HIGH PRECISION PHOTOGRAMMETRIC MEASUREMENT FOR BRIDGE BEAM LOAD FAILURE TEST

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ABSTRACT

Close - range photogrammetric techniques were used to measure the deformation of a 16 meter partly prestressed bridge beam in a load failure test. An analysis on the stability of the camera and the accuracy required for setting up camera are discussed in this paper. The author proposed the use of the ANOVA (analysis of variance) method to evaluated the stability of the camera's interior and exterior orientation between photographs. The experiment results showed the effectiveness of this method for the improving accuracy. An accuracy of +0.079 mm was achieved for measuring the Y component of movement.

INSTRUCTION

A load failure test of 16 meter partly prestressed bridge beam was carried out at the Railway Academic, Beijing, P.R. China to test the performance of partly prestressed beam. Close-range photogrammetric method was used to measure the displacement in horizontal and vertical direction on different positions of the beam under different load condition and critical broken condition.

The method of motion parallax is a simple and accurate method for measuring object movement in a plane. Single photo is taken for the object to be measured at different states. The photo taken at first state and the photo taken at second state constitute a stereopair. Stereoscopic vision takes place at the displacement parts of the object because of the x-parallax and y-parallax caused by the movements of object points. The movements of object can be calculated according to the x-parallax and y-parallax measured by stereocomparator or single-plate comparator. The principle of motion parallax is shown in Figure (1). In Figure (1), S is the center of the lens of camera, O plane is the object plane, P plane is film plane. The object point A has a image point a on the film before its movement. When the object point A moves to point A', its corresponding image point is a'. Suppose the film plane and the object plane is exactly parallel and the camera is absolutely stable during the time between the two exposures, the magnitude of motion ΔX, ΔY can be calculated from parallax P_x, P_y according to equation (1).

$$\Delta X = \frac{D}{f} p_x = M p_x$$
$$\Delta Y = \frac{D}{f} p_y = M p_y \qquad (1)$$

In equation (1), D is object distance, f is the focal length of the camera, and M is the denominator of the photo scale. The accuracy of the computed movement, ΔX and ΔY, depends on the validity of the two basic assumptions, on the precision of parallax measurement, and on the scale of the photography. (Wong and Vonderohe, 1981).

In this experiment, because of the tremendous load that breaks the beam, it is impossible to set up enough control points in object space for the correction of the variations of the camera's interior and exterior orientation between photography. For the purposes of improving the reliability and the accuracy, the author designed the target-pairs, which consist of two targets with a fixed distance between them, used to test the stability of the camera according to the ANOVA data analysis method. This method can be used to pick out the photo taken when the camera's interior or exterior orientation changes between photography among other photos. The experiment results showed the effectiveness of this method.

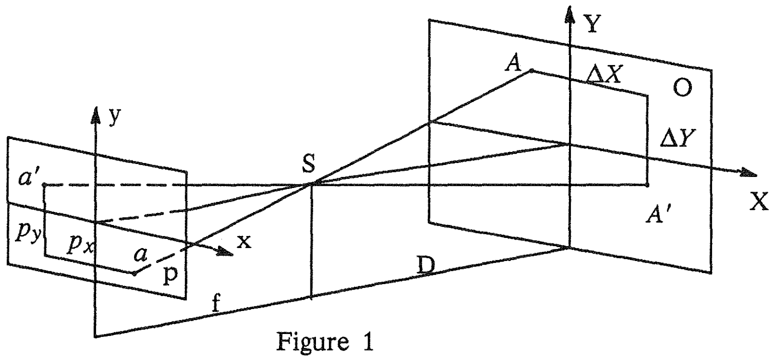

Figure 1

PHOTOGRAPHY DESIGN

(1) The choice of object distance D and and focal length f

From equations (1) , the accuracy for measuring movements by the motion parallax method can be derived as follows.

