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THE DETERMINATION OF DEFORMATION OF CONNECTING BEAM  
UNDER THE REPEATED LOADS BY USE OF THE NON-METRIC CAMERA

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**Abstract:** This paper introduces the problems of the refit and calibration of camera, the image measurement, the data processing and the accuracy test etc., which are related to the determination of deformation of the connecting beam under the repeated loads by means of the time-parallax using the non-metric camera.

### 1. Introduction

The wall bodies in the armoured concrete scissors wall are joined together by the connecting beam. The intensity, rigidity and deformational speciality of connecting beam have great influence on the supportability and anti-seismic capability of scissors wall. Thus a test of connecting beam was conducted by Research Institute of Structural Engineering, Tsinghua University. The purpose of test is, by model test, to study the destroyed shape and the speciality of the loading and deformation of connecting beam in armoured concrete scissors wall under repeated loads.

In conjunction with the test of Research Institute of Structural Engineering, an experiment of deformation determination of connecting beam by use of the close-range photogrammetry was performed by Institute of Seismology, State Seismological Bureau with the help of Survey Teaching And Research Section of Department of Civil Engineering, Tsinghua University.

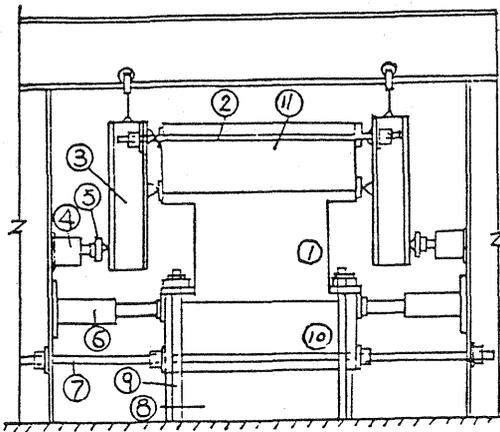
### 2. Brief Introduction To Loading Test of Connecting Beam

The test arrangement was shown as in Fig.1. The test sample is the model to one second scale and 50cmx50cm size. The thickening concrete blocks in ends of sample are used to load and fix the sample. The sample rotated 90° as the practical orientation in scissors wall is set over the loading plate. The under thickening block (10) is fixed by the fixing system composed of (6), (7), (8) and (9). The above thickening block (11) is loaded with loading system composed of (2), (3), (4) and (5).

The loading is applied left-right repeatedly. The displacement, the sliding and rotation of sample under various loads are measured using the traditional instruments such as the displacement meter and dipmeter etc.. At the same time the deformation of whole sample are recorded and determined by the close-range photogrammetry.

### 3. Close-Range Photogrammetric Method

Because the loading and displacement of sample are generated in a vertical plane and mainly in two dimensions, the data processings are done by the time-parallax using one camera to take a single photograph.



- (1) test sample
- (2) loading bar
- (3) loading beam
- (4) loading jack
- (5) load sensor
- (6) fixing jack
- (7) fixing bar
- (8) spacer
- (9) vertical fixing bar
- (10), (11) thickening block

Figure 1. Loading System

#### (1) Characteristics of Time-Parallax

Assuming that the first photograph is taken before deformation and the second one after deformation using a same camera (with the same external and internal orientation in the ideal case), the "parallax" will be produced on the point which had displaced in period between first and second exposure while two photographs are observed stereoscopically. This is called pseudo parallax or time-parallax.

If the object point is displaced from  $D_1$  to  $D_2$  parallel to plane XZ as be shown in Figure 2, the time-parallax  $\Delta x$  and  $\Delta z$  are:

$$\begin{aligned} \Delta x &= \frac{f}{Y} \Delta X \\ \Delta z &= \frac{f}{Y} \Delta Z \end{aligned} \quad (1)$$

