# MEASURING THE HARMONIC OSCILLATIONS OF TALL ANTENNAE BY PHOTOGRAMMETRY

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*ABSTRACT*
*Measuring the deformations resulting from forces operating on structures is a useful way to confirm or contradict theories used by construction engineers. In the case of tall antennae, the deformations, three rotations and two horizontal translations, usually produce harmonic oscillations with a time period which is sometimes much shorter than one second. A satisfactory recording frequency would require 50 or more observations per second, which can be done only by photogrammetry. A method using a 16 m"m cine camera with a 1000 m"m tele-objective was adapted for that purpose. Single photographs at intervals of 1/62 seconds were obtained and the antenna's deformations in that time interval were determined from the changes in the photo coordinates of control points and especially of control lines located in the upper part of the antenna.*

## *INTRODUCTION*

Finding a method suitable for measuring the harmonic oscillations of tall antennae seems at first glance to be impossible because of the strict requirements it has to fulfil.

- Two horizontal translations and three rotations should be measured simultaneousely.
- The frequency of the measurements is at least 50 times per second.
- The equipment which is used for the measurements should not cause additional deformations to the antenna or disturb in any way the regular operation of the antenna and the installations which are attached to it.
- The method of measuring should be applicable at any time and not require climbing up the antenna.

It is possible to measure the accelerations of the antenna by accelerometers, which are attached to the antenna's construction where oscillations should be measured. Three pairs of accelerometers, perpendicular to each other, are needed to record the oscillations, translatations and rotations in all the directions. Calculating the amplitudes and speeds from the accelerations can be very complicated and there is always a danger that some frequencies of the harmonic oscillations do not get recorded by the accelerometers.

Photogrammetry suggests an alternative solution which fulfils all the above mentioned requirements and measures the displacements themselves at very short time intervals.

### *CONCEPT OF THE METHOD*

The part of the construction whose harmonic oscillations should be recorded is photographed with a 16 mm cine camera equipped with a 1000 mm tele-objective. The cine camera, BOLEX H16 Reflex, has a spring motor which will take about 670 frames after each spring winding. The maximum shooting speed is 64 frames per second. It is sufficient for about 11 seconds of photograpy in which the antenna makes about 30 periodical oscillations. Usually the most interesting are the harmonic oscillations of the upper part of the antenna and therefore the usual inclination of the camera axis is between 30 to 45 degrees.

It is preferable to use elements of the antenna's construction as control. When this is impossible, a special control device is attached to the main construction. The control device moves together with the part of the antenna to which it is rigorously attached and of course will experience the same oscillations.

The relative position and orientation of the photographs and the control device are calculated for each moment of exposure by the usual procedures of space resection. Since the camera's position and orientation remain unchanged during the time of photography, the changes in the relative position and orientation of the control device between any two photographs are regarded to be the result of the linear and angular deformations of the antenna in the time interval between the exposures.

A photograph taken with a tele-objective has a large scale and a very narrow angle of coverage. The accuracy of determining the position of the camera exposure station is excellent in a plane perpendicular to the camera axis but it tends to be poor in the direction parallel to the axis. Due to the inclination of the camera axis this shortcoming has no effect on the accuracy of determining the antenna's linear displacements. There is definitely no vertical displacement which is not resulting from the antenna's periodical tilts. By adding this as a condition to the solution of the space resection, the accuracy obtained for the two horizontal displacements is very good.

The accuracy of determining the antenna's rotation about the camera axis is less good than the rotations about the perpendicular axes. Unfortunely there is no way to eliminate its effect on the accuracy of the calculated antenna's rotations about the vertical axis and especially the tilt about the horizontal axis in the direction of photography.

It is important to note that since the displacements are relatively small, while the principal distance is very long and the angle of coverage very narrow, the accuracy of the deformations is almost unaffected when using a non-metric cine camera.

### *THE CONTROL DEVICE AND THE METHOD OF COMPUTATION*

Let us choose a coordinate system to be referred to as the exterior system in which:
- The Y axis is vertical.