$$m_{\Delta X} = \Delta X \sqrt{\left(\frac{m_D}{D}\right)^2 + \left(\frac{m_{px}}{px}\right)^2 + \left(\frac{m_f}{f}\right)^2}$$
$$m_{\Delta Y} = \Delta Y \sqrt{\left(\frac{m_D}{D}\right)^2 + \left(\frac{m_{py}}{py}\right)^2 + \left(\frac{m_f}{f}\right)^2} \qquad (2)$$

On the right side of equation (2), the value of two terms $\frac{m_D}{D}$, $\frac{m_f}{D}$ is less than 1/10 of the value of $\frac{m_{px}}{px}$ at normal case. If only consider the error of photo measurement, equation (2) can be simplified as :

$$m_{\Delta X} = \frac{\Delta X}{P_x} m_{px} = \frac{D}{f} m_{px} = M\ m_{px}$$
$$m_{\Delta Y} = \frac{\Delta Y}{P_y} m_{py} = \frac{D}{f} m_{py} = M\ m_{py} \qquad (3)$$

According to the required accuracy for the measurement of the movement, $m_{\Delta x}$, and the accuracy of the photo measurement, m_{px} , the maximum permitted photographic distance $D_{\max}$ can be calculated from equation (3).

$$D_{\max} = f \frac{m_{\Delta x}}{m_{px}} \qquad (4)$$

In this experiment, $m_{\Delta x}$ = 0.05 mm, m_{px} = 0.002 mm, the UMK camera has four different lens, e.g. f = 65 mm, f = 100 mm, f = 200 mm and f = 300 mm. In this case, the values of $D_{\max}$ calculated from equation (4) for different lens are shown in Table (1).

Table (1)

f (mm)	65	100	200	300
$D_{\max}$ (mm)	1625	2500	5000	7500

The photogrammetric distance D is also determined by the width of the object to be measured, W, and the the photo width w as equation (5).

$$D = f \frac{W}{w} \qquad (5)$$

When W = 4000 mm, w = 166 mm, the photogrammetric distance D for four different camera angles calculated for equation (5) are shown in Table (2).

Table (2)

UMK	6.5 / 1318	6.5 / 1318	6.5 / 1318	6.5 / 1318
D (mm)	1566	2410	4819	7229

Considering the limit of experiment site, the stability of the camera and the permitted photogrammetric range, Camera UMK 20/1318, f = 200 mm, was chosen to use in this experiment. The selected photographic distance, D = 4.816 mm, is less than the maximum permitted photography distance $D_{\max}$ = 5000 mm.

(2) Targets

The beam has three photogrammetric planes, on which altogether 17 pair-targets were arranged. Targets 1, 2, 16, 17 were located at the firs plane, targets 3, 4, 5, 13, 14, 15 were located at the second plane and targets 6, 7, 8, 9, 10, 11 and 12 were located at the third plane. The distribution of targets is shown in Figure (2).

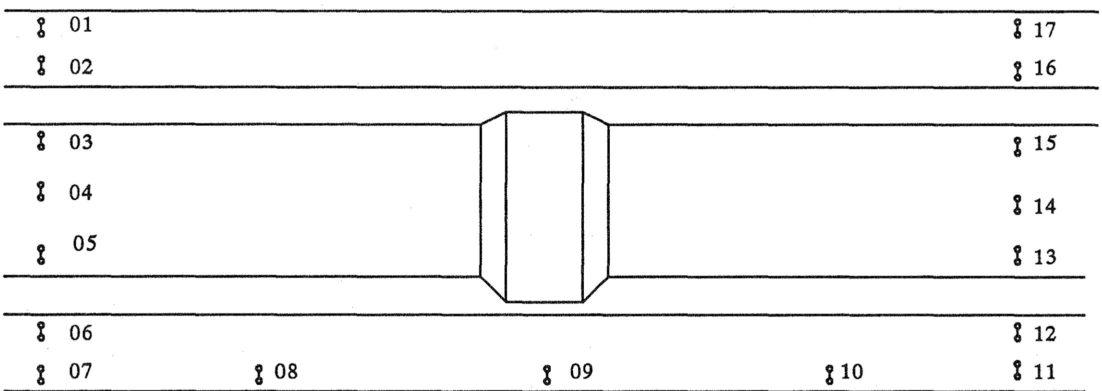

Figure (2)