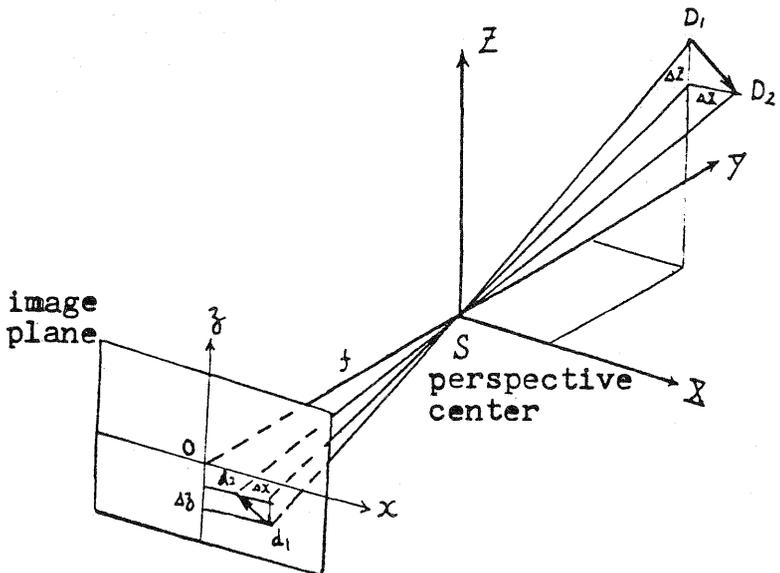


Figure 2

where  $\Delta X$ ,  $\Delta Z$ =displacement of object point;  $Y$ =distance from camera station to object point and  $f$ = focal length of camera. Measuring the parallax, the  $\Delta X$  and  $\Delta Z$  can be obtained if the scale of image point  $d$  is known. The scale can be determined from the known length on object space (assuming that the length is in a plane parallel to plane  $XZ$ ) and corresponding measurement on photograph or the focal length  $f$  and surveyed object distance  $Y$ .

## (2) Calibration of Camera

The one of main problems at present, which prevents the photogrammetry from expansive applying in industry, is considered to be the photogrammetric instruments. Because of the expensive photogrammetric instrument, the traditional techniques have to be used by the non-survey departments owing to the limited funds. In order to overcome the lack, a single non-metric camera (The China-made Seagull camera 4A 120) is used to photograph in this test.

In order to keep the stability of non-metric camera at the exposure, a connector fixing the camera on the base of the tripod is made specially (Fig.3). The camera can be also rotated by connector. The purpose of camera calibration is to determine the internal orientations and adjust the external orientations to make the image plane parallel to the facade of sample (in vertical position) when camera is fixed on tripod.

The Seagull camera 4A belongs to the model of double lenses and equips with a viewfinder. A set of graduated scale is stuck on each of four sides of the diffusing glass of viewfinder. A plummet and level which represent the centre-line of plane  $XZ$  in three dimensional control field of object space are marked. The camera is set in the vertical plane which is perpendicular to  $XZ$  plane and passes through centre-line. The photogrammetric distance approximates to one of the concrete sample. The readings of images of plummet and level in diffusing glass are estimated from graduated scale when the base of tripod is leveled and the image of camera is almost located vertically. Then the photographs of control field are taken. Then the internal and external orientations of camera are determined from the known space coordinates of control points using the space resection with a single photograph. The calibration equations are:

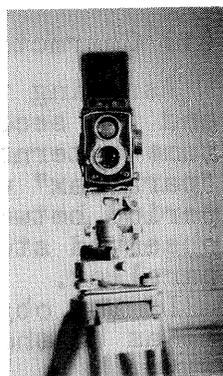


Fig.3

$$\begin{aligned} Fx &= [ (x+x_0)+\delta x ] M_2 + fM_1 = 0 \\ Fz &= [ (z+z_0)+\delta z ] M_2 + fM_3 = 0 \end{aligned} \quad (2)$$

Where  $[ M_1 M_2 M_3 ]^T = R [ (X-X_s) (Y-Y_s) (Z-Z_s) ]^T$ ;  $(X, Y, Z)$  = known coordinates of control points;  $(X_s, Y_s, Z_s)$  = unknown coordinates of the perspective center of camera lens;  $R$  = a  $3 \times 3$  rotation matrix consisted of external orientation elements  $\varphi, \omega, \kappa$ ;  $x, z$  = image coordinates;  $(x_0, z_0, f)$  = unknown basic parameters of internal orientation of photograph, the coordinates of principal point and focal length individually;  $(\delta x, \delta z)$  = corrections due to lens distortion and film deformation. For the distortion, a main term of symmetric lens distortions are only counted for. Therefore

$$\begin{aligned} \delta x &= (x-x_0)k_1r^2 + (z-z_0)(1+d_s)\text{sind}\beta \\ \delta z &= (z-z_0)k_1r^2 + (z-z_0)[(1+d_s)\text{cosd}\beta - 1] \end{aligned} \quad (3)$$

