EXTERNAL PROCEDURE OF OSCILLOSCOPE'S IMAGES REFINEMENT

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ABSTRACT

This paper suggests an external procedure of system calibration, under the actual operational conditions, of Oscilloscope images produced by equipments where the specifications of photogrammetry are not satisfied.
It states a general process of non-metric camera calibration and resolves the problem of determining the coordinates of an object having no control points without needing for any modification in the internal parts of either the camera or the instrument.
It explains with details the conditions to be considered and the steps of the refinement procedure which is based on the *successive elemination of the effect of each individual factor*. It derives the technical tools of the development of the mathematical model performed and analysed for each source of errors in order to determine the physical property of every parameter.

INTRODUCTION

To investigate the distortions performed by a certain system or to study the effect of an individual factor, the refinement procedure must be based on the process of *error's successive elemination of factor by factor* until that the remaining deformations will be due to that considered factor. Thus the final mathematical model where the parameters are functions of its effect ONLY may be obtained.
This process may be used in many fields as Televisions, Cathode Ray Tubes, oscilloscope and non metric cameras, X-Rays and ultrasonic. Its applications may be extended to the refinement of Satellite's images *considering that they all are sharing the same principles of image production* :

- emerged and reflected waves,
- electronics and processing,
- CRT display,
- Oscilloscope or non-metric camera (lens distortion and film deformation) -
- Mainly they are dealing with an object of completely unknown coordinates where no control points are available.
- Moreover, the successive steps of image production involve that the distortions occured are the results of the combined effect of several factors interferring with each others.

As a pilot example, the experiment was developed for an ultrasonic instrument, the Greatone III Sonograph using Polaroid films (Wishahy 1984), El Ghazaly & Wishahy 1988), its applicability to X-Rays films was technically verified. Compensation of systematic errors is carried out by dividing the factors producing image distortions into two main groups :

Group A : consisting of the camera of unknown inner orientation parameters and containing the lens and film deformations.

Group B: the ultrasonic instrument including the transducer, U.S. waves, C. R. T., the magnetic fields, electronics & processing..

Mathematical model for each group with its correction parameters has been derived from basic principles. The corresponding coefficients have been investigated to distinguish their meanings and understand their physical properties .

THE REFINEMENT PROCEDURE :

The experiment process is based on obtaining two photographs ; the first one is PHOTO I to express only the distortions of the elements of *Group A*. The second one is PHOTO II which will contain the errors caused by the combined effect of both *Groups A & B*.
This is achieved by selecting a standard Reseau formed by thin aluminium wires of the same dimensions as the screen and fixing it between the camera and the C. R. T. in the frame surrounding the screen . It will appear in both PHOTO I& PHOTO II . The Reseau has double role, that's of replacing the missing of fiducial marks as an **ARTIFICIAL** system of coordinates , in addition to its use as a **COMMON** system of coordinates to which are referring both the object and the photo coordinates to eleminate the effects of rotations, translations and scales .
PHOTO I : Before using the ultrasonic waves, the Reseau fixed in front of the C.R.T is photographed to produce PHOTO I on which the distortions are due to the elements of Group A only . Fig. (1.a)
PHOTO II : A standard 100 Millimeter Test Object is used for aligning, calibrating and measuring the performance of ultrasonic pulse-echoe apparatues as a whole. It is consisting of series of 0.75 mm. diameter stainless steel rods of known geometrical arrangement (tolerance ± .25 mm.) (Wishahy 1984, 1988) .
As soon as the transducer moves along the upper edge of the Test Model, the ultrasonic waves are propagated and reflected in the vertical plane of the transducer's motion (perpendicular to its face). This vertical cross section (Fig. 1-b) is displayed on the CRT screen behind the Reseau which still positionned in front of the the screen . This scanning is photographed either by Polaroid films or X-Rays films to give PHOTO II .
The distortions occured are the result of the combined effects of all the factors existing in both *Groups A & B*.
One expects that the pin's images will be points (cross sections of the rods) , but due to the principles of ultrasonic wave's propagation, described by Wishahy (1988) , the pin's images appear as horizontal curved lines as illustrated . The y coordinate of each pin's image is directly measured, but its x coordinate is considered in the middle of that line.
The refinement procedure is summarized in Figs. (1- a, b & 2) it follows the next steps :

- **Correction of the Camera's Distortions : (PHOTO I)**

(i) Measuring the object and photo coordinates of the Reseau ($\bar{X}_c$, $\bar{Y}_c$ & $\bar{x}_c$, $\bar{y}_c$) . respectively by a mono comparator .