The shape and size of targets are important in reduction of the pointing error. In this experiment, special targets, target-pairs, were designed for tow purposes: for the improvement of accuracy and for the evaluation of the stability of camera. The shape of target pair is shown in Figure (3). The target-pair consists of two targets which are connected with a fixed distance. The diameter of the outer circle is 24 mm. Its image is 1 mm in diameter, which can be seen clearly in the negative. The diameter of the center circle is 1.2 mm. The diameter of its image on negative is 0.05 mm, which is 5/3 the apparent diameter of the measuring mark of AC - 1 analytical plotter. Experiment showed this size of target can obtain highest accuracy for photo measurement.

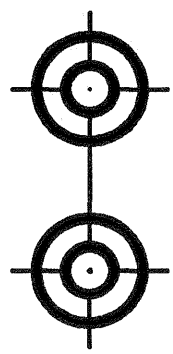

Figure (3)

The distance between the two targets of target-pair was measured by a comparator. An accuracy of 0.01 mm was obtained. The target-pairs were produced using photographic method. All the distances between the two targets of the target pairs are equal, which are used for evaluating the stability of camera according to the ANOVA method.

(3) The determination of the photo scale 1/M

The photo scale 1/M must be known for calculating the displacement of points according to equation (1). The photo scale can be determined either from the object distance D and focal length f, or from the distance L between to targets and its corresponding distance l on the photo according following equation.

$$\frac{1}{M} = \frac{l}{L} \qquad (6)$$

In this experiment, for the convenience of measuring distance, equation (6) was used to determine the photo scale. In this case, equation (2) can be expressed as

$$m_{\Delta X} = \Delta X \sqrt{\left(\frac{m_L}{L}\right)^2 + \left(\frac{m_{px}}{p_x}\right)^2 + \left(\frac{m_l}{l}\right)^2}$$
$$m_{\Delta Y} = \Delta Y \sqrt{\left(\frac{m_L}{L}\right)^2 + \left(\frac{m_{py}}{p_y}\right)^2 + \left(\frac{m_l}{l}\right)^2} \qquad (7)$$

The required accuracy for the measurement of distance L to be used for the calculation of photo scale can be estimated from equation (7). When the photographic distance D less than the maximum permitted distance $D_{\max}$, the required accuracy for measuring L can be determined from the value of $m_{\Delta Y}$, the required accuracy for the displacement measurement according to following equation.

$$m_L = L\frac{m_{\Delta Y}}{\Delta Y} \qquad (8)$$

According to theoretical calculation, the maximum vertical displacement at the middle section of the beam when it breaks, ΔY= 20 mm. In order to prevent the error of distance measurement from affecting the accuracy of the displacement measurement, $m_{\Delta Y}$ taking the value of 1/3 of the predetermined accuracy 0.05 mm, L taking the maximum distance between targets, L = 3800 mm, therefore, m_L equals 3.2 mm calculated from equation (8). Since the distance measurement on the beam does not need suspending the tape, the accuracy of 3.2 mm for m_L is easy to obtain.

The 16 meter prestressed beam has three photographic planes. Distances, 01 – 17, 02 – 16, 06 – 12 and 07 –11, were measured on first plane and second plane. For the second plane, the distance 04 – 14 can not be measured directly since the middle projecting part of the beam. Therefore the values of D_1, D_2, the distances between the second plane and the other two planes, were measured, and the scale of the third plane was calculated according to equation (9).

$$\frac{1}{M_3} = \frac{2}{M_1 + \frac{D_1}{f} + M_2 + \frac{D_2}{f}} \qquad (9)$$

where M_1 and M_2 are photo scale denominators of first and second plane respectively.