Where  $k_1$  = the symmetric lens distortion factor;  $d_s$  = the change of scale along axis  $z$ ;  $d\beta$  = the angular difference due to the non-normality of axis  $x$  and  $z$  and

$$r^2 = (x - x_0)^2 + (z - z_0)^2$$

The error equations gotten by the linearization of equations (2) tot up to 12 unknowns. Six control points are needed for resolution. The least square adjustment is used to resolve for more points. On the basis of the obtained external orientations  $\varphi$  and  $\omega$ , the readings of images of plummet and level on the diffusing glass of viewfinder are adjusted by rotating the connector in order to change the attitude of camera and make the image plane further vertical. Then photographing the control field by the adjusted camera again, resolving and further adjusting and so gradually approaching, it is at end that the  $\varphi$  and  $\omega$  of camera are near to zero. Then the screws of connector are fixed and the final reading of plummet on graduated scale is recorded.

For our Seagull camera (NO. 4A 2080286), the mean principal distance of camera  $f=86.54\text{mm}$  and mean lens distortion factor  $K_1=-4.22 \times 10^{-6}$  at the photogrammetric distance  $1.4\text{m}$  are computed from datum calculated by 12 films using above calibration program. The mean square errors of repeat-setting of external orientations in the vertical direction after three adjustments are  $m_\varphi = \pm 6.8'$ ,  $m_\omega = \pm 8.2'$ .

On the basis of the analysis of error theory of photogrammetry the setting tolerances of external orientation elements, which is required for paralleling the image plane to object plane, are

$$\begin{aligned} \Delta\varphi_t &= \frac{f \cdot \delta(dx)}{2x \cdot \Delta x} \\ \Delta\omega_t &= \frac{f \cdot \delta(dx)}{z \cdot \Delta x} \end{aligned} \quad (4)$$

Where  $\delta(dx)$  is the tolerant displacement error of image due to the setting error of external orientation;  $x$  is the biggest possible time-parallaxes (the biggest possible displacement of destroyed points). Letting  $\delta(dx)=0.01\text{mm}$ ,  $\Delta x=1\text{mm}$ ,  $x=z=25\text{mm}$ , knowing  $f=86.5\text{mm}$ , we obtain  $\Delta\varphi_t=59'$ ,  $\Delta\omega_t=1^{\circ}59'$ . It can be seen that the setting accuracy obtained by above calibration of camera is enough.

### (3) Photographic Arrangement of Testing Sample

The grid is drawn on sample using the chalk before photography. The intersections of grid are used as the measuring points of deformation. Three fixed points used as the controls are respectively set on each of above and under place of sample (three under fixed points are, in fact, the steel scale ruler). The camera is set in front of sample at a distance about  $1.4\text{m}$  (the photographic scale is about 1:16) and located in the vertical plane which is perpendicular to plane of sample and passes through the centre-line of grid. The reading of the centre-line of grid on the graduated scale of diffusing glass equals to the final reading of the plummet in control field of calibration of camera by means of the rotation of camera. This completes the orientation of camera (image plane parallel to object plane). First, the film of sample under zero load are taken then films under every load. One of the taken films is shown as Fig.4.

(4) Correction of Errors of Internal  
And External Orientation Elements

The small changes of internal and external orientation elements can be generated at different photographic period. The changes will cause the fixed points to product displacements  $\Delta x'$  and  $\Delta z'$ . These displacements can be expressed as function of small changes of internal and external orientation elements:

$$\Delta x' = \frac{x}{f} \Delta f - \frac{x^2}{f^2} \Delta x_0 + \frac{f}{Y} \Delta X_S - \frac{x}{Y} \Delta Y_S + (f + \frac{x^2}{f}) \Delta \varphi - \frac{xz}{f} \Delta \omega - z \Delta \kappa$$

$$\Delta z' = \frac{z}{f} \Delta f + \Delta z_0 - \frac{xz}{f} \Delta x_0 + \frac{f}{Y} \Delta Z_S - \frac{z}{Y} \Delta Y_S - \frac{xz}{f} \Delta \varphi - (f + \frac{z^2}{f}) \Delta \omega + x \Delta \kappa$$

