

**CALIBRATION OF AN AMATEUR
CAMERA FOR VARIOUS OBJECT DISTANCES**

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QUEBEC G1K 7P4 C A N A D A
Commission V

ABSTRACT

Self-calibration plays an important role in the application of non-metric cameras for precision photogrammetric works. An IBM-PC computer operated procedure has been developed for such an analytical calibration of cameras at the Photogrammetry Laboratories of Laval University, Quebec, Canada. The mathematical models used in this procedure are described. One Rollei SLX camera ($f=80.0\text{mm}$) was calibrated with respect to four object distances and various configurations. The calibrated focal lengths and the corresponding lens distortion patterns are presented for the various cases. The experience and the results indicate that the procedure is economical and convenient for most close-range applications demanding precision measurements.

INTRODUCTION

With more and varied use of photogrammetry for precision close-range applications, camera calibration plays an important role whereby non-metric cameras yield results enough to challenge metric cameras in view of their cost-effectiveness. Most realistic approaches for the calibration of a non-metric camera can be classified into two groups: (a) on-the-job calibration, and (b) self-calibration.

On-the-job calibration has been noted to require an object space control network (Anderson et al, 1975), where the number of necessary control points would be directly proportional to the desired number of involved parameters.

Unlike the classical procedures, self-calibration is based solely on the image point measurements without requiring absolute ground control. A strong geometrical configuration of multiple photographs over the same field of unknown object points would be desirable (Ghosh, 1988). Furthermore, self-calibration can also be "on-the-job". One would easily see that the self-calibration procedure has more advantages (Moriwa, 1977). This is why self-calibration is more welcome in practice, which is the rationale behind the present study.

BASIC MATHEMATICAL MODELS

The self-calibration approach is based on the augmentation of collinearity condition equations. There are however some differences among the mathematical models used in the self-calibration of various organizations.

These differences are mainly due to the different ways of modeling the lens distortions, the film deformation and the consideration of weighting the parameters in calibration (El-Hakim, 1979; Adiguzel, 1985; Fryer, 1984).

The basic mathematical models used in this study are as follows:

$$\bar{x} + dV_x = f \cdot \frac{M_{11}(X-X_0) + M_{12}(Y-Y_0) + M_{13}(Z-Z_0)}{M_{31}(X-X_0) + M_{32}(Y-Y_0) + M_{33}(Z-Z_0)} \quad (1)$$

$$\bar{y} + dV_y = f \cdot \frac{M_{21}(X-X_0) + M_{22}(Y-Y_0) + M_{23}(Z-Z_0)}{M_{31}(X-X_0) + M_{32}(Y-Y_0) + M_{33}(Z-Z_0)}$$

where,

dV_x, dV_y are the corrections for lens distortions,

$$dV_x = \bar{x} \cdot (k_1 r^2 + k_2 r^4 + k_3 r^6) + [P_1 (r^2 + 2 \cdot \bar{x}^2) + 2 \cdot P_2 \bar{x} \cdot \bar{y}] (1 + P_3 r^2)$$

$$dV_y = \bar{y} \cdot (k_1 r^2 + k_2 r^4 + k_3 r^6) + [P_2 (r^2 + 2 \cdot \bar{y}^2) + 2 \cdot P_1 \bar{x} \cdot \bar{y}] (1 + P_3 r^2)$$

$$\bar{x} = x - x_0 ; \quad \bar{y} = y - y_0$$

x and y are photo-coordinates with fiducial reference;

x_0, y_0 are the photo-coordinates of the principal point with fiducial reference;

f is the calibrated focal length;

k_1, k_2, k_3 are the coefficients for radial lens distortion model (polynomial);

P_1, P_2, P_3 are the coefficients for decentering lens distortion model;

M 's are the elements of the orientation matrix;

X, Y, Z are the ground coordinates of object points; and

X_0, Y_0, Z_0 are the ground coordinates of the camera perspective center.

