

# MULTI-TEMPORAL POINT ESTIMATION WITH PHOTOGRAMMETRIC FILTERING

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## ABSTRACT

Photogrammetry is used to determine the trajectories of moving object points through time. The information related to the motion of the object points has been integrated with the photogrammetric observation campaigns. This combined approach is equivalent to the estimation process of dynamic filtering. The mathematical formulation is presented and illustrated with a numerical example. This method improves the estimation of position and accuracy of the object points compared to a single photogrammetric solution.

## INTRODUCTION

Photogrammetry plays a major role among the geometric methods of displacement monitoring. Usually, a deformation area is represented by a series of detail points. Photographs taken at different time intervals (multi-temporal campaigns) provide instantaneous records of the three-dimensional positions of these points. Positional differences between those instants in time enable us to determine the movement of these points in the space domain.

The time factor is only used in relating the static "snapshots" which are used to determine the displacements (Figure 1). The position vectors  $r(t)$  and  $r(t + 1)$  are computed solely from individual photogrammetric campaigns without any interrelation between the two observation epochs.

However, the object points change their position progressively in time, stimulated by some physical cause. Thus in a time varying situation, the dynamic characteristics of the displacements can be taken into account by expressing changes in the parameter vector as a function of time in addition to changes due to new observations.

If the functional relationship between points at successive epochs is adequately known, we can compute a preliminary estimate of the position vector  $r(t)$  based on its previous spatial position  $r(t - 1)$  (see Figure 2). Similarly, the variance-covariance matrix of  $r(t)$  can be estimated based on the uncertainties of  $r(t - 1)$  and of the dynamic parameters expressing the transition to state  $r(t)$ .

## PHOTOGRAMMETRIC FILTERING

The position of a point at an instant in time is characterized by two models, namely the static photogrammetric model, and the dynamic prediction model. A combination of these two types of models leads to a photogrammetric filtering process, with which the physical situation at an instant in time can be described.

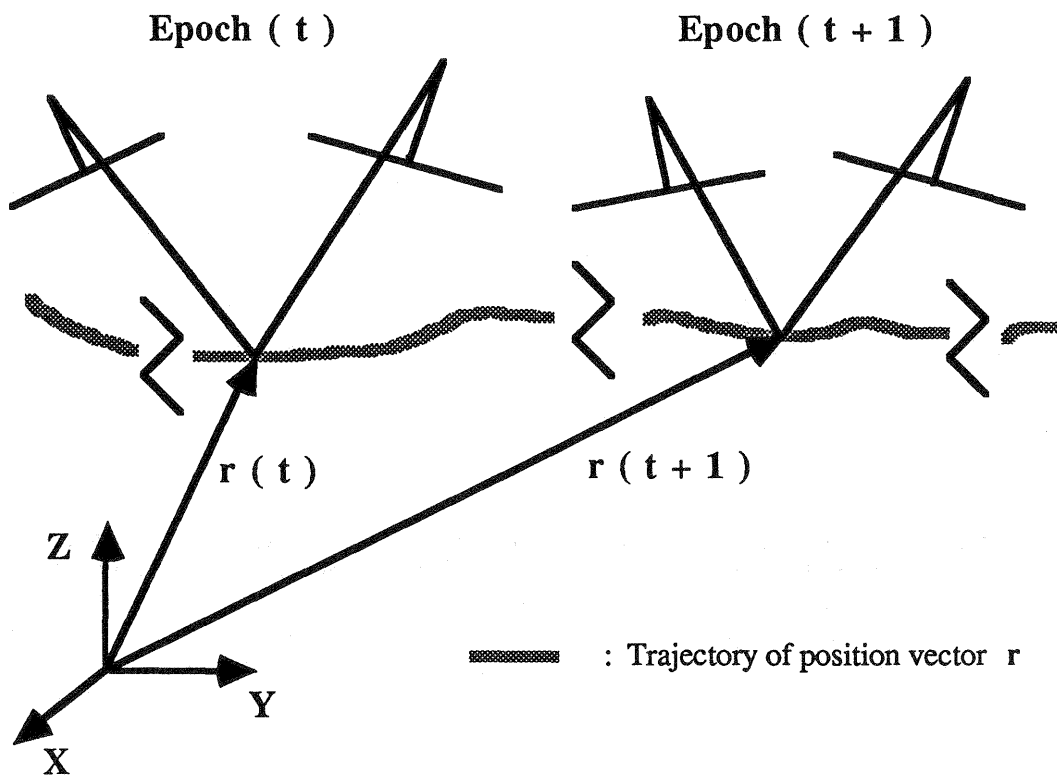


Figure 1 Determination of displacements in a static mode.