(4) Photograph measurement

Wild AC – 1 analytical plotter was used for photograph measurement. Every point was measured for three times independently. An accuracy of 2μ was achieved for the measurement of m_{px} and m_{py}.

ACCURACY ANALYSIS

(1) The accuracy for setting the camera's exterior orientation

When using the method of motion parallax to calculate the displacement according to equation (1), the camera photo plane and the object measuring plane must be kept parallel. For photography, at first, one should consider the permitted errors for setting the camera exterior orientation, ϕ , ω and κ . This errors belong to systematic error.

The first order expression for the effects of exterior orientation to the photo coordinates can be derived from the projective transformation equations as follows:

$$\begin{aligned} dx &= \left(f + \frac{x^2}{f} \right) d\phi + \; + \frac{xy}{f} d\omega - y d\kappa \\ dy &= \frac{xy}{f} d\phi + \left(f + \frac{y^2}{f} \right) d\omega + x d\kappa \end{aligned} \tag{10}$$

Equation (11) expresses the movements of photo point caused by the effect that the photo plane is not parallel to object plane. For the convenience of discussion, p_x , p_y are used to express the differentials of x and y, and $\delta(dx)$, $\delta(dy)$ are used to express the errors of photo point movements. From equation (10) we can get:

$$\begin{aligned} \delta(dx) &= \frac{2x\ p_x}{f} d\phi + \; + \frac{y\ p_x}{f} d\omega \\ \delta(dy) &= \frac{x\ p_y}{f} d\phi + \frac{2y\ p_y}{f} d\omega \end{aligned} \tag{11}$$

From equation (10) one can know that the photo point movements caused by κ is:

$$\begin{aligned} dx &= -y\ d\kappa \\ dy &= x\ d\kappa \end{aligned} \tag{12}$$

The error of photo point movements caused by the error of setting the angle ω can be derived from equation (12).

$$\begin{aligned} \delta(dx) &= p_y\ d\kappa \\ \delta(dy) &= p_x\ d\kappa \end{aligned} \tag{13}$$

According to equation (10) and (12), the required accuracy for setting the camera's exterior orientation ϕ , ω , κ can be determined according to $\delta(dx)$, $\delta(dy)$, the error of photo point displacement.

For the consideration of the displacement in X direction:

$$\begin{aligned} \Delta\phi\ limit &= \frac{f\ \delta(dx)}{2x\ p_x} \varrho \\ \Delta\omega\ limit &= \frac{f\ \delta(dx)}{y\ p_x} \varrho \\ \Delta\kappa\ limit &= \frac{\delta(dx)}{p_y} \varrho \end{aligned} \tag{14}$$

For the consideration of the displacement in Y direction:

$$\Delta\phi \ limit = \frac{f \ \delta(dx)}{x \ p_y} \varrho$$

$$\Delta\omega \ limit = \frac{f \ \delta(dy)}{2y \ p_y} \varrho \qquad (15)$$

$$\Delta\kappa \ limit = \frac{\delta(dy)}{p_x} \varrho$$

From equation(14) and (15) we can see that the accuracy for setting the the exterior orientation is inversely proportional to the value of p_x and p_y , that is, the greater the displacement, the higher the required accuracy for setting the camera. The accuracy for setting the exterior orientation in X direction is contrary to Y direction .

In this experiment, considering the maximum values of p_x, p_y, x and y, e.g., f = 200 mm, p_x = 0.2 mm, p_y = 1mm, x = 80 mm, y = 40 mm, the required accuracy for setting the angle ϕ , ω , κ calculated from equations (14) and (15) are shown in Table (3).

