{median}_i | v_i | = \text{Median Absolute Residual (MAR)}$$

$$(2) \hat{\sigma}_v = \text{median}_i | v_i - \text{median}(v) | \\ = \text{Median Absolute Deviation (MAD)}$$

$$(3) \hat{\sigma}_v = \text{MAD} / 0.6745$$

$$(4) \hat{\sigma}_v^2 = \hat{\sigma}_v^2 Q_v, \quad \hat{\sigma}_v = \text{MAR}, \quad Q_v = \text{Cofactor Matrix of } v$$

$$(5) \hat{\sigma}_v^2 = \hat{\sigma}_v^2 Q_v, \quad \hat{\sigma}_v = \text{MAD}$$

$$(6) \hat{\sigma}_v^2 = \hat{\sigma}_v^2 Q_v, \quad \hat{\sigma}_v = \text{MAD} / 0.6745$$

$$(7) \quad \sigma_v^2 = \sigma_o^2 Q_v, \quad \sigma_o^2 = v^T P v / df, \quad p_{ii} = f(v_i)$$

3. CASE STUDY

3.1 EXPERIMENT

Data for four photographs were generated using a fictitious photo data software (cf. Woolnough, 1973). An integrated and robustified bundle adjustment software was developed for this research. In order to test the variability in performance of the different weighting schemes in accurately detecting outliers, three different control point patterns were used, namely, high density control (pattern H), medium density control (pattern M) and sparse control (pattern S).

Andrews' sine wave robust M-estimator and Huber's M-estimator were used in the experiment. A tuning constant of 2.0 was used for the experiment in conformity with the conclusion arrived at in an earlier study reported in Faig and Owolabi (1988).

The study begins by running the program for all three types of control configurations, first with a 3 μ m blunder imbedded in one coordinate of an image point and then using all seven weighting schemes to process the data in turn. Next, a 10 mm blunder was imbedded and the procedure was repeated all over again. The root mean square errors at check points for all the cases grouped according to control configurations are shown in Tables 1 to 6. Studies have shown that robust estimation methods are capable of reproducing imbedded or naturally-occurring blunders which are displayed in the residuals (Kubik and Merchant, 1986; Faig and Owolabi, 1988). Thus, the magnitude of the recovered blunder provided another means of evaluating the effectiveness of the weighting schemes in the data processing (see Tables 1 to 6)

3.2 DISCUSSION AND CONCLUSION

Table 1 shows the root mean square values at the check points and the recovery of a $3\mu\text{m}$ imbedded blunder using control point pattern H. Clearly, all the scaling schemes seemed to give good recovery capability for the introduced blunder using both Andrews' and Huber's M -estimators. However, scaling schemes 3 and 7 produced the best results in accuracy among the methods considered. In particular, scaling scheme 3 has fewer iterations and consequently less computational time than the other methods. Scaling schemes 4 and 5 produced the worst results among the methods studied. This trend remained the same when the blunder was increased to 10mm using the same control point pattern H (see Table 2).

Tables 3 and 5, and Tables 4 and 6 have the same characteristics with Tables 1 and 2 respectively, except for variation in control point patterns. It can be seen that scaling schemes 3 and 7 consistently provided the best results with variation in control point patterns. However, the blunder recovery ability of Andrews' M -estimator decreased with decrease in density of control points. On the other hand, Huber's M -estimation method did not show any deviations with variation in control point density.

In conclusion therefore, only scaling schemes 3 and 7 may be considered for photogrammetric bundle adjustment in the presence of outliers. Scaling scheme 7 provided better accuracy than scaling scheme 3 since it computes the entire variance-covariance matrix of the residuals. On the other hand, scaling scheme 3 provided less computational time and should be favoured for very large photogrammetric blocks where the computation of the covariance matrix of the residuals would not be economically feasible.

TABLE 1 : Recovery of 3um Imbedded Blunder
Using Control Pattern H

SCALING SCHEME	RMS XY(MM)		RMS Z(MM)		RECOVERED BLUNDER(MM)		ITR		TIME (SEC)	
	A	B	A	B	A	B	A	B	A	B
1 MAR	4.1	4.1	10.0	10.0	3.2	3.0	12	9	6.4	4.9
2 MAD	4.1	4.1	10.0	10.0	3.2	3.0	12	8	6.6	4.6
3 MAD6	4.1	4.1	9.8	10.0	3.3	3.0	5	8	3.1	4.7
4 Q*MAR	5.9	4.8	15.0	11.1	3.3	3.0	15	30	12.1	26.1
5 Q*MAD	5.6	5.9	14.1	14.0	3.3	3.0	10	13	9.1	11.3
6 Q*MAD6	4.1	4.1	10.0	10.1	3.2	3.0	8	8	7.1	7.2
7 Q/VAR	4.1	4.1	9.9	10.2	3.0	3.0	6	8	5.3	7.0

Note: A = Andrews' estimator; B = Huber's estimator
RMS is in object unit Blunder is in image unit