(ii) Transforming them to ($\bar{X}$, $\bar{Y}$ & $\bar{x}$, $\bar{y}$) which are referred to the **COMMON** Resaeu system to eleminate the effects of scale, rotations and translations .

Where :
$\bar{X}$ axis is the line joining the points O_1 & S_1 of the Reseau *and not the wire.*
$\bar{Y}$ axis is the line perpendicular to $\bar{X}$ from point O_1 in space *and not the wire* .
$\bar{x}$ axis is the line joining points O & S the images of points O_1 & S_1 respectively .
$\bar{y}$ axis is the line perpendicular to $\bar{x}$ in space from point O .
Thus : *the $\bar{x}$ O $\bar{y}$ system is considered as the COMMON coordinate system, it is identical to the system $\bar{X}$ O_1 $\bar{Y}$.*

(iii) Resolving the mathematical model derived later in (equ. 5 a & b) between the two sets of coordinates ($\bar{X}$, $\bar{Y}$ & $\bar{x}$, $\bar{y}$)$_n$ of which the difference is due to the effects of Group A only. Thus the values of parameters (a_i, b_i, c_i, d_i, h_i, k) obtained by the least squares adjustment are deduced .

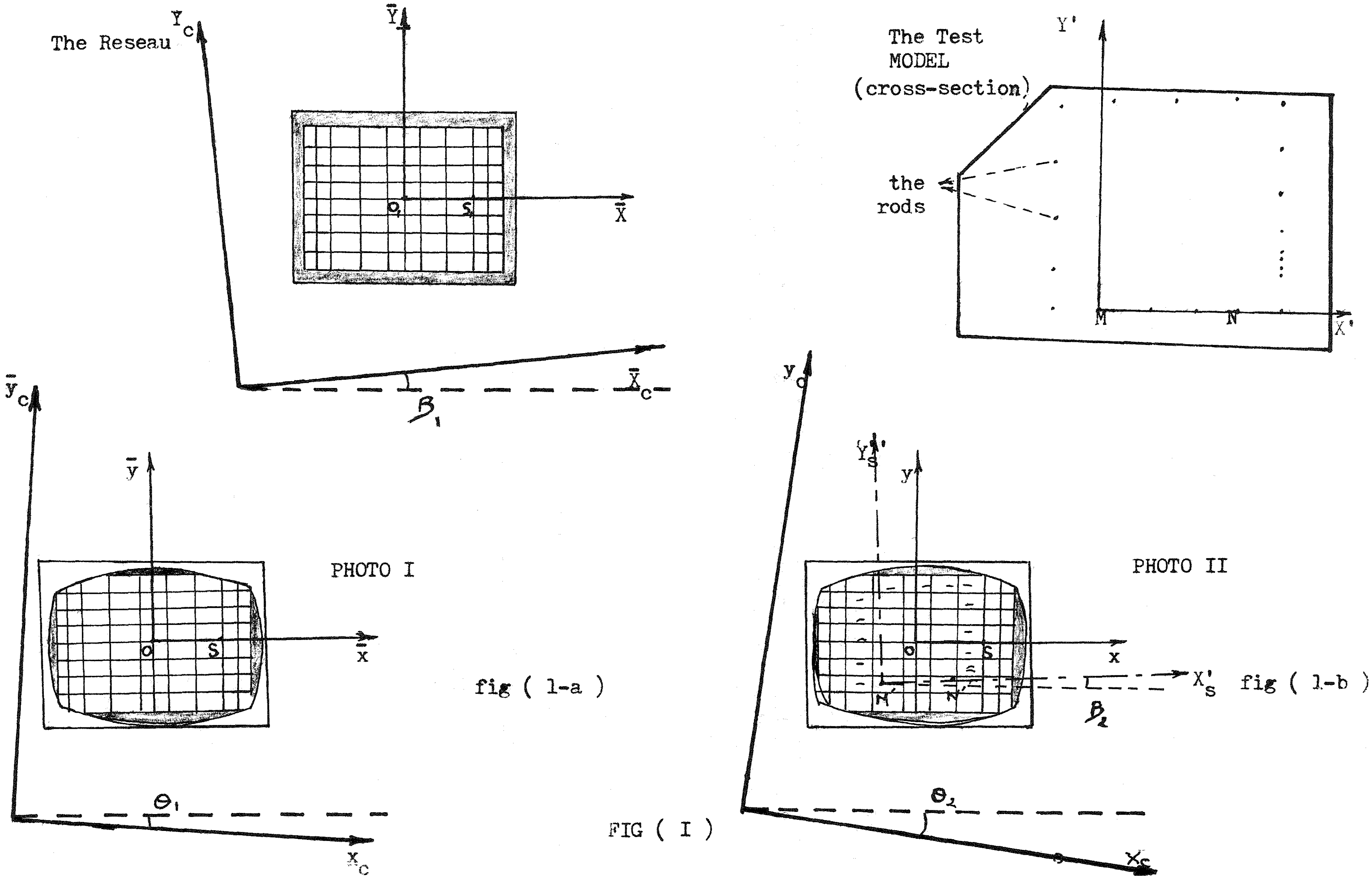
The Reseau
The Test
MODEL
(cross-section)
the
rods
M
N
X'
Y'
PHOTO I
PHOTO II
fig (1-a)
fig (1-b)