Table (3)

	$\Delta\phi_{limit}$	$\Delta\omega_{limit}$	$\Delta\kappa_{limit}$
δ (dx) $<$ $0.001mm$	21′ 29″	1° 25′	3′ 36″
δ (dy) $<$ $0.001mm$	8′ 35″	8′ 35″	17′ 11″

In Table (3), the minimum values of ϕ , ω , κ in both directions X an Y are $\Delta\phi_{limit}$ = 8′ 35″ , $\Delta\omega_{limit}$ = 8′ 35″ and $\Delta\kappa_{limit}$ = 3′ 36″ . For UMK camera, the minimum graduation for setting angles ϕ , ω , κ are 1′, 20″ and 30″ respectively. Therefore, it is quit easy to achieve the required accuracy for setting UMK camera in this experiment. For non-metric camera, simple geometric method can be used to set the the cameras exterior orientation.

(2) The stability of the camera's interior and exterior orientation

The stability of camera's exterior and interior orientation is very important for displacement measurement when using the method of motion parallax. Here the stability refers to the stability of camera during the whole procedure of the experiment and the stability of camera during the time between multi-exposures for a particular state of the object to be measured. In this case, the change of camera's exterior and interior orientation occurs at random. It is the fortuitous effects of systematic errors so such changes should be treated as random errors.

equations (10), (12) can be directly written into mean square error form. For the X direction:

$$m_{\Delta\phi} \ limit = \frac{m_{p_x}}{f + \frac{x^2}{f}} \varrho$$

$$m_{\Delta\omega} \ limit = \frac{f \ m_{p_x}}{xy} \varrho \qquad (16)$$

$$m_{\Delta\kappa} \ limit = \frac{m_{p_x}}{y} \varrho$$

For the Y direction:

$$m_{\Delta\phi\ limit} = \frac{f\ m_{py}}{xy}\ \varrho$$

$$m_{\Delta\omega\ limit} = \frac{m_{py}}{f + \frac{y^2}{f}}\ \varrho \qquad (17)$$

$$m_{\Delta\kappa\ limit} = \frac{m_{py}}{x}\ \varrho$$

From equations (16) and (17), we can see that the required stability for angle ϕ and ω is contrary in X and Y direction, and the stability for angle κ relates to the range of x and y. Taking m_{px}, m_{py} = 0.002 mm, and the same values of f, x, y as in Table (3), the values calculated from equations (16) and (17) are shown in Table (4).

Table (4)

	$m_{\Delta\phi\ limit}$	$m_{\Delta\omega\ limit}$	$m_{\Delta\kappa\ limit}$
$m_{px} < 0.002\ mm$	2″	26″	10″
$m_{py} < 0.002\ mm$	25″	2″	5″

In Table(4), taking the minimum values in both X and Y direction, $m_{\Delta\phi\ limit} = 2''$, $m_{\Delta\omega\ limit} = 2''$ and $m_{\Delta\kappa\ limit} = 5''$ as the required limits for the stability of camera's exterior orientation in this experiment. Obviously it is impossible to get such accuracy for UMK camera for changing glass plates between exposures. Therefore only adding exposures can not increase accuracy if no stable control points in object space for the correction of camera's variability.

The stability of camera can be obviously increased when aerial roll film is used for continuous exposure because it is not necessary to handle the camera. In this case, the variation of focal length caused by film unflatness is the main effect to the stability of camera.

The variation of focal length causes the proportional changes in photo coordinates which have the following relations.

$$df = \frac{f}{x}\ dx$$

$$df = \frac{f}{y}\ dy \qquad (18)$$

Substituting dx, dy with motion parallax p_x, p_y, the mean square error form can be derived from equation (18).

$$m_{f\ limit} = \frac{f}{x} m\ p_x$$

$$m_{f\ limit} = \frac{f}{f} m\ p_y \qquad (19)$$

For X direction, when x = 80 mm, m_{px} = 0.002 mm, the maximum limit for the variation of f is $m_{f\ limit}$ = 0.005 mm. For Y direction, when y = 40 mm, m_{py} = 0.002 mm, $m_{f\ limit}$ = 0.01 mm. Obviously it is very difficult for either glass plate or film to maintain such high accuracy for focal length.