After linearizing (1), and considering the other weighted function constraints for all the parameters, the mathematical model can be written as:

$$V + A_1 \Delta_1 + A_2 \Delta_2 + A_3 \Delta_3 + W = 0 \quad (2a)$$

$$V_1 + \Delta_1 + W_1 = 0 \quad (2b)$$

$$V_2 + \Delta_2 + W_2 = 0 \quad (2c)$$

$$V_3 + \Delta_3 + W_3 = 0 \quad (2d)$$

where,

V : residual vector of image coordinates,
 Δ_1 : unknown external orientation parameters,
 Δ_2 : unknown interior geometry parameters ($x_0, y_0, f, k_1, k_2, k_3, P_1, P_2, P_3$).
 Δ_3 : unknown object point coordinates,
 A_1, A_2, A_3 are the corresponding coefficient matrices,
 V_1, V_2, V_3 are the residual vectors of the three types of pseudo observations $\Delta_1, \Delta_2, \Delta_3$, respectively, and
 W, W_1, W_2, W_3 are the misclosure vectors in the equations.

The final estimations of the unknown parameters are based on the principle:

$$V^T P V + V_1^T P_1 V_1 + V_2^T P_2 V_2 + V_3^T P_3 V_3 \longrightarrow \min.$$

where,

P is the weight matrix of image coordinates, and
 P_1, P_2, P_3 are the weight matrices of the pseudo observations.

With regard to the present study, two characteristics of Eqs. (2) may be pointed out:

(1) Lens distortion is formulated by the well known odd-power polynomials. Non-linear film deformations are not considered, because the film format is small (56 x 56 mm), the linear part being automatically contained in the photo coordinate transformation. Usually, the effects of film deformation on image coordinates are very small for such small format films (Hatzopoulos, 1985).

(2) All unknown parameters are treated as pseudo observations. This includes the interior orientation parameters and the additional parameters for lens distortion. Proper weights can be assigned in practice to these pseudo observations.

The above characteristics have the following special advantages:

(a) All image points, including those appearing only on two photos can be used.

(b) Proper input of weights, plus good geometric conditions, can reduce or even eliminate the correlations among various parameters.

(c) Interaction amongst "standards" is avoided (eg., geodetic standard of ground control; manufacturers' standards for photogrammetric equipment and camera, etc.).

DATA ACQUISITION

One Rollei SLX camera ($f=80$ mm) was calibrated at four different object distances (0.25 m, 1.0 m, 2.5 m, 7.0 m) with a view to its use on objects at various distances. In the case of 0.25 m, an attachment had to be used for extending the lens tube in order to get a clear focus on the object (targets).

Three different test ranges had to be used for the calibration because of the differences in the corresponding ground coverages; one is for 0.25 m, one for 1.0 m and 2.5 m and the last one for 7.0 m. The designs of the three ranges are similar, except that the grid dimensions are different. Each range consists of more than 40 grid points. However, during the calibration, only 20 grid points were selected at random in each case. Four convergent photos were taken on the targets in each case. The geometry of the photography is depicted in Fig. 1.

Photo points were measured on both BC-1 Analytical Plotter and STK-1 Stereocomparator. The purpose of measuring the photos on the two instruments was to compare the results from different measuring tools. Software is available to store the image coordinates directly on disks to be used on the IBM-PC computer, with regard to both the instruments.

CALIBRATION RESULTS AND ANALYSES

The calibration results presented here are based on the image coordinates measured with the STK-1 Stereocomparator in mono-mode.

The interior orientation parameters in each case are listed in Table 1, where one can see that in cases 2, 3 and 4, the x_0 y_0 are near zero.

As mentioned before, in case 1, an attachment was used to obtain clear focus on the target. The x_0 and y_0 shifts in this case are conjectured to be due to the additional attachment, causing camera axis deviation.

According to the Gaussian optical law, the longer the object distance, the shorter should be the focal distance. The f values in Table 1 agree with this law, (see Fig. 2). The change seems to be systematic.

In case 4, the calibrated focal length has the best accuracy (0.28 mm), also suggesting that the camera has better focus at 7.0 m distance.

The lens distortion parameters of the camera are listed in Table 2, where from Table 2, one can see that the decentering lens distortions are negligible. With the film format 56 x 56 mm, the maximum tangential lens distortion components in x and y are less than 1 μm .

The radial lens distortion curves of this camera for different object distances are shown in Fig. 3. Table 2 and Fig. 3 indicate that (a) The radial lens distortions are significant in cases 1 and 2; and (b) In Cases 3 and 4, the lens distortions become very small (negligible in case 4).