TABLE 2 : Recovery of 10mm Imbedded Blunder
Using Control Pattern H

SCALING SCHEME	RMS XY(MM)		RMS Z(MM)		RECOVERED BLUNDER(MM)		ITR		TIME (SEC)	
	A	B	A	B	A	B	A	B	A	B
1 MAR	4.1	4.1	10.0	10.1	10.0	10.0	11	15	6.1	8.1
2 MAD	4.1	4.1	9.9	10.1	10.0	10.0	10	15	5.7	8.1
3 MAD6	4.1	4.1	9.8	10.0	10.0	10.0	6	14	3.6	7.5
4 Q*MAR	6.1	4.5	17.0	11.2	10.0	10.0	13	18	6.1	10.1
5 Q*MAD	5.5	6.0	16.1	14.5	10.0	10.0	14	18	6.3	10.3
6 Q*MAD6	4.1	4.1	9.9	10.1	10.0	10.0	13	18	6.1	10.1
7 Q/VAR	4.1	4.1	9.9	10.3	10.0	10.0	7	21	6.1	17.6

TABLE 3 : Recovery of 3um Imbedded Blunder
Using Control Pattern M

SCALING SCHEME	RMS XY(MM)		RMS Z(MM)		RECOVERED BLUNDER(MM)		ITR		TIME (SEC)	
	A	B	A	B	A	B	A	B	A	B
1 MAR	5.5	4.5	11.5	10.2	3.4	3.0	19	31	9.4	15.2
2 MAD	5.4	4.5	11.5	10.1	3.4	3.0	19	31	9.6	15.4
3 MAD6	4.1	4.1	8.8	9.1	3.2	3.0	5	7	3.2	4.2
4 Q*MAR	4.4	4.3	9.6	10.0	2.1	3.0	31	31	25.6	25.7
5 Q*MAD	4.3	4.2	11.4	9.8	2.1	3.0	31	31	26.4	26.2
6 Q*MAD6	4.3	4.5	8.5	10.0	3.0	3.0	31	31	26.4	26.4
7 Q/VAR	4.2	4.1	10.0	9.2	3.0	3.0	9	9	7.8	7.9

Note: A = Andrews' estimator; B = Huber's estimator
RMS is in object unit Blunder is in image unit

TABLE 4 : Recovery of 10mm Imbedded Blunder
Using Control Pattern M

SCALING SCHEME	RMS XY(MM)		RMS Z(MM)		RECOVERED BLUNDER(MM)		ITR		TIME (SEC)	
	A	B	A	B	A	B	A	B	A	B
1 MAR	5.2	4.5	11.1	10.4	10.0	10.0	23	31	11.9	15.1
2 MAD	5.2	4.6	11.1	10.3	10.0	10.0	27	31	14.4	15.1
3 MAD6	4.1	4.1	8.8	9.1	10.0	10.0	6	15	3.9	8.2
4 Q*MAR	4.4	4.3	10.0	10.2	7.2	10.0	31	31	25.7	25.9
5 Q*MAD	4.4	4.6	11.1	10.0	7.2	10.0	31	31	26.0	26.1
6 Q*MAD6	4.9	4.4	11.0	10.0	7.1	10.0	30	31	25.8	26.2
7 Q/VAR	3.9	4.1	8.3	9.0	10.0	9.8	6	21	5.3	17.9

Note: A = Andrews' estimator; B = Huber's estimator
RMS is in object unit Blunder is in image unit

TABLE 5 : Recovery of 3um Imbedded Blunder
Using Control Pattern S

SCALING SCHEME	RMS XY(MM)		RMS Z(MM)		RECOVERED BLUNDER(MM)		ITR		TIME (SEC)	
	A	B	A	B	A	B	A	B	A	B
1 MAR	5.4	4.9	10.6	10.1	2.1	3.0	15	31	7.6	14.9
2 MAD	5.4	4.9	10.6	9.9	2.1	3.0	15	31	7.4	15.2
3 MAD6	5.0	4.6	10.3	9.3	3.0	3.0	12	31	6.6	15.2
4 Q*MAR	5.1	4.7	10.2	9.6	2.1	3.0	31	31	26.4	25.5
5 Q*MAD	6.5	4.5	13.1	9.4	2.1	3.0	31	31	26.4	26.4
6 Q*MAD6	5.4	4.5	11.7	9.4	2.1	3.0	31	31	26.2	26.5
7 Q/VAR	4.9	4.5	10.1	9.3	3.0	3.0	13	9	11.0	7.7

TABLE 6 : Recovery of 10mm Imbedded Blunder
Using Control Pattern S

SCALING SCHEME	RMS XY(MM)		RMS Z(MM)		RECOVERED BLUNDER(MM)		ITR		TIME (SEC)	
	A	B	A	B	A	B	A	B	A	B
1 MAR	5.7	4.9	10.8	10.0	7.3	10.0	15	31	7.7	15.1
2 MAD	5.7	4.9	10.7	10.0	7.2	10.0	14	31	7.0	15.2
3 MAD6	5.0	4.6	10.3	9.4	10.0	10.0	13	31	7.1	15.2
4 Q*MAR	5.7	4.9	10.7	10.0	7.2	10.0	15	31	7.2	15.3
5 Q*MAD	5.2	4.6	10.4	8.4	7.1	10.0	31	31	25.8	26.1
6 Q*MAD6	5.5	4.3	10.3	9.0	7.2	10.0	31	31	26.1	26.1
7 Q/VAR	4.2	4.3	8.4	9.4	10.0	9.9	6	21	5.3	17.9

Note: A = Andrews' estimator; B = Huber's estimator
RMS is in object unit Blunder is in image unit

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