The objective of this research was to develop a model for generating a shade image of forested areas from multispectral LANDSAT observations by implementing a linear mixing model where shadow is considered as one of the primary components in a LANDSAT pixel. The shade images are hypothesized to be related to the observed variation in forest structure, i.e. forest type, age or tree crown cover. A constrained least squares method was used to generate shade images for eucalyptus, pine and cerrado using LANDSAT MSS and TM imagery over two study areas in Brazil, Mogi-Guaçu and Itapeva. The derived shade images were found to be indicative of species type differences in pine, of crown cover in cerrado and of age differences in eucalyptus.

1. INTRODUCTION

LANDSAT digital images in computer-compatible tapes (CCTs) are represented by dimensionless digital numbers ranging from 0 to 127 for bands 4, 5 and 6, from 0 to 63 for band 7 of MSS on current LANDSATs and from 0 to 255 for all TM bands (Markham and Barker, 1986). The value associated with each picture element, called a pixel, represents the average radiance from a small target on the ground in a corresponding spectral band. The ground spatial resolution of LANDSAT MSS and TM is approximately 0.45 hectares (57m x 79m) and 0.10 hectares (30m x 30m), respectively, at the Earth's surface. Hence, the radiation detected by a satellite is caused by a mixture of many different materials (e.g., soil, vegetation, rocks, water and others) plus the atmospheric contribution. Separating the individual components in a pixel becomes then an important problem in scene classification, and is known as the mixture problem (Horwitz et al., 1971; Detchmendy and Pace, 1972).

Often an important component in a mixed target's response is the shadow component. It is well known that shadowing is generally present in all natural scenes and represents a special case of the mixture problem. Several studies (Heimes,
1977; Ranson and Daughtry, 1987) have shown that shadowing has an important effect on scene spectral response, specially in forested areas. The proportion of shadow in a pixel provides additional data that can be used to improve the accuracy of the land-class acreage estimates. It can also serve to generate an additional data channel (texture channel) for machine-assisted image classification (Shimabukuro, 1987).

The objective of this paper is to develop a model for generating a "shade image" of forested areas from multispectral LANDSAT observations by implementing a linear mixing model where shadow is considered as one of the primary components in a pixel.

The model is then used to generate shade images for eucalyptus, pine and "cerrado", using LANDSAT MSS and TM imagery over two study areas in Brazil (Mogi-Guaçu and Itapeva). The shade image is then compared with field information from these two study areas.

2. LINEAR MIXING MODEL THEORY: A CONSTRAINED LEAST SQUARES METHOD

The linear mixture model can be written as follows:

\[ r_i = \sum_{j=1}^{n} (a_{ij} x_j) + e_i , \]  

where

\[ r_i = \text{mean spectral reflectance for the ith spectral band of a pixel containing one or more components;} \]
\[ a_{ij} = \text{spectral reflectance of jth component in the pixel for the ith spectral band;} \]
\[ x_j = \text{proportion value of jth component in the pixel;} \]
\[ e_i = \text{error term for the ith spectral band;} \]
\[ j = 1, 2, \ldots , n \ (n = \text{number of components assumed for the problem}); \]
\[ i = 1, 2, \ldots , m \ (m = \text{number of spectral bands for the sensor system}). \]

This model assumes that the spectral response of a pixel is a linear combination of the spectral response of each component within the pixel, including shadow.

For this problem, \( r_i \) and the reflectances \( a_{ij} \) are assumed to be known; the proportional value for each component (\( x_j \)) are the problem unknowns. System (1) is overdetermined, i.e., the number of equations is always (in practical cases) larger than the number of unknowns. The constrained least squares approach solves this system by minimizing the sum of the squares of the errors subjected to two conditions:
- for any pixel the proportion values \( x_i \) must be nonnegative;
- for any pixel the sum of the proportions should add to one.

Both conditions have an obvious physical meaning.

A quasi-closed solution method is proposed in this paper. This method is discussed for the three components case (\( n=3 \)), although the procedure can be generalized for any higher number of components in the mixture.