Figure 4

(5)

where  $\Delta f, \Delta x_0, \Delta z_0, \Delta X_S, \Delta Y_S, \Delta Z_S, \Delta \varphi, \Delta \omega$  and  $\Delta \kappa$  are the errors due to the changes of internal and external orientation elements respectively. Thus the errors of internal and external orientation elements can be obtained by above equations from the measured displacement of fixed points on films. The correction of other points, which are caused by changes of internal and external orientation elements, can be calculated from equation (5). Subtracting the correction from the measured parallax, the time parallax is obtained. Multiplying the time parallax by denominator of image scale, the practical displacement of grid points is determined.

Taking the zero load film as the left and other load film as right, as such "a pair of film" are formed to observe the parallax by means of the time-parallax. Because of lack of fiducial marks for non-metric camera, a pair of films are oriented from above and under fixed points near to the center of film (such as points 5 and 2 in Fig.5). Using the point 5 as an "origin", origins of two films coincide each other and films are oriented along the direction from 5 to 2. The practice proved that the orientation and measurement procedure are feasible because of the basic steady films at the exposure.

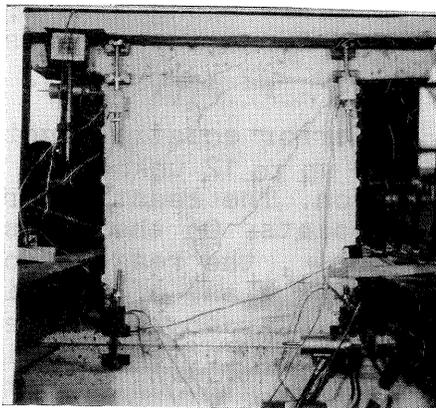
The object distance  $Y$  of each point can be considered to be near equal and constant in equation (5) as a result of image plane parallel to object plane. Thus the equation (5) can be reduced as follows:

$$\begin{aligned} \Delta x' &= a_0 - a_1 x + a_2 x^2 + a_3 x z - a_4 z \\ \Delta z' &= b_0 - a_1 z + a_2 x z + a_3 z^2 + a_4 z \end{aligned} \quad (6)$$

Two equations can be written for one point. In order to determine the coefficients  $a_0, b_0, a_1, \dots, a_4$ , at least three points are needed. The least square adjustment is used to resolve for more points. Then the error equation can be expressed in matrix notation:

$$AX - L = V \quad (7)$$

where



$$A = \begin{pmatrix} 1 & 0 & -x_1 & x_1^2 & x_1 z_1 & -z_1 \\ 0 & 1 & -z_1 & x_1 z_1 & z_1^2 & x_1 \\ 1 & 0 & -x_2 & x_2^2 & x_2 z_2 & -z_2 \\ 0 & 1 & -z_2 & x_2 z_2 & z_2^2 & x_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & -x_n & x_n^2 & x_n z_n & -z_n \\ 0 & 1 & -z_n & x_n z_n & z_n^2 & x_n \end{pmatrix}, \quad X = \begin{pmatrix} a_0 \\ b_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix}, \quad L = \begin{pmatrix} x_1 \\ z_1 \\ x_2 \\ z_2 \\ \vdots \\ x_n \\ z_n \end{pmatrix}, \quad V = \begin{pmatrix} v_{x1} \\ v_{z1} \\ v_{x2} \\ v_{z2} \\ \vdots \\ v_{xn} \\ v_{zn} \end{pmatrix},$$

$n$  is the number of fixed points. The normal equation and its resolution are:

$$A'AX - A'L = 0 \quad \text{or} \quad BX - C = 0 \quad (8)$$

$$X = B^{-1}C$$

The influence of lens distortion is not incorporated in correction equations (5) because the displacements of grid points on sample are considered to be not great and the influences of distortion are basically compensated each other during calculating the coordinate differences of the prior and rear film. For example, differentiating the equation of distortion (the first half part at right hand of equation (3)), the permissible displacement on film can be obtained in case that distortion influence is not considered;

$$\Delta r_p = \delta(\Delta r) / \sqrt{3} k_1 r^2 \quad (9)$$

Letting  $\delta(\Delta r) = 0.01\text{mm}$ ,  $r = 28\text{mm}$  and knowing  $k_1 = 4.22 \times 10^{-6}$ , we obtain  $\Delta r_p = 1.01\text{mm}$  which corresponds to practical displacement 16mm. The concrete sample in test could have been failed before the grid points achieve above displacement.

