

X-RAY PHOTOGRAMMETRY, STATE OF THE ART

Sandor A. Veress
University of Washington
Seattle, WA 98195
United States
IPRS Commission V

ABSTRACT

Geometry of X-ray photogrammetry is discussed as a function of maximum resolution and minimum radiation. Hardware configuration and calibration are described along with accuracy specification. Software possibilities are explored along with reliability, analysis, decision making, and data rejection. On-line analytical X-ray photogrammetry is detailed and specifications are given for hardware configuration and machine compatibility.

INTRODUCTION

The X-ray photogrammetry is basically utilized in the field of bio- and orthopedic engineering. In spite of the advantages the utilization is slow to advance in other fields, particularly in geotechnical engineering, where a maximum utilization is possible.

The biomedical applications are stereo and analytical X-ray photogrammetry. If pictorial information and analysis of the interaction between the soft tissue and skeleton are required, stereo X-ray photogrammetry is desirable. Due to the fact that the radiograms are shadows, better called shadographs, the stereo images are impaired and have limited accuracy capability. When high accuracy is the goal the analytical X-ray photogrammetry is required. This is the field where the major development took place during the last four years.

The analytical X-ray photogrammetry is used to increase the accuracy; while the stereo X-ray photogrammetry provides 1/12 - 1/15 base radiation distance ratio, the analytical X-ray photogrammetry is capable to $\frac{1}{2}$ ratio, thus considerably increasing the achievable accuracy. This is only possible by converging angles, thus the stereoscopic observation is impossible. Therefore, landmarks must be implemented into the bone prior to the exposure to x-rays.

This paper will address the development in the field of analytical X-ray photogrammetry. The geometrical efficiency is the key in minimizing the radiation a patient may receive. The mathematical model and data processing provide the diagnosis which must be positive and decisive. Finally, the development and need for online X-ray photogrammetry will be discussed in the light of required hardware, software and efficiency.

GEOMETRICAL EFFICIENCY

The geometry of the analytical X-ray photogrammetry is defined by the interrelation between the anodes, calibration frame and the photographic film.

A calibration frame is required to establish a three-dimensional reference system to determine the position of anodes. Once this task is accomplished, the x, y, z coordinates of discrete points in the patient can be computed from the known positions of the anodes and from the measured and reduced image coordinates.

Basic Geometries

There are two basic geometric configurations that have been published; the "bi-plane" geometry developed at Case Western Reserve University in Cleveland, and it has not been changed considerably since (Brown et al., 1976). The convergent system was developed at the University of Washington in cooperation with the Veteran's Administration Hospital in Seattle. The schematic view of these geometrics is exhibited in Figure 1A and B. These two geometrics are applicable to any part of the human body while others are usable only in smaller parts such as legs, arms, etc.