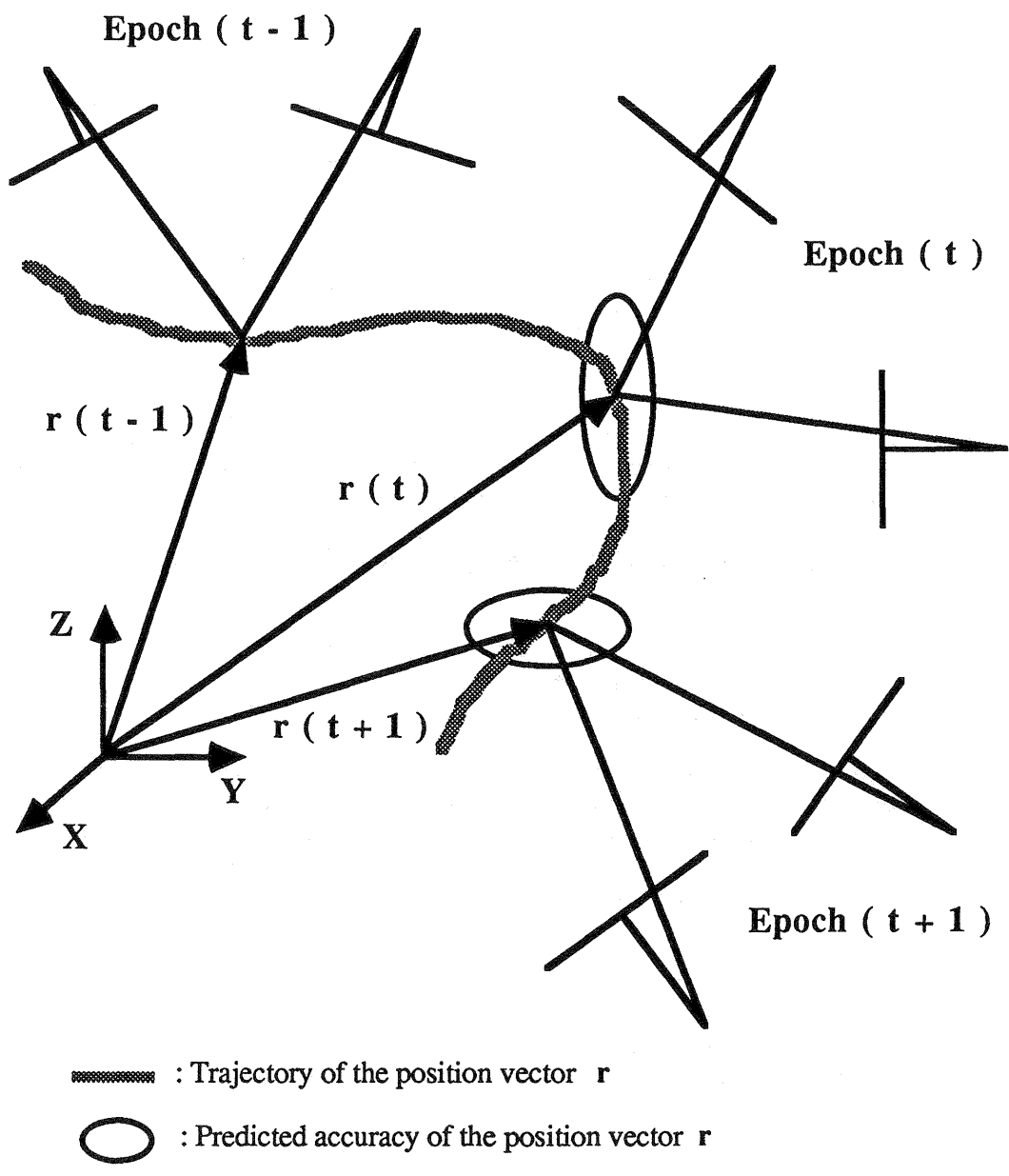


Figure 2 Determination of displacements in a sequential mode.