#### 4. Accuracy Examination And Result

In order to examine the accuracy of method, an experiment is conducted by use of the standard grid before the test of sample. A standard grid of polyester fibre paper plotted on the coordinate plotter is set on the vertical frame of object space of the repro-camera. The grid is photographed repeatedly using the calibrated and adjusted Seagull camera. During setting the camera, the photo-scale is approximated to one of concrete sample. The films are observed monocularly on Stocometer. The two dimensional projective transformation is used to transform the image coordinates:

$$x = \frac{L_1 x^* + L_2 z^* + L_3}{L_7 x^* + L_8 z^* + 1}$$

$$z = \frac{L_4 x^* + L_5 z^* + L_6}{L_7 x^* + L_8 z^* + 1} \quad (10)$$

where  $L_i (i=1, 2, \dots, 8)$  indicates the transformation factors. In fact because above equations are the two dimensional direct line transformation, they are also applied to non-metric camera. For eight factors in equation, at least four known points are needed. The least square adjustment is used to resolve for more points. Then the error equations will take the form:

$$\begin{aligned} V_x &= \frac{1}{\lambda} [L_1 x^* + L_2 z^* + L_3 - L_7 x^* - L_8 z^* - x] \\ V_z &= \frac{1}{\lambda} [L_4 x^* + L_5 z^* + L_6 - L_7 x^* - L_8 z^* - z] \end{aligned} \quad (11)$$

$$\lambda = L_7 x^* + L_8 z^* + 1.$$

Taking the standard image coordinates of six above and under grid points, which are reduced by photoscale from standard grid coordinates, as the fixed coordinates  $(x^*, z^*)$  (see fig. 5), the transformation factors  $L_i$  can be determined from equation (11). Then the transformation coordinates of other points are computed as follows:

$$\begin{bmatrix} L_1 - xL_7 & L_2 - zL_8 \\ L_4 - zL_7 & L_5 - zL_8 \end{bmatrix} \begin{bmatrix} x^* \\ z^* \end{bmatrix} = \begin{bmatrix} x - L_3 \\ z - L_6 \end{bmatrix} \quad (12)$$

Comparing the obtained transformation coordinates with the corresponding standard image coordinates, the displacement of each point can be computed. These displacements contain mainly the influence of lens distortion of camera.

The nominal picture area of Seagull camera is 5.5cmx5.5cm. In the case of application of central area 4cmx4cm, the standard errors of coordinate are evaluated from the displacements of each grid point as follows:

$$m_x = \pm 0.031 \text{ mm}, \quad m_z = \pm 0.035 \text{ mm}$$

the standard error of position:

$$m_s = \pm 0.046 \text{ mm}.$$

However it is the relative position errors that we concern about. Comparing the transformed grid coordinates of each film with the first film, the relative coordinate differences can be obtained. Thus the relative standard errors of coordinate and position are computed respectively:

$$m_x = \pm 0.014 \text{ mm}, \quad m_z = \pm 0.015 \text{ mm}, \quad m_s = \pm 0.021 \text{ mm}.$$

On the other hand, the grid coordinates on film are corrected by time-parallax. Supposing that the films are measured by the procedure in above section, the direction of points 5—2 is parallel to the axis y of comparator and image coordinates of each point are observed according to point 5 used as an "origin". Using the equations (6--8), the correction coefficients and the corrections of point concerned are calculated from the differences between the observed image coordinates of six above and under points and the corresponding standard grid coordinates. Comparing the corrected image coordinates with the image coordinates of standard grid and corrected image coordinates of first film with other films, the same result as above are obtained.

We can hold from the obtained result that the displacements 0.2mm—0.3mm or more on concrete sample can be detected using the non-metric camera by means of time-parallax. This is a relative accuracy of 1/7000 of photographic distance.

The vectors of distortion of grid points on sample at thirty-one load, which are corrected by five points (above one of six points is damaged), are shown in Fig. 6.