FIG (I)

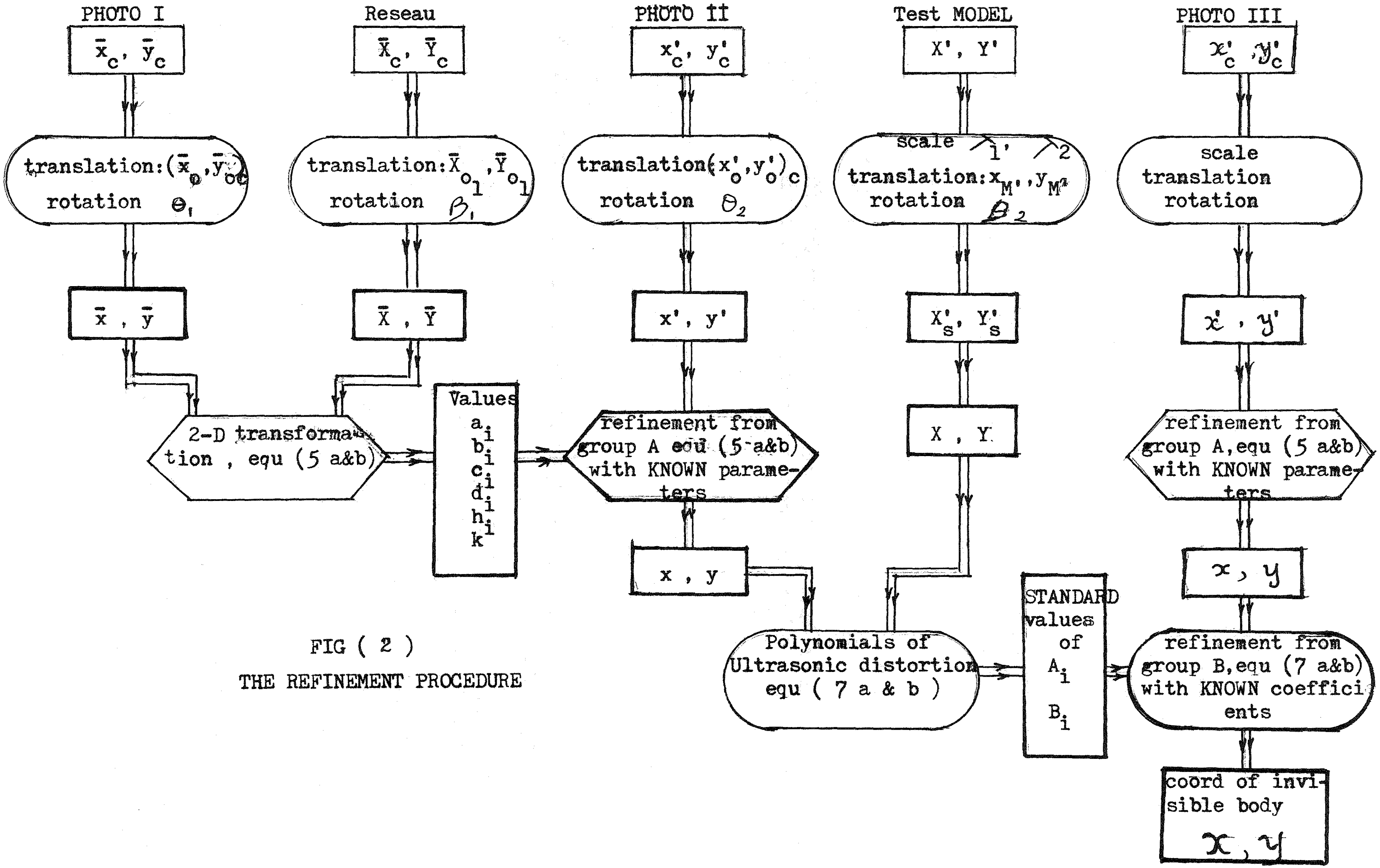

FIG (2)

THE REFINEMENT PROCEDURE

- **Refinement of distortions caused by the ultrasonic instrument (PHOTO II):**

(iv) Measuring the Test Model's photo coordinates (x'_c , y'_c) on photo II using the mono-comparator. Deducing their correspondings,Object coordinates (X', Y'), from the known distances between rods, then transforming them to (X'_S , Y'_S) , their alternative scaled coordinates .

<u>Where</u> :

X' axis is the line joining pin M and pin N ,

Y' axis is the line perpendicular to X' from point M .

$X'_S M' Y'_S$ is the alternative scaled (scale of PHOTO II) object coordinates system in the photo plane <u>where</u> :

$X'_S = \lambda_1 X'$ is the line joining M' to N' (images of M & N)

$Y'_S = \lambda_2 Y'$ is the line perpendicular to X'_S from M'

λ_1 & λ_2 are the scale factors of PHOTO II in the X and Y directions respectively.

(v) Transforming them to (X, Y & x', y') which are referred to the COMMON Reseau system to eleminate the effects of scales, rotations and translations.
The (x' ,y') contain the errors of all the factors of both groups A&B .

(vi) Obtaining the refined photo coord. (x , y) from the effect of group A as the L. H. S. of the mathematical models (5 a & b) having as input in the R.H.S. the KNOWN values of parameters a_i , b_i, ... k deduced at step (iii) and the (x', y') coordinates .