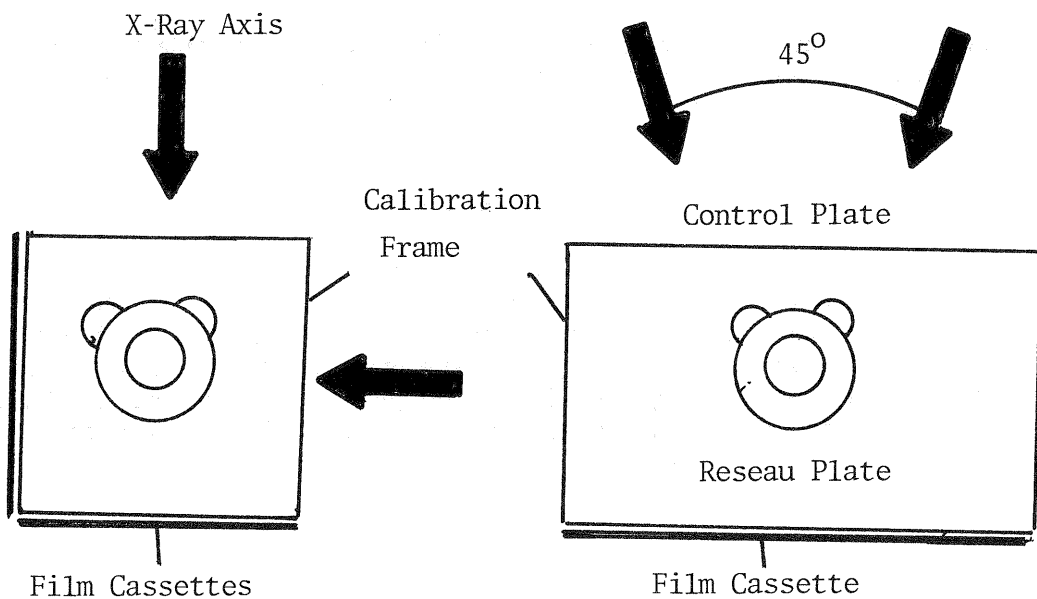


Figure 1. Geometrics

Both of these solutions have serious disadvantages. These are the difficulties in patient orientation and deterioration of resolution due to the angle of convergency.

Incident Angle and Resolution

The angle of incident is regarded as the angle between the plane of the film and the radiated rays. In order to reduce the amount of radiation, the X-ray film has emulsion on both sides of the emulsion carrier. The thickness of the film is 0.2mm, therefore the X-ray must strike the film at nearly a 90 degrees incident angle of radiation of the bi-plane system is advantages.

The X-ray photographs are more or less dense shadows of the object depending on the atomic weight. The medical analytical X-ray photogrammetry is most frequently used in orthopedics to determine prosthesis loosening. The prosthesis is made of stainless steel in the case of complete hip or knee replacement. The landmarks are also stainless steel balls; thus, if the landmarks fall in the shadow of the prosthesis, they will not be visible on the radiograms. This means that several radiograms must be made of the patient in order to place him/her in a position where all or most of the landmarks are visible. Consequently, the patient will receive an excess amount of radiation, thus the criterion of minimum radiation is not maintained.

This problem is eliminated by the convergent geometry. However, the axis of radiation is 70° of incident angle of the convergent system, thus the resolution is deteriorated, which prevents the achieving of maximum accuracy.

It can be concluded that neither of the presently practices systems is capable of fulfilling the desirable goal - that is, to obtain maximum accuracy with minimum radiation. Both systems have medically usable radiograms equally usable for technical purposes.

Optimum Geometry

Qassim in 1984 studied the effect of various geometries. The study was conducted at the University of Washington and sponsored by the Veteran's Administration. A mechanical specimen was studied which was constructed such that changes in distances could be measured by micrometer and by X-ray photogrammetry. The specimen consists of a plexiglass in which there are two pistons separated by a spring. Both of the pistons have steel balls implanted, one of the pistons is fixed, the other is movable by a micrometer. The arrangement is shown in Fig. 2.

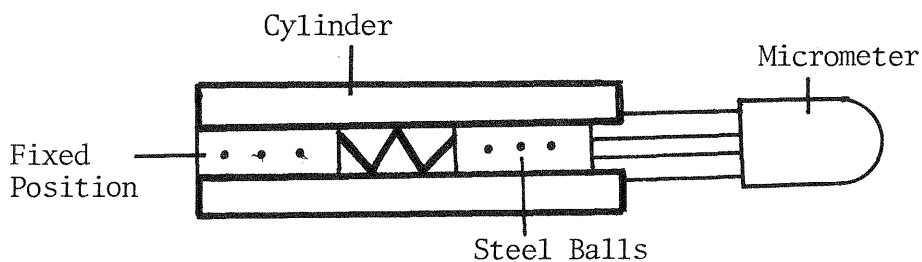


Figure 2. Mechanical Specimen

There are 25 distance combinations available. The movement of the piston and thus the changes in distances was controlled by the micrometers in three settings; therefore, 75 observations were made in x, y and z direction. The conclusion is drawn from 225 observations.

The standard error of the coordinates of the right and left anodes was found to be: in x direction ± 0.012 y direction ± 0.013 mm and ± 0.017 mm in the direction of radiation. The average distance standard error is ± 0.027 mm.