- a) The "static" photogrammetric model consists of three types of observation equations together with the relevant weight matrices.
- i) The photo-coordinate measurements are related to the unknown parameters by the extended collinearity equations, in a photo-variant mode:

$$F_1(\hat{x}_I, \hat{x}_E, \hat{x}_O) = \ell_P; P_P \quad (1)$$

where

- $\hat{x}_I$  is the vector of the unknown parameters of interior orientation
- $\hat{x}_E$  is the vector of the unknown parameters of exterior orientation
- $\hat{x}_O$  is the vector of the unknown parameters of object coordinates
- $\ell$  is the vector of the observed photo-coordinates
- $P_P$  is the weight matrix of the observations.

- ii) The ten elements defining the interior orientation for each photo-frame, namely  $x_0, y_0, c, a_1, \dots, a_7$  are introduced as weighted parameter constraints to avoid ill-conditioning:

$$F_2(\hat{x}_I) = \ell_I, \text{ with } \ell_I = x_I^{(0)} \text{ and } P_I \quad (2)$$

where

- $\ell_I$  is the vector of the initially given elements of interior orientation;
- $P_I$  is the weight matrix of the observations.

- iii) The coordinates of the object are also treated as weighted parameter constraints. Although this is necessary for the required control points to avoid datum indeterminacies, all points can be utilized as such. The observation equations have the general form

$$F_3(\hat{x}_O) = \ell_0, \text{ with } \ell_0 = x_0^{(0)}; P_0 \quad (3)$$

where

- $\ell_0$  is the vector of the initially given object coordinates and
- $P_0$  is the weight matrix of the observations.

- b) The "dynamic" prediction model consists of two equations.

- i) The propagation of the parameter vector through time is determined as:

$$\hat{x}(t) = T(t, t-1) \hat{x}(t-1) + \hat{z}(t, t-1) \quad (4)$$

where

- $\hat{x}(t)$  is the predicted parameter vector at time  $t$  (current state)
- $x(t-1)$  is the estimated parameter vector at time  $t-1$  (previous state)
- $T(t, t-1)$  is the transition matrix, which maps  $\hat{x}(t-1)$  onto  $\hat{x}(t)$  in the time interval  $\delta t = (t) - (t-1)$
- $\hat{z}(t, t-1)$  is the vector describing the uncertainty (residual or noise) of the system.

ii) The propagation of errors of the parameter vector through time is determined by:

$$C_{\hat{x}(t)} = T(\delta t) C_{\hat{x}(t-1)} T(\delta t)^T + Q(t) \quad (5)$$

where

$C_{\hat{x}(t)}$  is the predicted covariance matrix of  $\hat{x}(t)$   
 $C_{\hat{x}(t-1)}$  is the covariance matrix of the previous state parameter  $\hat{x}(t-1)$   
 $Q(t)$  is the covariance matrix of  $\hat{z}(t, t-1)$  and generally expresses the uncertainty of the prediction model.

At a particular instant of time we have an a priori knowledge of the parameters whose a priori weight matrix is non-zero (see equations (4) and (5)). Thus we can write

$$\hat{x}(t) = x^0(t) + \delta \hat{x} \quad (6)$$

Omitting the time subscript  $t$  and using the subscript 1 for the dynamic model, equation (6) becomes

$$\hat{x}_1 = x_1 + \delta \hat{x}_1 \quad (7a)$$

$$\delta \hat{x}_1 + x_1 - \hat{x}_1 = 0 \quad (7b)$$

or in a general linearized form

$$A_1 \delta \hat{x}_1 + w_1 = 0, \quad (A_1 = I) \quad (7c)$$

where

$A_1 = \frac{\partial f}{\partial x}$  is the first design matrix,

$\delta \hat{x}_1$  is the solution (correction) vector

$w_1$  is the misclosure vector.