Synthesizing the above analysis on the stability of camera's exterior and interior orientation, we can see that the stability of camera is an important factor to be considered for high accuracy photogrammetric measurement. At the experiment site, there are many factors which is difficult to estimate for affecting the stability of camera. Since the limit of experiment site, usually it is difficult to lay out stable control points in object space for the effective correction for the variation of camera exterior and interior orientation. Therefore an effective data analysis method for evaluating the stability of camera, to effective distinguish the effects of systematic errors from multi-exposures, is very useful for practical application.

ANOVA DATA ANALYSIS METHOD FOR THE CAMERA'S STABILITY

ANOVA (analysis of variance) method was used to evaluate the stability of camera's exterior and interior orientation in this experiment. This method can be used to pick out the photo with systematic errors among multi-exposures taken at the same state of the object. This photo may result from the variation of camera's exterior and interior orientation between continuous exposures or from the photo measurement with orientation blunders.

The principle of ANOVA method is the statistical inferential method based on analysis of variance. Given multi-photos were taken at one state for the object to be measured. For the photo j, n_j target-pairs were measured and n_j distances between target-pairs were obtained. The population distribution for photo j is N(μ_j , σ^2), j = 1, 2, ... r, μ_j and σ^2 are unknowns. Since all the photos were measured with the same instrument, the population variance of the r photos was equal. Under the condition of sample independent, the distance d_{ij} between the target-pair calculated from the photo coordinates of targets satisfies the following mathematical model:

$$d_{ij} = \mu_j + \epsilon_{ij} \qquad \epsilon_{ij} \sim N(0, \sigma^2) \qquad i = 1, 2, \cdots n_j; \quad j = 1, 2, \cdots, r \tag{20}$$

where d_{ij} expresses the distance between the target-pair i of the photo j and μ_j , σ^2 are constants. For the multi-exposures, the variation of μ_j reflects the effect of variation of the camera's exterior and interior orientation and ϵ_{ij} reflects the random errors.

The value of overall mean for all the sample data

$$\mu = \frac{1}{n} \sum_{j=1}^{r} n_j \mu_j \tag{21}$$

where, $n = n_1 + n_2 + \cdots + n_r$.

writing

$$\delta_j = \mu_j - \mu \qquad j = 1, 2, \ldots, r \tag{22}$$

where δ_j is the effect of each photo, which expresses the influence of systematic error caused by the variation of camera's exterior and interior orientation. The model (20) can be written as following form.

$$d_{ij} = \mu + \delta_j + \epsilon_{ij} \qquad \epsilon_{ij} \sim N(0, \sigma^2) \qquad i = 1, 2, \cdots, n_j \quad j = 1, 2, \cdots, r \tag{23}$$

If the camera is stable between multi-exposures, the sample treatment mean of the distances between target-pairs of each photo, $\mu_1, \mu_2, \ldots, \mu_r$, is equal with each other. Therefore the value of each δ_j calculated from equation (22) should be equal to zero; conversely, if the value of δ_j is not equal to zero, we can say that the camera's exterior or interior orientation changed between multi-exposures. Thus the stability of camera can be expressed by the photo effect, the value of δ_j .Therefore, hypothesis testing for the means of the r photo's distributions, $N(\mu + \delta_1, \sigma^2)$, $N(\mu + \delta_2, \sigma^2)$, ... , $N(\mu + \delta_r, \sigma^2)$, is used for the inference of camera's stability.

$$H_0 : \quad \delta_1 = \delta_2 =, \ldots, = \delta_r = 0 \tag{24}$$

The corresponding alternative hypothesis is that the values of $\delta_1, \delta_2, \cdots, \delta_r$ are not equal, or that at least one pair of the δ 's differs.