- The Z axis is horizontal and nearly in the same vertical plane as the camera axis.
- The origin is in the point in which the linear displacements are measured instead of the camera station as is done usually. It is preferable to choose it so that the correlations between the angular and the horizontal deformations should be a minimum.

At the moment t the coordinates of a point in the positive photograph coordinate system are:

$$\begin{pmatrix} {}_tX \\ {}_tY \\ {}_tZ \end{pmatrix} = {}_tM \begin{pmatrix} X + {}_tdX \\ Y + 0 \\ Z + {}_tdZ \end{pmatrix} + \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix} \qquad (1)$$

where:

${}_tX$, ${}_tY$, ${}_tZ$, are the coordinates in the positive coordinate system at the moment t. The origin of the positive coordinate system is at the exposure station and its Z axis coincides with the camera axis and oriented towards the photograped objects. The X and Y axes are parallel to the photograph's frame lines.

X, Y, Z are the coordinates of the same point in the exterior coordinate system.

$X_0$, $Y_0$, $Z_0$, are the coordinates of point O the origin of the exterior coordinate system in the positive coordinate system, at the moment t = 0.

${}_tdX$, ${}_tdZ$, are the horizontal displacements of point O, the origin of the coordinate system at the moment t. They are given in directions parallel to the orginal horizontal axes of the exterior coordinate system at the moment t = 0.

${}_tM$ is the matrix of direction cosines at the moment t.

For the moment t = 0 we obtain:

$$\begin{pmatrix} {}_0X \\ {}_0Y \\ {}_0Z \end{pmatrix} = {}_0M \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix} \qquad (2)$$

The matrix of direction cosines of the antenna's rotations at the moment t is:

$${}_tR = {}_tM \cdot {}_0M^{-1} \qquad (3)$$

Because of the change in the position of the origin of the exterior coordinates system the well known projective transformation equations are also changed to the following form.

$$\frac{{}_t x}{f} = \frac{{}_t m_{11}(X + {}_t dX) + {}_t m_{21} Y + {}_t m_{31}(Z + {}_t dZ) + X_o}{{}_t m_{13}(X + {}_t dX) + {}_t m_{23} Y + {}_t m_{33}(Z + {}_t dZ) + Z_o} \qquad (4)$$

$$\frac{{}_t y}{f} = \frac{{}_t m_{12}(X + {}_t dX) + {}_t m_{22} Y + {}_t m_{32}(Z + {}_t dZ) + Y_o}{{}_t m_{13}(X + {}_t dX) + {}_t m_{23} Y + {}_t m_{33}(Z + {}_t dZ) + Z_o}$$

where:

${}_t x$, ${}_t y$, are the photo coordinates of the point in the photograph taken at the moment t.

$f$ is the camera's focal length.

${}_t m_{11}$, ${}_t m_{21}$, .....${}_t m_{33}$ are elements of the matrix ${}_t M$

Additional control elements which prove to be very useful are control lines. These are lines whose mathematical equations are known in the exterior coordinate system.

$$\frac{X-U}{A} = \frac{Y-V}{B} = \frac{Z-W}{C} \qquad (5)$$

where: U,V,W are the spatial coordinates of a point on the line.
A,B,C are the direction cosines of the line.

$$(A^2 + B^2 + C^2)^{1/2} = 1$$

By eliminating X,Y,Z, the coordinates of the point from equations (4) and (5) we obtain the following determinant which is an equation of a line in the photo coordinate system.

$$\begin{vmatrix} x & y & f \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix} = 0 \qquad (6)$$

where:

$$\begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix} = {}_tM \begin{pmatrix} A \\ B \\ C \end{pmatrix} \qquad \begin{pmatrix} Q_x \\ Q_y \\ Q_z \end{pmatrix} = {}_tM \begin{pmatrix} (U + {}_t dX) \\ (V + 0) \\ (W + {}_t dZ) \end{pmatrix} + \begin{pmatrix} X_o \\ Y_o \\ Z_o \end{pmatrix}$$

The equation of the same line can be determined also from two or more points located on it by the usual procedures of analytic geometry The equation is.

$$I x + J y + K f = 0 \qquad (7)$$

From equations (6) and (7) we obtain two equations which are equivalent to the projective transformation equations.

$$\frac{P_y{*}Q_z - Q_y{*}P_z}{I} = \frac{P_z{*}Q_x - Q_z{*}P_x}{J} = \frac{P_x{*}Q_y - Q_x{*}P_y}{K} \qquad (8)$$

At first glance the control lines seems to be less efficient than the control point because a single control point provides two equations like those in eq. (4) for each pair of photo coordinates while at least two pairs of photo coordinates are needed for equivalent equations by the method of control lines. On the other hand the control lines have three advantages which compensate for this inefficiency.