The weight matrix corresponding to equation (7c) is

$$P_1 = (C_{x(t)})^{-1} \text{ or } P_1 = C_1^{-1} \quad (7d)$$

At the same instant of time  $t$  the photogrammetric measurement model has the following general linearized form

$$A_2 \delta \hat{x}_2 - v_2 + w_2 = 0 \quad (8a)$$

where

$$A_2 = \begin{bmatrix} A_{PI} & A_{PE} & A_{PO} \\ 0 & 0 & A_{OB} \\ A_{II} & 0 & 0 \end{bmatrix}$$

and

$$A_{PI} = \frac{\partial F_1}{\partial x_1}, \quad A_{PE} = \frac{\partial F_1}{\partial x_E}, \quad A_{PO} = \frac{\partial F_1}{\partial x_0}, \quad A_{II} = \frac{\partial F_2}{\partial x_I}, \quad A_{OB} = \frac{\partial F_3}{\partial x_0}$$

$$v_2 = [v_P^T \ v_0^T \ v_I^T]^T,$$

$v_P, v_I, v_0$  are the residual vectors corresponding to observation equations  $F_1, F_2, F_3$ , respectively.

$$w_2 = [w_P^T \ w_0^T \ w_I^T]^T,$$

$w_p, w_I, w_0$  are the misclosure vectors corresponding to the observation equations  $F_1, F_2, F_3$ , respectively.

The weight matrix corresponding to equation (8a) is

$$P_2 = \begin{pmatrix} P_P & 0 & 0 \\ 0 & P_O & 0 \\ 0 & 0 & P_I \end{pmatrix} \text{ or } P_2 = C_2^{-1} \quad (8b)$$

The combined linearized mathematical model based on equations (7c) and (8a) is

$$A \delta \hat{x} - v + w = 0 \quad (9a)$$

where

$$A = \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \begin{pmatrix} I \\ A_2 \end{pmatrix}, \quad v = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \quad w = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} \quad (9b)$$

Assuming logically that there is no correlation between the two sets of measurements, the combined weight matrix for the model of equation (9a) is

$$P = \begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix} \quad (9c)$$

The solution vector  $\delta \hat{x}$  is not partitioned because both subsets of observations are related to the same unknown parameters. It can be estimated by applying the least squares criterion:

$$v^T P v = \delta \hat{x}_1^T P_1 \delta \hat{x}_1 + v_2^T P_2 v_2 = \min \quad (10)$$

In reality, measurements become available sequentially, and/or a priori estimates of the solution vector may be available (e.g., equ. (4)). Therefore, it is preferable and practical to determine new estimates based on the new measurements (e.g., equ. (1), (2), (3)) in terms of previous solutions. This is possible by deriving sequential expressions of the least squares solutions [Wells and Krakiwsky, 1971; Junkins, 1978]. Hence,

$$\delta \hat{x} = -N_1^{-1} q_1 - N_1^{-1} A_2^T k_2 \quad (11a)$$

where

$$k_2 = (P_2^{-1} + A_2 N_1^{-1} A_2^T)^{-1} (-A_2 N_1^{-1} q_1 + w_2) \quad (11b)$$

$$N_1^{-1} = (A_1^T P_1 A_1)^{-1} = P_1^{-1} \quad (11c)$$

$$q_1 = A_1^T P_1 w_1 = P_1 w_1 \quad (11d)$$

It is known that

$$-N_1^{-1} q_1 = w_1 = \delta \hat{x}_1 \quad (12)$$

where  $x_1$  is the solution for the parameter vector when the dynamic model only is used. If we set  $\delta \hat{x}_2 = \delta \hat{x}_1$  which means that the unknown parameters are estimated after considering the photogrammetric observation model and using eqs. (11b) - (12), then equ. (11a) becomes

$$\delta \hat{x}_2 = \delta \hat{x}_1 + \Delta x \quad (13a)$$

where

$$\Delta x = -C_1 A_2^T (C_2 + A_2 C_1 A_2^T)^{-1} (A_2 \delta \hat{x}_1 = w_2) \quad (13b)$$