The geometry is presented in Fig. 3 with the proper dimensions. The advantage of this geometry is that it requires a minimum radiation, and provides the maximum accuracy, due to the nearly 90 degrees incident angle. However, the radiograms are double exposed; the images are in different scales and the medical use is very difficult.

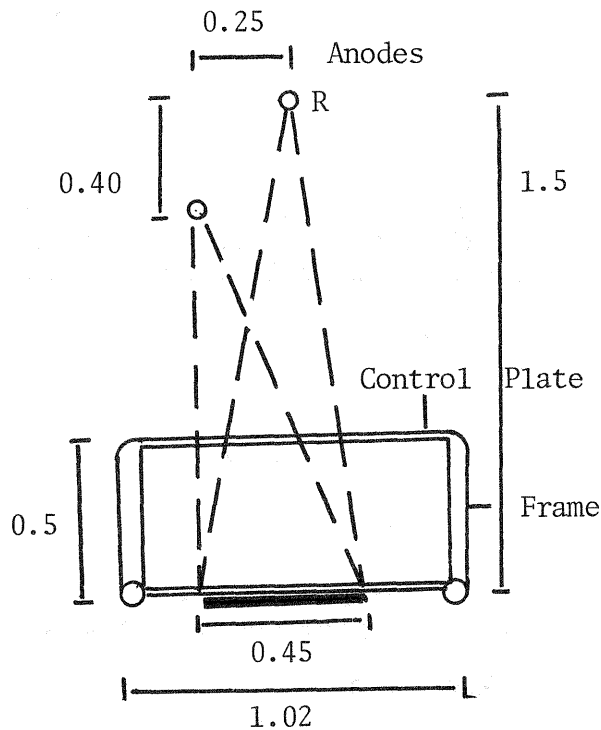


Figure 3. New Geometry
(Numbers are in m)

These experiments show that the resolution is one of the most important factors to achieve high accuracy. Thus the incident angle of the rays should be close to 90 degrees, which also reduces the number of radiograms, and thus the radiation the patient receives, to the minimum.

MATHEMATICAL MODEL AND DATA PROCESSING

Establishing the Image Coordinates

The images of the radiograms must be digitized. These images are larger than the actual object; therefore, a digitizer with least-reading of 10 micrometers is sufficient. The digitized image coordinates must be transformed into the coordinate system of the control frame by using the reseau crosses. The plane of the film is not parallel to the reseau plane; therefore, a perspective transformation is most suitable. That is, if the image coordinates are xy and the control frame coordinates and XY , the linearized form of the equations is:

$$\begin{aligned} X &= (a_0 + a_1x + a_2y)/(c_1x + c_2y + 1) \\ Y &= (b_0 + b_1x + b_2y)/(c_1x + c_2y + 1) \end{aligned} \quad (1)$$

where a_0, a_1, \dots, c_2 are the transformation co-efficient determined from the known coordinates of the reseau crosses and the measured coordinates of their images.

Mathematical Model

There are a number of mathematical models that have been established based on various principles. A single mathematical model can be used to determine the coordinates of the anodes and the coordinates of the landmark. This is the equation of the line:

$$(X_o - X_i)/\ell = (Y_o - Y_i)/n = (Z_o - Z_i)/m$$

where $m, n,$ and ℓ are the direction cosines.

$$\begin{aligned} \ell &= (X_{ii} - X_i)/d, \quad m = ((Y_{ii} - Y_i)/d, \quad n = (Z_{ii} - Z_i)/d \\ \text{and} \\ d &= [(X_{ii} - X_i)^2 + (Y_{ii} - Y_i)^2 + (Z_{ii} - Z_i)^2]^{\frac{1}{2}} \end{aligned}$$

where $X_i, Y_i,$ and $Z_i,$ the coordinates of i target point in the control frame system X_{ii}, Y_{ii} and Z_{ii} are transformed image coordinates. The expansion of equation (2) leads to

$$\begin{bmatrix} m & -\ell & 0 \\ n & 0 & -\ell \end{bmatrix} \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} - \begin{bmatrix} mX_i - Y_i \\ nX_i - \ell Z_i \end{bmatrix} = 0 \quad (3)$$

or in form of observation equation: $AX - L = V$

where V is the correction vector, which is minimized through the standard least squares of adjustment. The coordinates of the anodes computed as

$$X = (A^T A)^{-1} A^T L \quad \dots(4)$$

The same equations used for intersections where the X_0, Y_0 and Z_0 coordinates are replaced by X_i, Y_i and $Z_i.$

If better accuracy is required this computation can serve as an input to a more rigorous solution. The computed coordinates of th targets considered as approximate values ($X^0 Y^0 Z^0$).