(vii) Resolving the mathematical models (7 a&b) describing the performance of the ultrasonic instrument's effects (excluding all the other factors), group B only, between the (X, Y) Model Object coordinates and the previous (x, y) coord., both referred to the COMMON system. Thus the deduced correction parameters A_i , B_i will be **theoritically standard** for all the following photographs obtained by the same camera and the same equipment .

(viii) Now the unknown CORRECT coord. of an invisible body having no CONTROL points appearing on PHOTO III can be DIRECTLY obtained (LHS) by resolving the STANDARD (theoritically) mathematical models having as input (R.H.S) the KNOWN parameter's values and the body's photo coord. (x , y) which have been refined from the camera's distortions and referred to the **COMMON** Reseau system.

DERIVATION OF THE MATHEMATICAL MODELS OF:

A. The Camera's Distortion :

The difference between ($\bar{X}$,$\bar{Y}$) & ($\bar{x}$, $\bar{y}$) is due to the systematic errors caused by the camera lens distortion and the film deformation in addition to random errors (assuming that no tilt angle exists between the Reseau and the film plane) .The mathematical model describing this relationships is derived from the basic three dimensional transformation :

$$\begin{bmatrix} \bar{X} - \bar{X}_0 \\ \bar{Y} - \bar{Y}_0 \\ \bar{Z} - \bar{Z}_0 \end{bmatrix} = \lambda \begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{bmatrix} \begin{bmatrix} \bar{x} - \bar{x}_p \\ \bar{y} - \bar{y}_p \\ f \end{bmatrix}$$