Since we assumed non-linear models for the sake of generality, the final solution  $x$  is determined from both the dynamic and the combined model. When the dynamic model is used, then

$$\hat{x} \hat{=} \hat{x}_1 = x^{(0)} + \sum_{i=1}^k \delta \hat{x}_{1i} \quad (14)$$

where  $k$  is the required number of iterations.

When the combined model is used, then,

$$\begin{aligned} \hat{x} \hat{=} \hat{x}_2 = x^{(0)} &= \sum_{i=1}^m \delta \hat{x}_{2i} = x^{(0)} + \sum_{i=1}^m (\delta \hat{x}_{1i} + \Delta x_i) \\ \text{or } \hat{x} \hat{=} \hat{x}_2 &= \hat{x}_1 + \sum_{i=1}^m \Delta x_i \end{aligned} \quad (15)$$

with  $m$  being the required number of iterations.

If we substitute the expression  $\Delta x$  from equ. (13b) into equ. (15) and linearize each time about the most recent estimate, we obtain an important recursive formula for the  $i$ th iteration.

$$\hat{x}_{2,i} = \hat{x}_1 - C_1 A_2^T [C_2 + A_2 C_1 A_2^T]^{-1} [A_2 (\hat{x}_{2,i-1} - \hat{x}_1) + w_2] \quad (16)$$

where  $\hat{x}_{2,0} \hat{=} \hat{x}_1$  and  $\hat{x}_1 = x^{(0)}$  (value at which linearization occurs)

This expression states clearly that when a new set of observations is added for the determination of the parameter vector, the resulting new parameter vector is equal to the parameter vector estimated from all previous observation equations plus a correction term. Applying the law of error propagation to equ. (16), a sequential form of the variance-covariance matrix  $C_x$  of the estimated parameter  $\hat{x}$  is derived. Thus,

$$C_x \hat{=} [I - C_1 A_2^T (C_2 + A_2 C_1 A_2^T)^{-1} A_2] C_1 \quad (17)$$

The dynamic model provides a recursive estimation process through time for the unknown vector of parameters. Therefore, in the above sequential expressions, the time is considered when the parameter vector  $\hat{x}_{2,i}$  changes not only as new observations become available (term  $\Delta x$ ) but also as function of cause in time (term  $\hat{x}_1$ ). In terms of modern optimal estimation theory this represents a filtering process, referred to here as photogrammetric filtering. An examination of equs. (16) and (17) derived from the sequential weighted least squares adjustment with time consideration reveals, that they have the same appearance and therefore are mathematically equivalent to the expressions given for the iterated extended Kalman filter for non-linear dynamic systems (Gelb, 1974).

Examining also the structure of these equations with respect to existing familiar forms and computational aspects involved, we observe the following for the term  $(C_2 + A_2 C_1 A_2^T)^{-1}$ ; Firstly, the sequence of matrices involved does not resemble the well known form of the coefficient matrix  $A_2^T C_2^{-1} A_2$  of the unknown parameters of the least squares adjustment. Secondly, the order  $n$  of the matrix to be inverted is much larger than the order  $u$  used in a regular photogrammetric bundle block adjustment ( $n, u$  are the numbers of observations and unknowns respectively).

At this point we invoke a matrix inversion identity given in Henderson and Searle, (1981) which has been found also in Mikhail and Helmering, (1973) and Kratky, (1980), namely:

$$(C_2 + A_2 C_1 A_2^T)^{-1} = C_2^{-1} [I - A_2 (C_1^{-1} + A_2^T C_2^{-1} A_2)^{-1} A_2^T C_2^{-1}] \quad (18)$$

Also, a new notation is adopted to conform with Gelb's (1974) notation. This provides a better understanding of the time factor (Schwarz, 1983) and a more explicit distinction between predicted and updated estimates. The subscript  $t$  implies the final estimation at time  $t$ , which is obtained after applying the contribution of the measurements of the second model. The symbol  $(-)$  indicates predicted values based on the dynamic model immediately prior to time  $t$ . The symbol  $(+)$  indicates updated values due to the contribution of the observations immediately following the time  $t$ .