If equation 2 is linearized, the combined condition and observation equation can be used for least squares adjustment.

$$BV + AX + W = 0 \quad (5)$$

or in detail for a single line i :

$$B_i = \left[\begin{array}{cccc} (X^o - X_{ii}) / (X_{ii} - X_i)^2, & (Y_{ii} - Y^o) / (Y_{ii} - Y_i)^2, & 0, & (X_i - X^o) / (X_{ii} - X_i)^2, & (Y^o - Y_i) / (Y_{ii} - Y_i)^2 \\ (X^o - X_{ii}) / (X_{ii} - X_i)^2, & 0, & (Z_{ii} - Z^o) / (Z_{ii} - Z_i)^2, & (X_i - X_o) / (X_{ii} - X_i)^2, & (Z^o - Z_i) / (Z_{ii} - Z_i)^2 \end{array} \right]_i$$

$$A_i = \left[\begin{array}{ccc} 1 / (X_{ii} - X_i), & -1 / (Y_{ii} - Y_i), & 0 \\ 1 / (X_{ii} - X_i), & 0, & -1 / (Z_{ii} - Z_i) \end{array} \right]_i$$

and

$$V_i^T = (V_{X_i}, V_{Y_i}, V_{Z_i}, V_{X_{ii}}, V_{Y_{ii}}, V_{Z_{ii}})_i$$

$$W_i = \left[\begin{array}{c} (X^o - X_i) / (X_{ii} - X_i), (Y^o - Y_i) / (Y_{ii} - Y_i) \\ (X^o - X_i) / (X_{ii} - X_i), (Z^o - Z_i) / (Z_{ii} - Z_i) \end{array} \right]_i$$

The normal equation is

$$A^T (BP^{-1} B^T)^{-1} A \bar{X} = A^T (BP^{-1} B^T)^{-1} W \quad (6)$$

where $\bar{X}^T = (\Delta X, \Delta Y, \Delta Z)$ are the corrections to be added to the approximate coordinates to obtain their most probable values. P is the weight matrix of the observations.

The posteriori precision of the parameters is:

$$\Sigma_{XX} = \sigma_0 (A^T (BP^{-1} B^T)^{-1} A)^{-1} \quad (7)$$

If the x-ray photogrammetry is used to determine prosthesis loosening distances between the prosthesis and the landmark and their accuracies must also be computed.

Dataflow

The dataflow consists of four steps. One, the reduction of measured image coordinate to the system of the control frame. Two computation of the spatial position of the anodes by resection and computation of coordinates of targets by intersection.

The computation may be terminated here if no higher accuracy is desirable. Three, the computed coordinates of the target regarded as approximate values and the rigorous least-square adjustment is performed. The result is the most probable values of the coordinates and their standard error.

Four, the utilization of the spatial coordinates. Computation of distances in load and unloaded position, computation of rotational angles etc., should be followed.

ONLINE X-RAY PHOTOGRAMMETRY

The data management of analytical x-ray photogrammetry can be sequential and online. The sequential data management may use different individual equipment without inter-connection such as a digitizer, computer, printer, etc. However, a very serious problem is the large number of data handling, and the constant problem of erroneous data.

The effective data management requires an online solution. The radiograms are developed in the x-ray room after the exposure and the development and drying time is only 95 seconds. It is therefore logical that the technical evaluation of these radiograms be performed in the x-ray room by the technician. Suphavitai 1985 studied the the problem of online x-ray photogrammetry at the University of Washington.