The function to be minimized is:

\[
\sum_{i=1}^{m} e_i^2 = e_1^2 + e_2^2 + \ldots + e_m^2 = \sum_{i=1}^{m} \left( r_i - \sum_{j=1}^{n} a_{ij} x_j \right)^2. \tag{2}
\]

For the case of three pixel components (\( n=3 \)) this function takes the form:

\[
V_1 x_1^2 + V_2 x_2^2 + V_3 x_3^2 + V_4 x_1 x_2 + V_5 x_1 x_3 + V_6 x_2 x_3 + V_7 x_1 + V_8 x_2 + V_9 x_3 + V_{10},
\]

where the coefficients \( V \)'s are functions of the mean spectral reflectance \( r_i \) for the pixel in band "i" and of the spectral reflectance for each of the "j" the pixel components \( (a_{ij}) \).

Function (2) is to be minimized subject to the constraints:

\[
\begin{align*}
x_1 + x_2 + \ldots + x_n &= 1 \\
0 &\leq x_i \leq 1 \\
&\vdots \\
0 &\leq x_n \leq 1.
\end{align*}
\tag{3}
\]

For three components (\( n=3 \)) the first constraint becomes:

\[
x_1 + x_2 + x_3 = 1 \quad \text{or} \quad x_3 = 1 - (x_1 + x_2)
\]

and the function to be minimized becomes:

\[
F = e_1^2 + \ldots + e_6^2 = U_1 x_1^2 + U_2 x_2^2 + U_3 x_1 x_2 + U_4 x_1 x_3 + U_5 x_2 x_3 + U_6
\tag{4}
\]

Coefficients \( U \) through \( U_6 \) are functions of the pixel's reflectance \( (r_i) \) and of the components reflectances \( (a_{ij}) \) and can easily be derived. The remaining constraints \( (x_1, x_2, x_3) \) within the interval \([0,1]\) can be satisfied if we impose the condition that the solution for \( x_1 \) and \( x_2 \) must lie inside the triangular shaped area as depicted in Figure 1.
The unconstrained minimum for function (4) can be found by solving the linear system:

\[ \frac{\partial F}{\partial x_1} = 2U_1 x_1 + U_3 x_1 + U_4 = 0, \]
\[ \frac{\partial F}{\partial x_2} = 2U_2 x_2 + U_3 x_2 + U_5 = 0. \]  

The solution can easily be calculated:

\[ x_1 = \frac{(U_3 U_5 - 2 U_4 U_4)/(4 U_1 U_2 - U_2^2)}, \]
\[ x_2 = \frac{(U_3 U_4 - 2 U_4 U_5)/(4 U_1 U_2 - U_2^2)}. \]

Combining \( x_1 \) and \( x_2 \) with the remaining constraints we come to five possible outcomes, depending upon the position of \( x_1 \) and \( x_2 \) in Figure 1 (see Table 1):

**TABLE 1**

POSSIBLE OUTCOMES FOR \( n=3 \) COMPONENTS IN A PIXEL

<table>
<thead>
<tr>
<th>Outcomes</th>
<th>Within Values to be the region recalculated</th>
<th>( x_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 POS POS YES -</td>
<td>1( -x_1 - x_2 )</td>
<td></td>
</tr>
<tr>
<td>2 POS POS NO ( x_1 ) and ( x_2 )</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>3 NEG POS NO ( x_2 ) (( x_1=0 ))</td>
<td>1( -x_2 )</td>
<td></td>
</tr>
<tr>
<td>4 NEG NEG NO (( x_1=x_2=0 ))</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>5 POS NEG NO ( x_1 ) (( x_2=0 ))</td>
<td>1( -x_1 )</td>
<td></td>
</tr>
</tbody>
</table>
Outcome 1: All constraints are satisfied; no further action is required.

Outcome 2: Minimum of function $F$ lies outside the region and $x_1$ and $x_2$ are positive. In this case, the constrained minimum lies on the line $x_1 + x_2 = 1$, resulting $x_3 = 0$. In this case $x_2$ can be expressed in terms of $x_1$. Equations (5) reduce then to a single one:

$$\frac{\partial F}{\partial x_1} = 2(U_1 - U_2 - U_3) x_1 + (U_3 + U_4 - U_5 - 2U_2) = 0,$$
resulting:

$$x_1 = -\frac{(U_3 + U_4 - U_5 - 2U_2)}{2(U_1 + U_2 - U_3)},$$

if $x_1 > 1$ then: $x_1 = 1$ and $x_2 = x_3 = 0$,

if $x_1 < 0$ then: $x_1 = 0$ and $x_2 = 1$, $x_3 = 0$.