Where :
$[\bar{X}_0, \bar{Y}_0, \bar{Z}_0]$ are the coord. of point 0 w.r.t. the system $\bar{X}\,0_1\bar{Y}$
$\bar{x}_p, \bar{y}_p$ are the two shifts between the principal point p and point 0
$(e_{ij})^{i=1\ \text{to}\ 3}_{j=1\ \text{to}\ 3}$ are the direction cosines of the rotation matrix

- Or the <u>special conditions</u> of the experiment involve the following :
  (1) The Reseau is an object of flat surface where the thin aluminium wires are placed in one plane : i.e. Z = constant
  $\lambda(Z-Z_0)$ = constant = D **where :** λ is the scale factor

$$\bar{X}-\bar{X}_0 = \frac{D.(e_{11}\bar{x}+e_{12}\bar{y} \;+\; (e_{13}f.-e_{11}\bar{x}_p-e_{12}\bar{y}_p)}{e_{31}\bar{x}+e_{32}\bar{y} \;+\; (e_{33}f-e_{31}\bar{x}_p-e_{32}\bar{y}_p)} \quad \text{..........} \quad (1\text{-a})$$

$$\bar{Y}-\bar{Y}_0 = \frac{D.(e_{21}\bar{x}+e_{22}\bar{y} \;+\; (e_{23}f.-e_{21}\bar{x}_p-e_{22}\bar{y}_p)}{e_{31}\bar{x}+e_{32}\bar{y} \;+\; (e_{33}f-e_{31}\bar{x}_p-e_{32}\bar{y}_p)} \quad \text{..........} \quad (1\text{-b})$$

Put : $(e_{33}f-e_{31}\bar{x}_p-e_{32}\bar{y}_p) = R$
Then :

$$\bar{X}-\bar{X}_0 = \frac{\frac{D.e_{11}}{R}.\bar{x}+\frac{D.e_{12}}{R}.\bar{y}+\frac{D}{R}(e_{13}f.-e_{11}\bar{x}_p-e_{12}\bar{y}_p)}{\frac{e_{31}}{R}.\bar{x}+\frac{e_{32}}{R}.\bar{y}+1} \quad \text{........} \quad (2\text{-a})$$

$$\bar{Y}-\bar{Y}_0 = \frac{\frac{D.e_{21}}{R}.\bar{x}+\frac{D.e_{22}}{R}.\bar{y}+\frac{D}{R}(e_{23}f.-e_{21}\bar{x}_p-e_{22}\bar{y}_p)}{\frac{e_{31}}{R}.\bar{x}+\frac{e_{32}}{R}.\bar{y}+1} \quad \text{........} \quad (2\text{-b})$$

(2) The case that the two systems of coordinates are identical to each others as explained before , produces that :
*) $\bar{X}_0 = 0$ $\bar{Y}_0 = 0$ equ. (2) turn to the <u>PROJECTIVITY</u> equation:

$$\bar{X} = \frac{a_1''\bar{x}+b_1''\bar{y}+c_1''}{a_0\bar{x}+b_0\bar{y}+1} \quad \text{......} (3\text{-a}) \qquad \bar{Y} = \frac{a_2''\bar{x}+b_2''\bar{y}+c_2''}{a_0\bar{x}+b_0\bar{y}+1} \quad \text{......} (3\text{-b})$$

**) $e_{11} = \cos\bar{X}\bar{x} = e_{22} = \cos\bar{Y}\bar{y} = e_{33} = \cos\bar{Z}\bar{z}, \quad = \cos 0^\circ = 1$

***) all the remaining direction cosines of the M matrix will be equal to $\cos 90^\circ = 0$

<u>We obtain</u>: $R = +f$

$a_0 \simeq 0$ $\qquad$ $b_0 \simeq 0$

$a_1'' = D.e_{11}/R \simeq \lambda(\bar{Z}-\bar{Z}_0)/f$ $\qquad$ $a_2'' = D.e_{21}/R \simeq 0$

$b_1'' = D.e_{12}/R \simeq 0$ $\qquad$ $b_2'' = D.e_{22}/R \simeq \lambda(\bar{Z}-\bar{Z}_0)/f$

$c_1'' = D(-e_{11}\bar{x}_p)/R \simeq \lambda(\bar{Z}-\bar{Z}_0)(-\bar{x}_p)/f$ $\qquad$ $c_2'' = D(-e_{22}\bar{y}_p)/R \simeq \lambda(\bar{Z}-\bar{Z}_0)(-\bar{y}_p)/f$