F distribution can be used to test the hypothesis (24).

If

$$\frac{(n-r)\ S_2}{(r-1)\ S_1} > F_\alpha\ (r-1,\ n-r) \qquad (25)$$

then H_0 can be rejected at the level α , which shows the the camera's exterior and interior orientation changed; otherwise H_0 can be accepted, which shows the camera is stable .

EXPERIMENT RESULTS

Because of the limits of the experiment site and experiment condition, It was very difficult to lay out enough stable control points in object space for the correction of variation of camera's exterior and interior orientation. Only a two meter invar bar was set under the beam to check the stability of the camera.

Film continuous exposures were used to increase the camera's stability. Three exposures were taken for every state of the beam and three measurements were did for each photo. ANOVA method was used to make analysis on the camera's stability according to the data obtained from three successive photos.

The results of ANOVA analysis on two group photos for load level 24 and load level 26 are shown in Table (5) and Table (6).

ANOVA TABLE Table (5)

Source	Sum of Squares	Degrees of Freedom	Mean Square	F
S_1	0.03016	2	0.01508	2.57338
S_2	0.28126	48	0.00586	
Total	0.31142	50	0.00623	

ANOVA TABLE Table (6)

Source	Sum of Squares	Degrees of Freedom	Mean Square	F
S_1	0.05156	2	0.02578	4.39932
S_2	0.28111	48	0.00586	
Total	0.33267	50	0.00665	

For the three photos taken at level 24, $F_{0.05}(2.58) = 3.23$, which shows no significant difference at the level α = 0.05, therefore the camera is stable. For the three photos taken at 26 level, the difference is significant. The main reason is supposed to be the effect of photo unflatness.

The two targets at the ends of the invar bar can be used to check the accuracy of the photogrammetric measurements. The movements of fixed targets should be zero. If the movement values of fixed targets do not equal zero, the movement values reflect the photogrammetric errors, which include the influence of the camera's stability. The movement values of the two invar bar targets calculated from 3 photos taken at 24 level and 26 level are shown in Table (7).

Table (7)

	ΔX (mm)	ΔY (mm)	Number of point	Number of photos
24 level	0.060	0.133	2	3
26 level	0.274	0.048	2	3

The value of ΔX at 26 level obviously increases, which shows that the camera's exterior and interior orientation changes. This result is consistent with the ANOVA analysis. Picking out one photo and then recalculate the movements, $\Delta X = 0.082$ mm and $\Delta Y = 0.040$ mm, the accuracy obviously increased. This shows the effect of ANOVA method.

If no significant difference between photos after ANOVA analysis on the three successive photos, the m_d, the mean square error of photogrammetric measurement of the distance between target-pair, can be expressed as follows:

$$m_d = \sqrt{\frac{S_T}{(n-1)}} = 0.078 \ mm$$

Target-pairs were set up along vertical direction, so m_d can be used to express the accuracy of photogrammetric measurement for displacement in Y direction.

Using the two targets fixed on the invar bar as a check, we get

$$m_{\Delta x, \Delta Y} = \sqrt{\frac{[\Delta\Delta]}{n}} = 0.086 \ mm$$

Electronic displacement sensors were set up at the bottom of the beam. The difference between photogrammetric measurement and sensor measurement compared at the middle section of the beam is 0.153 mm. Consistent results were achieved for two different methods in this experiment.

SUMMARY

(1) The method of motion parallax is a simple and accurate method for the measurement of displacement in a plane and the stability of camera is an important factor for accuracy. The accuracy required for setting up camera and an analysis on the camera's stability are discussed in this paper.

(2) Under the condition that is impossible to lay out enough control points for the correction of the variation of camera's exterior and interior orientation, the accuracy can not be increased if only increase the number of photos. The analysis on the camera's stability is required for high accuracy photogrammetric measurement.

(3) ANOVA data analysis method is effective for the analysis on camera's stability.

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