Outcome 3: Minimum lies outside the region with $x_1 < 0$ and $x_2 > 0$. In order to satisfy the constraints, $x_1$ has to be made equal to 0 and Equations (5) become:

$$\frac{\partial F}{\partial x_2} = 2U_2 x_2 + U_5 = 0,$$
resulting:

$$x_2 = -\frac{U_5}{2U_2},$$

if $x_2 > 1$ then: $x_2 = 1$ and $x_1 = x_3 = 0$,

if $x_2 < 0$ then: $x_1 = 0$ and $x_3 = 1$.

Outcome 4: Minimum lies outside the region with $x_1 < 0$ and $x_2 < 0$. In this case $x_1 = x_2 = 0$ and $x_3 = 1$.

Outcome 5: Minimum lies outside the region with $x_1 > 0$ and $x_2 < 0$. In this case, $x_2 = 0$ and Equations (5) become:

$$\frac{\partial F}{\partial x_1} = 2U_1 x_1 + U_4 = 0,$$
resulting:

$$x_1 = -\frac{U_4}{2U_1},$$

if $x_1 > 1$, then: $x_1 = 1$ and $x_2 = x_3 = 0$,

if $x_1 < 0$, then: $x_1 = 0$ and $x_2 = 0$, $x_3 = 1$.

3. EXPERIMENTS

3.1 - STUDY AREAS

The model developed in Section 2 was applied to two study areas in Brazil, to generate shade images. LANDSAT MSS and TM subscenes are used. The first study area, Mogi-Guaçu, is located at $22^\circ 15'S$ and $47^\circ 10'W$, in the State of São Paulo. It is representative of pine and eucalyptus plantations common to that region. The major Pinus species are *Pinus elliottii* and
Pinus taeda. Other species such as Pinus caribaea, Pinus bahamensis, Pinus oocarpa and Pinus palustris are also planted in small amounts. The major Eucalyptus species are Eucalyptus alba and Eucalyptus saligna. In the LANDSAT MSS image (September 13, 1975) the eucalyptus trees ranged in age from eight months to approximately twenty years (Shimabukuro et al., 1980). The study area "Itapeva" is located at 20° 30'S and 53° 20'W in the State of Mato Grosso do Sul. This site includes eucalyptus plantation and a large area covered by the native vegetation called "cerrado". Brazilian "cerrado" is the general name of xeromorphic woodland, scrub, savanna and grassfield vegetation of central Brazil (Eiten, 1978).

LANDSAT digital numbers were converted to reflectance data to allow for a more correct analysis from a physical viewpoint. The conversion from digital numbers to physical values (radiance and/or reflectance) is treated in detail in Robinove (1982) and Markham and Barker (1986).

Study area Mogi-Guaçu

Based on a work done by Shimabukuro et al. (1980), two experiments were performed, one with respect to forest composed of eucalyptus and the other in a pine forest. The primary components of the mixture in the pixels are eucalyptus, soil and shadow for the first case and pine, soil and shadow for the second. The reflectance of eucalyptus and pine were extracted from previous image analysis work done by Shimabukuro et al. (1980). The reflectance values for soil were extracted from the LANDSAT image itself. It comes out that the soil reflectance for the eucalyptus forest is not identical to the one for the pine forest. This may not be caused by a different nature of the soil itself, but rather by the distinct type of understory present in these two forest types. The shadow reflectance was obtained by Shimabukuro (1987) using field plot data available from Heimes (1977). The shade images generated by the mixture model for both types of forest were compared with the ground data available in Shimabukuro et al. (1980).