An analytical x-ray photogrammetric program can be run on IBM type micro computer family. The computer should run under the PC-Disk Operating System (DOS) and MS-DOS. the minimum requirement would be one disk drive and 512Kb random access memory. The recommended solution however is 512Kb random access memory, two floppy drives and an 8087 numeric coprocessor by the Intel Corporation. The 8087 math chip increases the processing speed by about five fold.

A standard E size digitizer should be used as an input. The digitizer must have a back light unit, which is required to illuminate the transparent radiograms. The digitizer must have a minimum of 0.35 x 0.45 mm active surface in order to accommodate the standard x-ray film. There are a number of such units available such as Tektronics, Hitachi, etc.

To use the digitizer with the IBM PC or compatibles the computer must have at least one serial communication port; or "Res 232-C asynchronous communication port." some IBM compatible computers such as AT&T 6300, Tandy 1000 or Zenith 151 the serial port is standard thus these computers are ready to be connected to the digitizer. The digitizer must be equipped with RS 232-C interface and capable of transmitting data in ASCII Binary Coded Decimal (BCD) format.

The system must be connected to a printer in order to obtain hard copy output. A matrix printer is sufficient for this purpose.

CONCLUSION

The analytical x-ray photogrammetry in the practice, must fulfill the technical need of the practitioner of medicine. this can only be done by obtaining high technical quality image even if the pictorial (medical) interpretation is impaired. In this regard the new overlapping geometry provides the most promise, for maximum accuracy is needed for determining prosthetic loosening particularly where moment forces are involved.

The mathematical model is very effective and not much improvement is required in this regard. The online x-ray photogrammetry is the correct solution for the routine practice. The system should be transferred into the x-ray room and should be practiced by the x-ray technician. The capability of the micro computers should be extended by the "Enhanced Graphic Adapter" (AGA) board which is standard for the business graphic hardware. If the AGA is associated with color monitor this would visually improve the diagrams for the medical doctors particularly in cases of prosthesis loosening.

REFERENCES

- Agnoletto, E.B., 1974. X-ray photogrammetry with special reference to the treatment of uterine cancer. Proceedings of the Biostereometrics 74 Symposium, Washington, D.C.
- Aronson, A.S., Holt, L., and Selvick, G., 1974. An instrument for insertion of bone markers. Radiology 113: pp. 733-734.
- Brown, R.H., Bustein, A.H., Nash, C.L., and Schock, C.C., 1976. Using a three dimensional radiographic technique. Journal Biomechanic Vol. 9: pp. 355-365.
- Jonason, C., and Hindmarsh, J., 1975. Stereo X-ray photogrammetry as a tool in studying scoliosis. Proceedings of ASP Symposium on Close-Range Photogrammetric System, Illinois.
- Lippert, F.G., Veress, S.A., Takamoto, T., and Spoleck, G.A., 1975. Experimental studies on patellar motion using X-ray photogrammetry. Proceedings of ASP Symposium on Close-Range Photogrammetric System.
- Moffitt, F.H., 1972. X-ray photogrammetry applied to orthodontic measurements. Invited paper prepared for Working Group II, Commission V, of the 12th Congress of the International Society of Photogrammetry, Ottawa, Canada.
- Qassim, A.J. Abdullah, 1984. Reliability aspect and data rejection in X-ray photogrammetry. Ph.D. Dissertation, University of Washington.
- Suphavitai, R., 1986. Development in simultaneous least square adjustment to achieve on-line analytical X-ray photogrammetric system. Ph.D. Dissertation, University of Washington.
- Takamoto, T., 1976. X-ray photogrammetric analysis of skeletal spatial motions. Ph.D. Dissertation, University of Washington.
- Veress, S.A., and Lippert, F.G., 1978. A laboratory and practical application of X-ray photogrammetry. Proceedings of ASP 44th Annual Meeting, Washington, D.C., pp. 409-421.
- Veress, S.A., and Lippert, F.G., and Takamoto, T., 1977. An analytical approach to X-ray photogrammetry. Photogrammetric Engineering and Remote Sensing, Vol. 43/12.