- It is easier to identify a line on the photograph than a single point, especially when they are not marked previously.
- It is possible to achieve good accuracy in the measurements of deformations although the accuracy of the coordinates of the control points is poor, if the same control points appear and are measured in all the photographs. In this case the residuals are relatively big and there is a strong correlation between the residuals of the same coordinate in all the photographs. There is always a possibility that a control point may be hidden from the camera by a moving part of the construction. The probability that a line will be hidden is much lower.
- As a result of the second advantage the equations of the control lines can be taken from the construction plans while a control device based on control points must be measured precisely.

*ACCURACY TEST*

In theory the RMS of determining a linear displacement of the antenna by a photograph taken with 1000 mm tele-objective from a point 50 meters away is about 1 mm. The RMS of measuring an angular deformation is estimated to be 0.3 to 0.5 milli-radian about axes perpendicular to the camera axis and about 1 to 2 milli-radians about an axis parallel to it. This estimation was checked by the following experiment:

A control device was attached rigorously to a Wild T16 theodolite (see Fig. 1) stationed on the roof of a tall building. It was photographed from a point about 50 meters from the building and about 30 meters below the control device with a 35 mm camera equiped with the same 1000 mm tele-objective. The orientation of the device was changed after each shooting by rotating the alidade and the new direction was measured on the theodolite. The inclinations and the position of the control device on the theodolite remain unchanged. The photographs were used to determin the angular and linear displacements by the photogrammeric method. The differences between the results and the true values confirm the theoretical estimation of the accuracy.

Later the displacements of a 30 meter antenna were measured by the same photogrammetric method and simultaneously its accelerations were measured by accelerometers. The differences between the results obtain by the photogrammetric method and those obtain by the accelometers were smaller than expected.

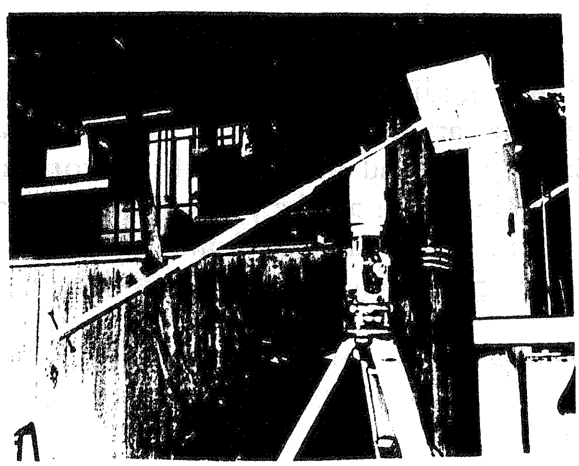

Fig. 1. The control device mounted upon the theodolite

## CONCLUDING REMARKS

The accuracy of determining the linear and angular dispacements is very satisfactory. On the other hand the photogrammetric measurments take too much time. The time needed for measuring one frame is about one minute but since many frames are involved, two days of measuring photo coordinates are needed in order to obtain the displacements of a single point on the antenna during about 11 seconds. In order to make the photogrammetric method more practical, the photography must be done with a video camera operating on small batteries which makes good quality pictures, in the field. In addition there is a need for equipment and procedures which will make the video tapes available for automatic detection of photo images and measuring their photo images. With these means it will be possible to save the laborious and slow procedure of measuring the photographs with a mono-comparator. In this case the control lines have another advantage because it is much easier and more accurate to automate the detection of lines on the photograph and determine their equations than to automate the search and measuring of the photo coordinates of single points.