Study area Itapeva

Three LANDSAT images were used to test the proposed model: MSS Image of July 27, 1978, MSS and TM images of July 18, 1984. Using the 1978 MSS image and based on the work done by Hernandez Filho et al. (1978), experiments were performed on a forested area which consists of eucalyptus and over an area covered by "cerrado". The three primary components for the forest of eucalyptus were assumed to be eucalyptus, soil and shadow, whereas for the "cerrado" vegetation, "cerrado", soil and shadow were taken as the mixture components. The eucalyptus and "cerrado" reflectances as well as the soil reflectance for both areas were extracted from the image itself ("pure pixels" were assumed) based on the work done by Hernandez Filho et al. (1978). Similarly to the Mogi-Guaçu study area, shadow reflectance was obtained by Shimabukuro (1987).
The 1984 LANDSAT image available from both MSS and TM sensors, which provide different ground resolutions. Aerial photographs (June, 1984) covering this area are also available. The availability of these data made possible a second experiment over the same area. Eucalyptus, soil and shadow were assumed as the primary components within the pixels. Reflectance values for eucalyptus and soil were extracted from the satellite images, based on the aerial photographs and available reforestation maps. The shadow reflectance was obtained by Shimabukuro (1987). Reflectance values obtained in this fashion are available in Shimabukuro (1987).

These data were used as input to the Constrained Least Squares Model and shade images were generated from the available satellite imagery (MSS and TM).

3.2 - RESULTS

Figure 2 depicts the Mogi-Guacu subarea selected for eucalyptus forest. Figure 3 shows the corresponding shade image for this subarea. Comparing this image and the ground truth information available for the forest of eucalyptus, it becomes apparent that the shade image shows clearly the difference in concentration of shadow for the two age groups of eucalyptus plantations. The shade image shows that the young eucalyptus presents smaller amount of shadow than the old eucalyptus. This is in good agreement with the ground truth: young eucalyptus (age ranging from eight months to two years) presents a uniform canopy cover and heights, providing little amount of shadow. On the other hand, as the eucalyptus forest becomes older, the canopy cover becomes less uniform, producing a larger amount of shadow inside the forest. Hence, the shade image may explain the forest structure. Similar shade image was generated for the pine forest. Comparing this image and the information originated from ground truth, it can be shown that variations in shadow concentrations can be associated with different species of pine in the study sites.

Tests were also performed using the two images (MSS and TM) covering the "Itapeva" study area. Two different shade images were generated: one for the forest of eucalyptus and another for the "cerrado". Comparing the resulted shade images and the ground information available in Hernandez Filho et al. (1978), the shade image shows the difference in concentration of shadow in the two age groups of eucalyptus plantations. The results confirm the conclusions drawn from the Mogi-Guacu study site: young eucalyptus present a smaller amount of shadow than the old eucalyptus. Shade image developed for the "cerrado" vegetation presents variations in shadow concentration, caused by canopy cover structure. This canopy structure may be associated with the difference in tree heights and/or species composition.
Fig. 2 - Location of the study sites for eucalyptus in the study area "Mogi-Guaçu" (LANDSAT MSS-5).

Fig. 3 - Shade image for forest of eucalyptus in the study area "Mogi-Guaçu".
The last experiment made use of the July, 1984 MSS and TM images over an area also covered by aerial photographs which were used to extract ground information. The corresponding shade images for this area were also generated using CLS method. Comparison of these shade images to the aerial photographs makes apparent the good qualitative agreement between the information contained in the aerial photo and the one coming from the TM and MSS shade images.

4. CONCLUSIONS

In this paper, a linear mixture model which implements a constrained least squares approach considering shadow as one of the primary components has been developed. The input data required by the model are the pixel's reflectance and the individual component reflectances (vegetation, soil and shadow) in each of the spectral bands (or channels) available. The component reflectances for vegetation and soil were obtained from the image itself by assuming the existence of "pure pixels" for both components. "Pure pixels" were located with the help of ground information or aerial photograph. Shadow reflectance was obtained by Shimabukuro (1987) using the composite reflectance and component proportion data available from Heimes (1977).

LANDSAT TM and MSS imagery covering forested areas of distinct nature were used to test the CLS method. Results were in good agreement with ground or large scale aerial photographs derived data. Shade images proved to be indicative of species types differences in pine, of crown cover in "cerrado" vegetation and of age differences in eucalyptus.

5. REFERENCES


MARKHAM, B.L. and BARKER, J.L. 1986. LANDSAT MSS and TM Post-Calibration Dynamic Ranges Exoatmospheric Reflectances and At-Satellite Temperatures, LANDSAT Technical Notes, EOSAT, LANDSAT User Notes, Lanham, Maryland.