<u>Note</u>:

The sign (=) is only correct if the perfect conditions of perpendicularity and parallelity between the Reseau and the film plane are satisfied, otherwise the sign (=) has to be replaced by the sign ($\simeq$)

The pervious relationship (3 a&b) has been derived without considering any deformations therefore, the object coordinates $\bar{X}$, $\bar{Y}$ have to be modified to absorb the effect of lens distortion and film deformation:

$$\bar{X} = \frac{a_1''\bar{x} + b_1''\bar{y} + c_1''}{a_0\bar{x} + b_0\bar{y} + 1} + a_1'\bar{x} + b_1'\bar{y} + c_1' + d_1\bar{x}\bar{y} + k(\bar{x}^3 + \bar{x}\bar{y}^2) + \alpha_1 \quad \text{...... (4-a)}$$

$$\bar{Y} = \frac{a_2''\bar{x} + b_2''\bar{y} + c_2''}{a_0\bar{x} + b_0\bar{y} + 1} + a_2'\bar{x} + b_2'\bar{y} + c_2' + d_2\bar{x}\bar{y} + k(\bar{y}^3 + \bar{y}\bar{x}^2) + \alpha_2 \quad \text{...... (4-b)}$$

<u>Where</u>:

$b_1', c_1', d_1,$ / b_2', c_2', d_2: account the film deformation

k: for the effect of symmetric lens distortion.

a_1', a_2': absorb the two effects

α_1, α_2: are terms of higher order to be verified by the experimental test. They were found to be $h_1 x^4$&$h_2 y^4$ for the ultrasonic instrument and Polaroid film (El Ghazali & Wishahy 1988).

Rearranging and substituting by:

$-a_1 = a_1'' + a_1'(a_0\bar{x} + b_0\bar{y} + 1)$

<u>Or</u> $\quad a_0 \simeq 0$, $b_0 \simeq 0$ & a_1 is very small quantity

<u>Then</u> $\quad a_1 = a_1'' + a_1'$. it accounts the projectivity relationship in addition to lens and film deformation.

<u>Consequently</u>:

$-a_2 = a_2'' + a_2'$ $\qquad$ or $a_2'' \simeq 0$

<u>Then</u> $\quad = a_2'$ $\qquad$ effect of film deformation

$-b_1 = b_1' + b_1''$ $\qquad$ or $b_1'' \simeq 0$

$= b_1'$ $\qquad$ effect of film deformation

$-b_2 = b_2'' + b_2'$

$-c_1 = c_1'' + c_1'$ $\qquad$ effect of film deformation added to the projectivity relation

$-c_2 = c_2'' + c_2'$

Finally we obtain :

$$\bar{X} = \frac{a_1\bar{x} + b_1\bar{y} + c_1}{a_0\,\bar{x} + b_0\bar{y} + 1} + d_1\,\bar{x}\,\bar{y} + k\,(\bar{x}^{\,3} + \bar{x}\,\bar{y}^{\,2}) + h_1\,\bar{x}^{\,4} \qquad \text{............ (5-a)}$$

$$\bar{Y} = \frac{a_2\bar{x} + b_2\bar{y} + c_2}{a_0\,\bar{x} + b_0\bar{y} + 1} + d_2\,\bar{x}\,\bar{y} + k\,(\bar{y}^{\,3} + \bar{y}\,\bar{x}^{\,2}) + h_2\,\bar{y}^{\,4} \qquad \text{............ (5-b)}$$

B. Mathematical Modeling of the Ultrasonic instrument's Distortions:

As the acoustic imagery is produced by displaying on a screen the reflected pulses which have been emitted and received through the transducer, then analysed and processed by the instrument's electronics, thus the errors occured are owing to two main sources:

1. **The distortions due to the ultrasonic waves properties,** concerning their frequency, intensity , resolution and reflection principles, this distortion called ϵ occuring through the x direction was investigated by Wishahy (1988) and it was found to be of the form .

$$(\Delta x)_\epsilon = m_1 x + m_2 y .$$

Where :

$m_1 x$ = accounts the effect of the transducer's diameter and the frequency

$m_2 y$ = is introduced to compensate the effects of the sound beam divergence which is proportional to the vertical distance y between the transdcer's and the point.