Applying equ. (18) and this notation on equs. (16) and (17), the final updated expressions are derived (Armenakis, 1987):

$$\hat{x}_{t,i}(+) = \hat{x}_t(-) - C_t(-) A_t^T C_t^{-1} G [A_t (\hat{x}_{t,i-1}(+) - \hat{x}_t(-)) + w_t] \quad (19)$$

and  $C_t(+) = [I - C_t(-) A_t^T C_t^{-1} G A_t] C_t(-) \quad (20)$

where  $G = I - A_t (C_t(-)^{-1} + A_t^T C_t^{-1} A_t)^{-1} A_t^T C_t^{-1} \quad (21)$

These equations represent one formulation of the iterated extended Bayes filter. The use of a different matrix identity (equ. (18)) results in a different expression for the so-called Bayes filter (Morrison, 1969; Vanicek and Krakivsky, 1986).



## SEQUENTIAL ESTIMATION OF THE STATE INFORMATION

Practical considerations and computational efficiency led to the use of a reduced measurement model, i.e. the object coordinates have been selected to serve as state parameters for the filtering algorithm (eqs. (19), (20)). More details about the different situations and approaches examined can be found in Armenakis, (1987) and Armenakis and Faig, (1987). This solution concentrates on the main interest, the determination of the trajectories of the object points. It involves the following general steps:

- Step 1. Solve for the parameters  $\hat{x}_E$ ,  $\hat{x}_I$  of the exterior and interior orientation by the extended space resection.
- Step 2. Consider  $\hat{x}_E$  and  $\hat{x}_I$  as known and form the reduced photogrammetric model where only the coordinates  $\hat{x}_O$  of the object points are unknown parameters.
- Step 3. Determine the optimal position and accuracy estimates of the current object coordinates using the prediction and reduced observation models in the final updated equations (19) and (20).

The above steps are executed in an iterative manner.