In order to study the degree of importance of lens distortion on the calibration results, case 2 was studied as an example for the following instances:

- i Calibration without considering any of the six lens distortion parameters (k_1 , k_2 , k_3 , P_1 , P_2 and P_3).
- ii Calibration by considering the six lens distortion parameters.
- iii Calibration by considering only k_1 , k_2 and k_3 .
- iv Calibration by considering only P_1 , P_2 and P_3 .

The results are summarized in Table 3, from which one can draw the following conclusions:

(1) By comparing the results of instances i and ii, one can see that the accuracy is improved significantly by considering the lens distortion parameters into the calibration.

(2) By comparing the results of case i with those in case iv, or case ii with case iii, one can find that the decentering lens distortion has no effect on the calibration output accuracy.

Tests about the effect of the lens distortions on relative orientation of two photos taken by the camera were also performed. Each relative orientation was done in two cases, one without considering the lens distortions, the other by considering the lens distortions. The results of one such test are shown in Table 4, where one can see that the accuracy of the relative orientation is improved significantly by considering the radial lens distortion.

In order to study the accuracy improvement with regard to the number of photos used, the calibrations in the four cases were also carried out with 4, 3 and 2 photos each. Some illustrative results are presented in Table 5 and Fig. 4. Table 5 indicates that with the reduction of photographs, not only the redundancy of observations is decreased, but also the geometry is weakened, consequently, the calibration accuracy becomes poorer.

So far, the presented results are all from the calibrations based on the image coordinates measured at the STK-1 Stereocomparator. As mentioned earlier, the photo coordinates are also measured at the BC-1 Analytical Plotter. The calibration results from the STK-1 measurements are almost identical to these from the BC-1 observations. These are not, therefore, presented here.

CONCLUSIONS AND RECOMMENDATIONS

(1) The camera has significant radial lens distortion. This distortion has strong effect when the camera is used on close-range objects. (2) Radial lens distortion effects seem to be reduced as the object distance is increased. (3) Decentering lens distortions are negligible for this camera (for all object distances). (4) Measurement data from the STK-1 Stereocomparator and from the BC-1 Analytical Plotter gave similar calibration results, indicating thereby that the STK-1 is good enough for this

type of camera calibration. (5) The calibrated focal lengths change significantly when the object distances change. The calibrated focal length values can be interpolated (Fig. 2) for specific applications with regard to the object distances. (6) The calibration results show that the accuracy of the calibrated focal length is also a function of the object distance. The calibration at 7.0 m object distance gave the best accuracy of the focal length. (7) During the calibration tests, it was found that the correlations among the parameters are negligible. According to our case study, with the intersection geometry as used (Fig. 1), i.e. a good convergent multi-photo configuration, such correlations are "broken". This establishes very stable and reliable geometric configuration. (8) The attachments of the camera should be used with caution. Otherwise, it may produce annoying effects. (9) To obtain a good estimation of lens distortions, object points should be widely distributed so as to cover the entire photo in each case.

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Fig. 1
 GEOMETRY OF THE PHOTOGRAPHY

Note: 1,2,3 and 4 are
 the camera stations;
 Object distances to the
 center of test area
 (S) are: 0.25 m, 1.0 m,
 2.5 m and 7.0 m.

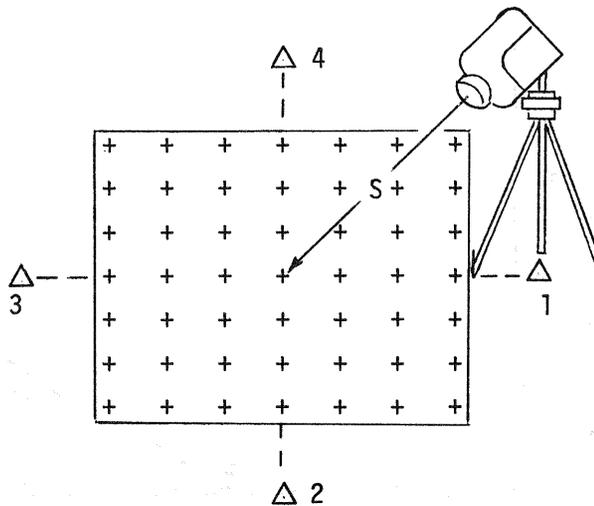
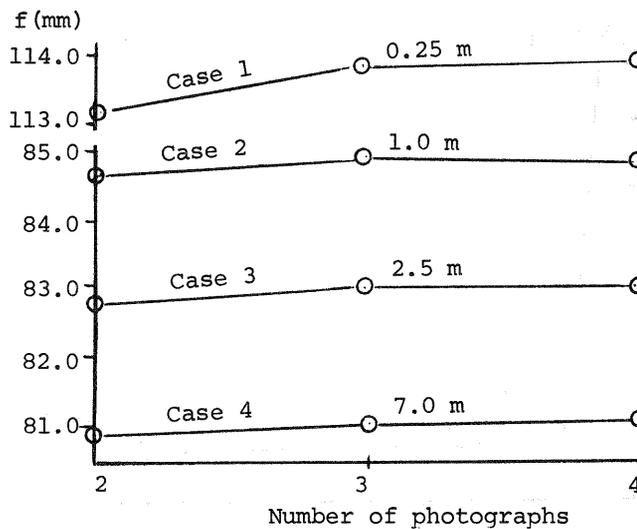