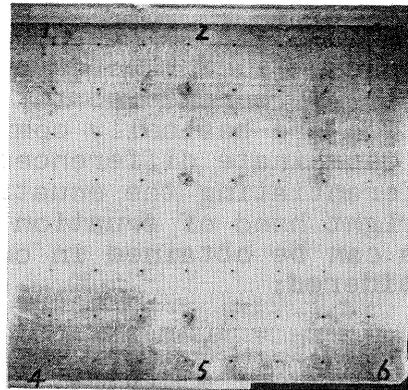


Figure 5

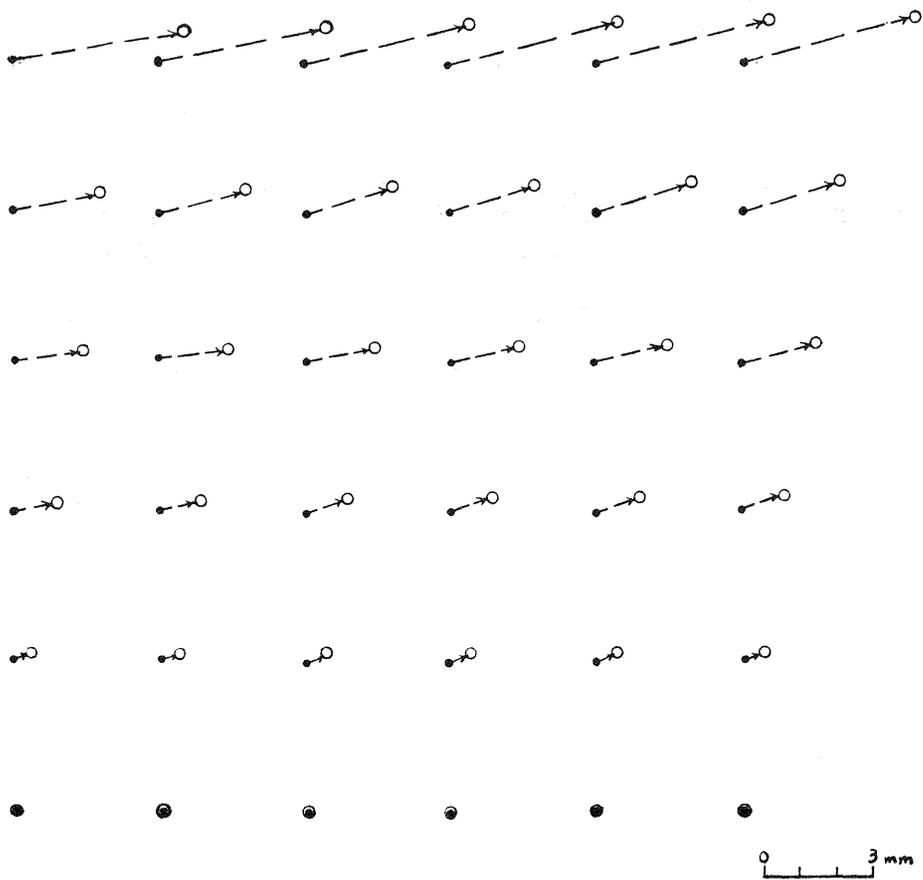


Fig. 6

### 5. Conclusion

Though the higher accuracy can be obtained using the traditional instruments to determine the deformation of sample (for example, the displacement meter can measure up to 0.01mm or more), it has the disadvantage that a unit can only measure a single point and displacement along one direction. The photogrammetric method can determine the deformation of whole sample simultaneously. When the targets on sample are arranged in array form as shown in Fig.4, the stress and strain can be analyzed by structural specialists from the deformation vectors of position of array points by means of the matrix method. The disadvantage of photogrammetric method is that the accuracy is lower than one of direct method and the data can't be obtained immediately. However the data of overall deformation supplied by photogrammetric method are important and great valuable to mechanical analysis of sample after the event.

It is possible that the accuracy of deformation determined by the time-parallax using non-metric camera is close to the level of metric camera. The photographic system in this test is simpler, lighter and obtainable easy. The Seagull camera 120 and computer PC-1500 are more common in the general productive and scientific departments. The image coordinates on film of small picture area can be observed using the optical measuring tool such as the universal tool microscope instead of comparator. Therefore this pho-

togrammetric system is easy to spread in the non-survey departments.

#### References

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