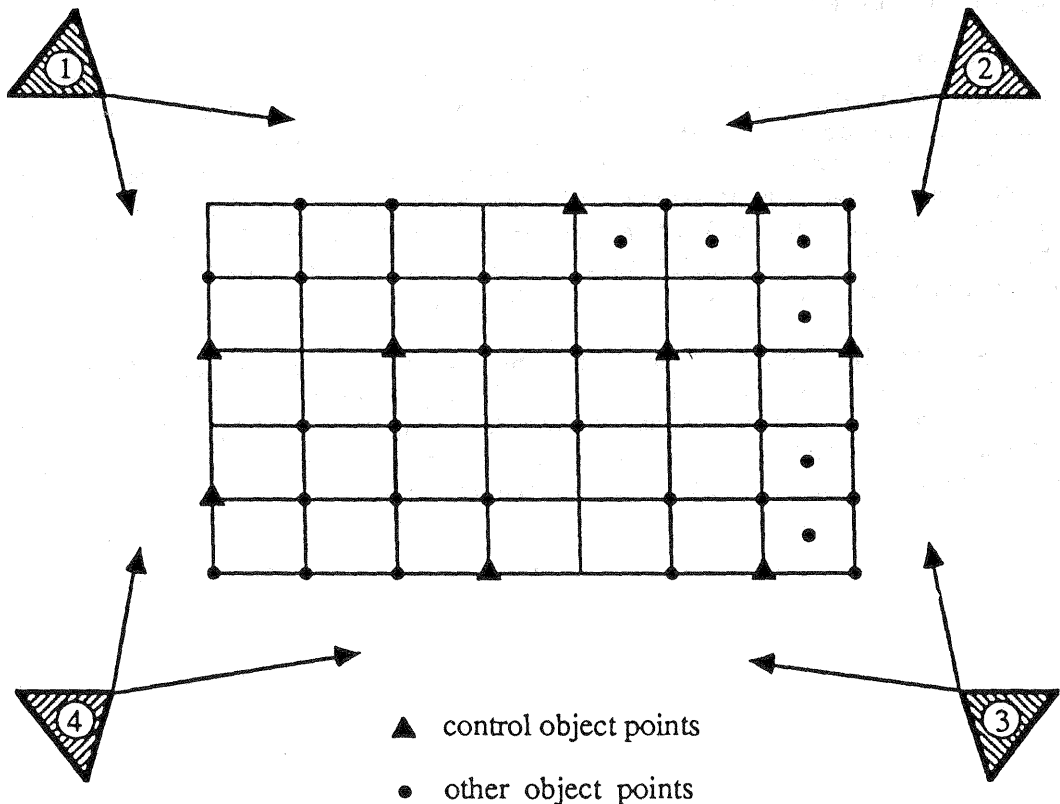


Figure 3 Plan diagram (not to scale) of the object-camera configuration.

## NUMERICAL EXAMPLE

The photogrammetric filtering process was incorporated into the program SPDM (Sequential Photogrammetric Displacement Monitoring; Armenakis, 1987) in order to perform the multi-temporal point estimation.

The system was evaluated with the aid of a laboratory test. The test model consists of five parts and has the following dimensions 1.40 m x 0.90 m x 0.25 m. Each part can accommodate a different deformation, while several points can be moved individually.

Two photogrammetric observation epochs were utilized. For the second epoch, single point displacement and subsidence deformation was introduced into parts of the model. The maximum magnitude of displacements was 1 cm.

Convergent photography with 100% overlap was taken from above the four corners of the test field with a Canon AE-1 non-metric camera with standard lens ( $f=50$  mm) with an approximate photo-scale of 1:45 (see Figure 3).

The photo-coordinates of the image points of all eight photographs were measured on the precision analogue stereo-plotter Wild A-10. Two sets of measurements were performed resulting in an average accuracy for both x- and y- photo-coordinates of  $\pm 12$   $\mu$ m. The accuracy of the surveyed object points was  $\pm 1-2$  mm.

The bundle adjustment program PTBV (Photogrammetric Triangulation by Bundles-Photo-Variant; Armenakis, 1987) was utilized to estimate the position and accuracy of the object points in epoch 1. For the second epoch, the predicted object coordinates were estimated through approximate photogrammetric means (without the use of additional parameters) due to lack of a systematic and controlled mechanism causing the displacements. Their uncertainty was introduced via the diagonal uncertainty matrix  $Q$  ( $q_{ii} = 2.25E-04m^2$ ).

The sequential estimation of the state information (position and accuracy) was carried out using photogrammetric filtering with SPDM. Besides SPDM, the program PTBV was run as well. The statistical information obtained from both the combined and single epoch bundle block adjustment approaches is given in Table 1.

Table 1: Statistical information of photogrammetric adjustments (Epoch 2)

Residuals	Mean Value		Standard Deviation	
	(SPDM)	(PTBV)	(SPDM)	(PTBV)
photo x(mm)	-0.001	0.000	±0.010	±0.007
photo y(mm)	-0.002	0.000	±0.012	±0.010
check-points X (m)	0.000	0.000	±0.002	±0.002
check-points Y (m)	0.001	0.001	±0.002	±0.002
check-points Z (m)	0.002	0.002	±0.009	±0.009
control-points X (m)	0.000	0.000	±0.001	±0.001
control-points Y (m)	0.000	0.000	±0.001	±0.001
control-points Z (m)	0.000	0.000	±0.003	±0.004
a-posteriori variance factor				
(SPDM): 0.181E-09		(PTBV): 0.191E-09		
mean value of the standard deviations of the non-control points				
(SPDM):	$\bar{\sigma}_X = \pm 0.3$ mm	$\bar{\sigma}_Y = \pm 0.3$ mm	$\bar{\sigma}_Z = \pm 0.4$ mm	
(PTBV):	$\bar{\sigma}_X = \pm 0.8$ mm	$\bar{\sigma}_Y = \pm 0.8$ mm	$\bar{\sigma}_Z = \pm 1.2$ mm	