Fig. 2
 FOCAL LENGTHS FOR
 VARIOUS OBJECT DISTANCES



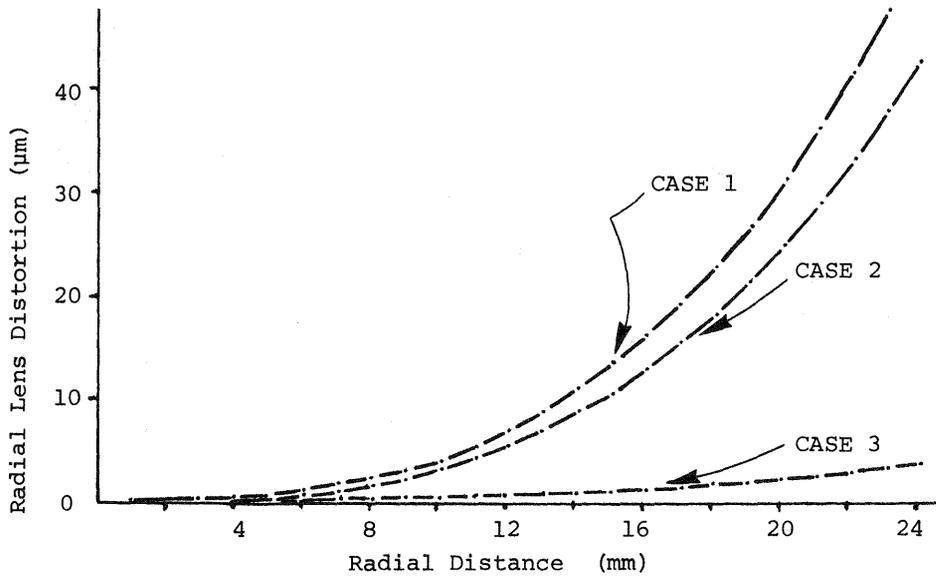


Fig. 3 : RADIAL LENS DISTORTION

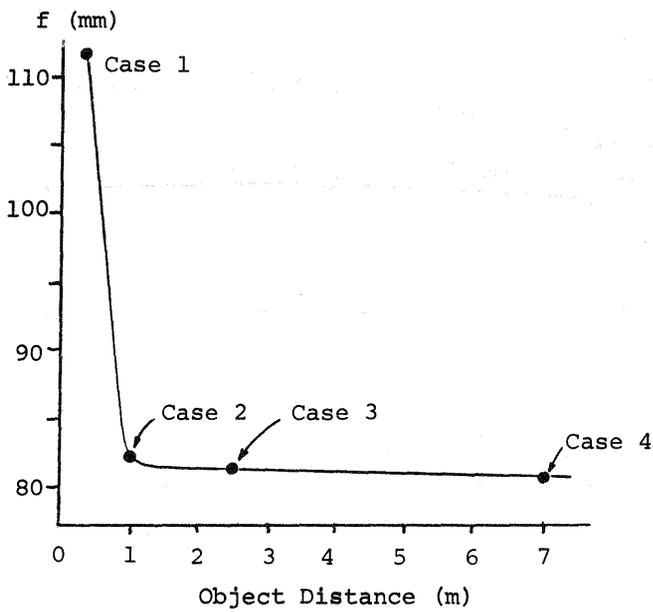


Fig. 4
CALIBRATED FOCAL LENGTHS

Table 1. I.O. Parameters of the Camera

Case no.	x_0 (mm)	σ_{x0} (mm)	y_0 (mm)	σ_{y0} (mm)	f (mm)	σ_f (mm)
1	-0.52	0.39	1.02	0.39	113.99	0.76
2	0.04	0.09	-0.59	0.62	84.63	0.91
3	0.03	0.13	-0.46	0.56	82.95	0.49
4	-0.02	0.26	-0.37	0.65	80.90	0.28

Note: Cases 1, 2, 3 and 4 correspond to 0.25 m, 1.0 m, 2.5 m and 7.0 m, object distances, respectively.