2. **The deformations resulted from the scanning and the displaying processes** on the Cathode Ray Tube , were interpretated by K. wong (1968 , 1969 , 1974, 1975) . they included :

* *Symmetrical radial distortions* :

Caused by the effects of : electron and magnetic lens systems, non-uniform magnetic field and the curvature of the CRT screen

$$(\Delta x)_s = x\,(k_0 + k_1 r^2 + k_2 r^4 + k_3 r^6 + \text{-------})$$
$$(\Delta y)_s = y\,(k_0 + k_1 r^2 + k_2 r^4 + k_3 r^6 + \text{-------})$$

* *Asymmetric tangential and radial distortions* :

Caused by the decentering of lens elements from the optical axis and expressed as :

$$(\Delta x)_d = -\,(g_1 r^2 + g_2 r^4 + g_3 r^6 + \text{-------})\sin\theta$$
$$= -\,(h'_1 (x^2 + y^2) + h'_2 (x^2 + y^2)^2 + \text{--------})$$

$$(\Delta y)_d = (g_1 r^2 + g_2 r^4 + g_3 r^6 + \text{-------})\cos\theta$$
$$= h'_3 (x^2 + y^2) + h'_4 (x^2 + y^2)^2 + \text{--------}$$

Where :

$h'_1 = g_1 \sin\theta$, $h'_2 = g_2 \sin\theta$,

$h'_3 = g_1 \cos\theta$, and $h'_4 = g_2 \cos\theta$

* *Orientation Distortion* :
They are due to mis alignment of the optical axis of the camera from the optical axis of the Cathode Ray tube and equal to :

$$(\Delta x)_0 = p_0 + p_1 xy + P_2 x^2 - P_3 y$$
$$(\Delta y)_0 = p_4 + p_2 xy + P_3 x + P_1 y^2$$

* *Composite Distortion equations* :
The combined distortion components due to the ultrasonic instrument will be:
$(\Delta x)_u = (\Delta x)_s + (\Delta x)_d + (\Delta x)_0 + (\Delta x)_\epsilon$
$(\Delta y)_u = (\Delta y)_s + (\Delta y)_d + (\Delta y)_0$
or $X = x + (\Delta x)_u$, $Y = y + (\Delta y)_u$
the first two terms of each polynomial are considered and the terms of higher order are neglected
Then :

$$X = P_0 + x(1 + k_0 + m_1) + y(m_2 - P_3) + P_1 xy + x^2(P_2 - h_1) + y^2(-h_1') + k_1(x^3 + xy^2) - h_2'(x^2 + y^2)^2 \quad \text{.......... (6-a)}$$

$$Y = P_4 + y(1 + k_0) + x(P_3) + P_2 xy + y^2(P_1 + h_3) + x^2(h_3') + k_1(y^3 + yx^2) + h_4'(x^2 + y^2)^2 \quad \text{.......... (6-b)}$$

We attain to the final mathematical model describing the performance of the ultrasonic system expressed by the non-conformal polynomials :

$$X = A_0 + A_1 x + A_2 y + A_3 xy + A_4 x^2 + A_5 y^2 + A_6(x^3 + xy^2) + A_7(x^2 + y^2)^2 \quad \text{.......... (7-a)}$$

$$Y = B_0 + B_1 y + B_2 x + B_3 xy + B_4 y^2 + B_5 x^2 + B_6(y^3 + yx^2) + B_7(x^2 + y^2)^2 \quad \text{.......... (7-b)}$$

Where :

- $A_0 = P_0$, $B_0 = P_4$ — are the effect of the orientation of the camera
- $B_1 = 1 + k_0$ — is the effect of symmetrical lens distortion
- $A_1 = 1 + k_0 + m_1$ — is B_1 added to the ultrasonic effect .
- $B_2 = P_3$ — is the effect of the camera's orientation
- $A_2 = m_2 - P_3$ — is B_2 substracted from the ultrasonic effect
- $A_3 = P_1$, $B_3 = P_2$ — are the effect of the camera's orientation
- $A_4 = P_2 - h_1'$, $B_4 = P_1 + h_3'$ — are the effect of asymmetric lens distortion and the camera's orientation .
- $A_5 = -h_1'$, $B_5 = h_3'$ — are the effect of asymmetric lens distortion

- $A_6 = k_1$
- $B_6 = k_1$

are the effect of symmetric lens distortion

- $A_7 = -h_2$
- $B_7 = h_4'$

are the effect of asymmetric lens distortion

STATISTICAL RESULTS :

The χ^2 test was used to verify the significance of the coefficients a_i, b_i, c_i, d_i, h_i and k of the equations (5- a&b). The results (El Ghazaly and Wishahy 1988) showed that they all are significant at a significance level = 0.05.