Finally, the differences in displacements between geodetic results and photogrammetric ones (from SPDM) were compared at 16 points, resulting in average differences of

$$\bar{\delta} X = -0.4 \text{ mm}, \quad \bar{\delta} Y = 0.1 \text{ mm}, \quad \bar{\delta} Z = -0.9 \text{ mm}$$

#### CONCLUSIONS

The comparison and evaluation of the results between the single-epoch bundle adjustment and photogrammetric filtering based on this experiment and a number of others described in detail in Armenakis (1987) led to the following conclusions:

- 1) When a strong and well-controlled bundle geometry exists, the estimated positional parameters (orientation elements of the exposure stations as well as object coordinates) tend to be similar. This is illustrated in Table 1 where the mean values and the standard deviations of the examined residuals and the a-posteriori variance factors do not significantly differ. The imposition of additional object constraints is reflected in the slightly larger standard deviations of the photo-residuals as well as in slight differences in the estimated elements of interior orientation.
- 2) The estimated positional parameters tend to differ for the two approaches in cases of weak bundle geometry and/or poor datum definition. If the predicted information (object coordinates and their accuracies) is reliable, then photogrammetric filtering provides better absolute results.
- 3) The use of photogrammetric filtering provides a significant improvement to the absolute accuracy of the object coordinates. This is shown in Table 1 where the mean values of the standard deviations of the non-control points are smaller than their corresponding values from the single-epoch bundle adjustment approach.

Generally, it can be stated that photogrammetric filtering contributes to a better estimation of position and accuracy, since each of the underlying models plays the role of a safeguard and complements the other. The recursive nature of the approach has great potential in real-time photogrammetric applications.

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#### REFERENCES

1. Armenakis, C. (1987). "Displacement Monitoring by Integrating On-Line Photogrammetric Observations with Dynamic Information." Ph.D. Dissertation, Dept. of Surveying Engineering, University of New Brunswick, Fredericton, NB, Canada, 284 pp.
2. Armenakis, C., W. Faig (1987). "Sequential Photogrammetry for Monitoring Displacements." Proc. of the 1987 ASPRS-ACSM Annual Convention, Vol. 7, pp. 62-70.
3. Gelb, A. (1974). "Applied Optimal Estimation." The MIT Press, Massachusetts Institute of Technology, Cambridge, Massachusetts. USA, 274 pp.
4. Henderson, H.V., S.R. Searle (1981). "On Deriving the Inverse of a Sum of Matrices." SIAM Review, Vol. 23, No.1, pp. 53-60.
5. Junkins, J.L. (1987). "An Introduction to Optimal Estimation of Dynamic Systems." Sijthoff & Noordhoff International Publishers, B.V., Alphen van den Rijn, The Netherlands, 339 pp.
6. Kratky, V. (1980). "Present Status of On-Line Analytical Triangulation." Inter. Archives of Photogrammetry, Vol. XXIII, B3, Com.III, Hamburg, W. Germany, pp. 279-388.
7. Mikhail, E.M., R.J. Helmering (1973). "Recursive Methods Methods in Photogrammetric Data Reduction." Photogrammetric Engineering, Vol. 39, No. 9, pp. 983-989.
8. Morrison, N. (1969). "Introduction to Sequential Smoothing and Prediction." McGraw-Hill Inc., USA, 645 pp.
9. Schwarz, K.-P. (1983). "Kalman Filtering and Optimal Smoothing." Papers for CIS Adjustment and Analysis Seminars, edited by E. J. Krakiwsky, CISM, pp. 230-264.
10. Vanicek, P., E. Krakiwsky (1986). "Geodesy: The Concepts." 2nd edition, Elsevier Science Publishing Company, Inc., New York, USA, 697 pp.
11. Wells, D.E., E.J. Krakiwsky (1971). "The Methods of Least Squares." Lecture Notes 18, Dept. of Surveying Engineering, University of New Brunswick, Fredericton, NB, Canada, 180 pp.