Table 2. Lens Distortion Parameters of the Camera

	k_1	k_2	k_3	P_1	P_2	P_3
1	3.907	0.006	0.000	0.000	0.000	0.000
2	3.121	0.003	0.000	-0.128	-0.079	0.000
3	0.278	0.000	0.000	-0.025	0.000	0.000
4	0.109	0.000	0.000	-0.002	0.000	0.000

Note: 1, 2, 3 and 4 are the cases for 0.25 m, 1.0 m, 2.5 m and 7.0 m, object distances, respectively.

Table 3. Accuracy Outputs for the Camera

Instance	σ^2	σ_f (mm)	σ_x (μm)	σ_y (μm)	σ_x (mm)	σ_y (mm)	σ_z (mm)
i	2.02	1.18	13.4	11.6	0.044	0.045	0.086
ii	1.54	0.91	10.2	10.2	0.037	0.038	0.072
iii	1.54	0.91	10.2	10.2	0.037	0.038	0.072
iv	2.02	1.18	13.4	11.6	0.044	0.045	0.086

Note: σ^2 : The estimated variance of unit weight.
 σ_f : Standard deviation of the calibrated focal length.
 σ_x and σ_y : Standard deviations of image coordinates.
 $\sigma_x, \sigma_y, \sigma_z$: Standard deviations of adjusted object coordinates

Table 4. Results of Relative Orientation (independent method)

Photo	rotations/(standard deviations)			P _y (average) (μm)	σ (μm)	Comments
	ω (gra)	Φ (gra)	χ (gra)			
1	38.859 (0.019)	-2.097 (0.019)	3.077 (0.112)	8.5	3.5	No lens distortions considered
2	-37.315 (0.025)	1.891 (0.019)	3.069 (0.102)			
1	36.858 (0.019)	-2.106 (0.019)	3.636 (0.111)	5.4	2.4	Lens distortions considered
2	-37.431 (0.025)	1.891 (0.019)	3.587 (0.102)			

Note: Here P_y is the average residual y parallex after relative orientation; and σ is the normalized standard deviation of unit weight in the least-squares adjustment for orientation.

Table 5. Calibration Results for Different Number of Photos

Object Dist.	No. of photos	σ _{x0} (mm)	σ _{y0} (mm)	f (mm)	σ _f (mm)	σ _x (μm)	σ _y (μm)	σ _x (mm)	σ _y (mm)	σ _z (mm)
0.25m	4	0.39	0.39	113.99	0.76	6.5	8.4	0.020	0.019	0.071
	3	0.49	0.49	113.99	0.82	6.5	8.7	0.023	0.024	0.078
	2	0.61	0.65	113.96	0.90	7.0	8.8	0.028	0.026	0.084
1.0m	4	0.09	0.62	84.63	0.91	10.2	10.2	0.037	0.038	0.072
	3	0.10	0.60	84.71	0.91	10.1	10.1	0.037	0.038	0.072
	2	0.13	0.65	84.61	1.00	10.1	10.5	0.041	0.040	0.080
2.5m	4	0.13	0.56	82.95	0.49	7.0	10.6	0.031	0.031	0.060
	3	0.15	0.58	82.99	0.52	7.1	11.2	0.032	0.032	0.063
	2	0.18	0.60	82.78	0.54	7.1	11.5	0.033	0.033	0.064
7.0m	4	0.26	0.65	80.90	0.28	5.0	13.4	0.103	0.104	0.135
	3	0.31	0.66	80.90	0.30	5.3	13.5	0.105	0.105	0.138
	2	0.38	0.70	80.75	0.35	5.4	14.0	0.110	0.110	0.145

Note: σ_{x0} and σ_{y0} are standard deviations of principal point coordinates. For the rest, see Note for Table 3.