The Fisher " F-test " applied to the mathematical models (7- a&b) proved that the terms (A_2y, A_3xy, A_4x^2, A_5y^2, A_6x^3 and A_7y^4) of the X polynomial and the terms (B_3xy, B_4y^2, B_5x^2, B_6y^3, B_7x^4)of the Y polynomial are insignificant at a significance level = 0.05.

Thus finally we obtain :

$$X = A_0 + A_1x + A_6xy^2 + A_7x^4$$
$$Y = B_0 + B_1y + B_2x + B_6yx^2 + B_7y^4$$

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APPENDIX

The Photographs PHOTO I and PHOTO II produced by Polaroid films

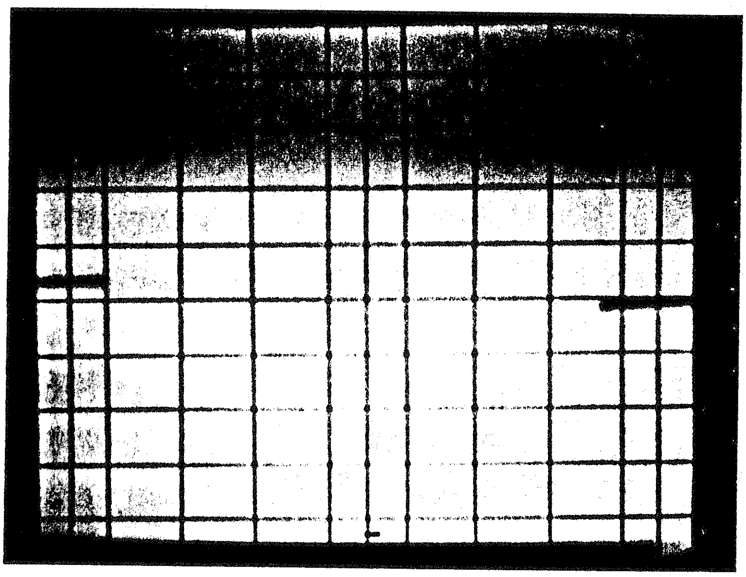

PHOTO I
The Reseau

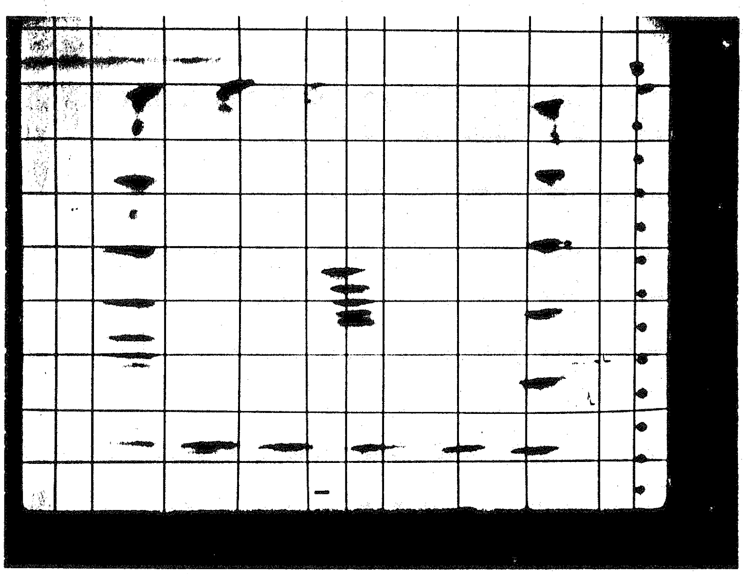

PHOTO II
The Reseau in front of the scanning
of the 100 Millimeter Test MODEL
(the distortions are magnified)